HOMEWORK 6: STRUCTURAL DYNAMIC DISCRETE CHOICE

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ECONOMICS 628: TOPICS IN ECONOMETRICS

Consider a dynamic model of import decision:

$$V(\omega_{t-1}, d_{t-1}) = \max_{d_t \in \{0,1\}} W(\omega_{t-1}, d_{t-1}, d_t) + \xi_t(d_t) \tag{1}$$

$$W(\omega_{t-1}, d_{t-1}, d_t) = \mathrm{E}\left[\tilde{\pi}_\theta(\omega_t, d_t) - FC_\theta(d_t, d_{t-1}) + \beta V(\omega_t, d_t) | \omega_{t-1}, d_{t-1}\right]. \tag{2}$$

where ω_t is productivity, $(\xi_t(0), \xi_t(1))$ are import cost shocks, d_t is a binary import decision, $\pi_t(\cdot)$ is the profit after maximizing out the variable inputs,

$$FC_\theta(d_t, d_{t-1}) = (\delta_1 + \delta_2(1 - d_{t-1}))d_t$$

is the fixed cost of importing materials where δ_1 is per-period fixed cost while δ_2 is one-time sunk cost of importing. The stochastic process of ω_t and $(\xi_t(0), \xi_t(1))$ are given by

$$\begin{aligned} \omega_t &= \rho\omega_{t-1} + \epsilon_t \\ \epsilon_t &\sim_{iid} N(0, \sigma^2), \\ (\xi(0), \xi(1)) &\sim_{iid} \text{Type 1 Extreme value distribution.} \end{aligned}$$

We specify

$$\tilde{\pi}_\theta(\omega_t, d_t) = \exp(\tilde{\alpha}_0 + \tilde{\alpha}_1\omega_t + \alpha_2 d_t)$$

and

$$\begin{aligned} \mathrm{E}\left[\tilde{\pi}_\theta(\omega_t, d_t) | \omega_{t-1}, d_{t-1}\right] &= \mathrm{E}\left[\exp(\tilde{\alpha}_0 + \tilde{\alpha}_1\omega_t + \alpha_2 d_t) | \omega_{t-1}, d_{t-1}\right] \\ &= \exp(\alpha_0 + \alpha_1\omega_{t-1} + \alpha_2 d_t) \equiv \pi_\theta(\omega_{t-1}, d_t) \end{aligned}$$

with $\alpha_0 = \tilde{\alpha}_0 + 0.5\sigma^2$ and $\alpha_1 = \tilde{\alpha}_1\rho$, where the last equality follows from $\omega_t = \rho\omega_{t-1} + \epsilon_t$ and $\mathrm{E}[\exp(\epsilon_t)] = \exp(0.5\sigma^2)$.

Using the property of extreme value distributed variable, (1) and (2) can be written as

$$V(\omega_{t-1}, d_{t-1}) = \text{Euler's constant} + \ln\left(\sum_{d_t \in \{0,1\}} \exp\left(W(\omega_{t-1}, d_{t-1}, d_t)\right)\right) \tag{3}$$

$$W(\omega_{t-1}, d_{t-1}, d_t) = \pi_\theta(\omega_{t-1}, d_t) - FC_\theta(d_t, d_{t-1}) + \beta \int V(\rho\omega_{t-1} + \epsilon', d_t)(1/\sigma)\phi(\epsilon'/\sigma)d\epsilon'. \tag{4}$$

Once the fixed point of (3) and (4) is solved, then we can compute the conditional choice probabilities by the logit formula as:

$$\mathrm{P}_\theta(d_t = 1 | \omega_{t-1}, d_{t-1}) = \frac{\exp\left(\pi_\theta(\omega_{t-1}, 1) - FC_\theta(1, d_{t-1}) + \beta \mathrm{E}_{\epsilon'}\left[V(\rho\omega_{t-1} + \epsilon', 1)\right]\right)}{\sum_{d' \in \{0,1\}} \exp\left(\pi_\theta(\omega_{t-1}, d') - FC_\theta(d', d_{t-1}) + \beta \mathrm{E}\left[V(\rho\omega_{t-1} + \epsilon', d')\right]\right)} \tag{5}$$

Read "HW6_data" into Matlab, which contains $N \times (T+1)$ panel data for firm's discrete import decisions ("d_m") and productivity shock ("omega"), where there are $N = 1976$ firms across

$T+1 = 7$ years. Estimating the AR(1) process for ω_{it} , we have $\hat{\rho} = 0.5955$ and $\hat{\sigma}^2 = 0.0657$. In this exercise, you are asked to estimate the parameters

$$\theta = (\alpha_0, \alpha_1, \alpha_2, \delta_1, \delta_2)$$

while fixing $\rho = 0.5955$ and $\sigma^2 = 0.0657$ by maximizing the log likelihood function:

$$\mathcal{L}(\theta|\{d_{it}, \omega_{it}) = \sum_{i=1}^{N}\sum_{t=1}^{T_i} \ln\left(\mathrm{P}_\theta(d_{it}|\omega_{i,t-1}, d_{i,t-1})\right), \tag{6}$$

where

$$\mathrm{P}_\theta(d_{it}|\omega_{i,t-1}, d_{i,t-1})$$

is given by (5).

