

Philosophy 220A: Symbolic Logic I

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1. What is logic?

Logic encompasses many different kinds of study, so that one might wonder what the common thread is. Some claim that logic is the study of *truth*, and is thus the most basic and fundamental science. While every science aims at truth, logic is the science of truth itself. It tries to discover the truth about truth. Logic studies truth in the same way that biology studies living things.

Others say that logic is concerned with *thought*, and tries to discover the “laws of thought”. These laws do not describe the way people actually think, for that is the task of psychology. Rather, they prescribe the way people *ought* to think; they describe the way a perfect mind thinks. Logic is like ethics and morality, in that it separates right from wrong. Logic, one might say, is the ethics of thought and belief.

Others say that logic is essentially concerned with *language*, in some way. Logic tries to understand the “logical form” of statements, and certain structural relations between sentences. It is certainly true that logicians spend a lot of time studying languages, especially the artificial languages that logicians themselves have devised. Much of this course will consist of learning how to use artificial languages. Logicians also study natural languages, such as English, French, Mandarin and so on.

In my view, logic is concerned with all three of these, with thought, with language, and with truth. This is possible, since these three things are closely connected. But how? This is a controversial area in philosophy, which we shall explore a little.

1.1 Meaning

Let’s start with language. A language consists of symbols, which are usually marks on paper or spoken sounds. (They may be other things, such as smoke signals.) These symbols are linked, via convention, with meanings, so that language can be used for communication. When words are spoken, the meaning is conveyed from speaker to listener. It should be noted that language has a variety of uses, such as to express emotions, to give commands, and to ask questions. In this course however we are only concerned with one function of language, namely the attempt to *convey information*.

The first thing to recognise about language is that a sentence is the smallest piece of language that makes up an act of communication, or conveys information. Smaller linguistic units, such as individual words, do not communicate anything by themselves. They do not express statements. The meaning of a word comes from its contribution to

the meanings of sentences that contain that word. Thus the fundamental kind of meaning is the meaning of a sentence. Since logic is concerned with information, the only kind of sentence we are interested in is the kind that makes a claim, or an assertion of fact. (This is called a declarative sentence.) Declarative sentences may state mere opinions, as well as known facts. They may also state things that are false – even things that the speaker knows to be false. A sentence is declarative if it says something that a person *could* believe.

Consider, for example, the sentence “I hereby resign from this committee”. It does convey information since, when uttered by (say) Janet, it tells the audience that Janet is resigning from the committee. But it does more than this: it also *performs the act* of resigning from the committee. It is in saying those words that Janet severs her ties to that committee. For this reason, the sentence is not declarative.

In a similar way, questions are not declarative sentences, since they request information. Even if they sometimes supply information as well, they do not *merely* assert (alleged) facts as a declarative sentence does.

Exercise 1.1

[N.B. All the exercises in these notes are optional, being designed to help you learn the concepts involved. Do not hand them in.]

1. Which of the following sentences are declarative sentences, i.e. aim merely to convey information and do no other work?

- a. Yesterday, I resigned from the committee.
- b. Why did you do such a crazy thing?
- c. Pierre is the capital of South Dakota.
- d. I order you to leave.
- e. The blue car over there is mine.
- f. I bring you greetings from the Socialist Party of Finland.
- g. Get lost, eh.
- h. I am very happy.
- i. Hooray!
- j. I'm out of gas.

2. Sometimes it looks as if a single word can convey information. For instance, someone asks: “How old are you?”, and you reply “Twenty”. Here