Part 1. Solve the fixed point of (4) by approximating the state space of ω by a set of $N_\omega = 61$ discrete points: $\mathcal{W} = \{-1.5, -1.45, -1.4, ..., -0.05, 0, 0.05, ..., 1.4, 1.45, 1.5\}$. Let $\omega^1 = -1.5, \omega^2 = -1.45, ..., \omega^{61} = 1.45$, and $\omega^{N_\omega} = 1.5$. Let

$$q_j = (\omega_j + \omega_{j+1})/2$$

for $j = 1, ..., N_{\omega-1}$ so that q_j is the middle point between ω_j and ω_{j+1} . The AR(1) process of ω is then approximated on the grid as:

$$\mathrm{P}(\omega_t = \omega^j|\omega_{t-1} = \omega^i) = \begin{cases} \Phi((q^1 - \rho\omega^i)/\sigma) & \text{if } j = 1 \\ \Phi((q^j - \rho\omega^i)/\sigma) - \Phi((q^{j-1} - \rho\omega^i)/\sigma) & \text{if } 1 < j < N_\omega \\ 1 - \Phi((q^{N_\omega} - \rho\omega^i)/\sigma) & \text{if } j = N_\omega. \end{cases}$$

Compute $Pr(\omega_t = \omega^j|\omega_{t-1} = \omega^i)$ for $i, j = 1, ..., N_\omega$ and construct a $N_\omega \times N_\omega$ transition matrix, called M_ω , of which (i, j)-th element is $\mathrm{P}(\omega_t = \omega^j|\omega_{t-1} = \omega^i)$. We set $\beta = 0.9$.

Part 2. Once the state space is discretized, both the value function V and the conditional choice probabilities $\mathrm{P}_\theta(d_t = 1|\omega_{t-1}, d_{t-1})$ can be expressed by $N_\omega \times 2$ matrices. Write a function m-file which takes θ, M_ω , and $\{\omega^1, ..., \omega^{N_\omega}\}$ as inputs and then produces the fixed point V and the conditional choice probability $\mathrm{P}_\theta(d_t = 1|\omega^j, d_{t-1})$ for $j = 1, ..., N_\omega$.

Part 3. "Discretize" the data on productivity shock, $\{\omega_{it}\}$, by setting ω_{it} equal to the closest grid point among $\{\omega^1, ..., \omega^{N\omega}\}$, i.e.,

$$\omega^*_{it} = \begin{cases} \omega^1 & \text{if } \omega_{it} \le q^1 \\ \omega^j & \text{if } q^j < \omega_{it} \le q^{j+1} \\ \omega^{N_\omega} & \text{if } q^{N_\omega} < \omega_{it} \end{cases}.$$

Part 4. Write a function m-file which takes θ, M_ω , and $\{\omega^1, ..., \omega^{N_\omega}\}$, $\{\omega^*_{it}\}$, and $\{d_{it}\}$ as inputs and produces the negative value of log-likelihood function (6) as output. Estimate θ by maximum likelihood. Report the estimates and their standard errors.

Part 5. The conditional choice probabilities $\mathrm{P}_\theta(d_t = 1|\omega_t, d_{t-1})$ and the transition matrix of ω^*_t jointly characterize the stochastic process of (d_t, ω^*_t). Using the estimate $\hat{\theta}$ and the transition matrix M_ω , compute the stationary joint distribution of (d_t, ω^*_t) as well as the conditional distribution of d_t given ω^*_t in the long-run.

Part 6. Compute the empirical transition matrix of d_t and report it in the 2×2 table, where the $(1,1)$-th element is a fraction of firms with $d_t = 0$ given $d_{t-1} = 0$. Compute the predicted transition matrix of d_t under the estimated parameters and compare it with the empirical transition probability. Does the estimated model successfully replicate the dynamic patterns of import status?

Part 7. Suppose that a government unexpectedly and permanently introduces import subsidies in the form of one-time lump-sum transfer for the "first-time" importers at $t = 0$and, as a result, the value of δ_2 decreases by 50%. Analyze how a fraction of importers change over time after the introduction of import subsidies starting from the stationary distribution of (ω_t^*, d_t) by plotting a fraction of importers as y-axis and time as x-axis.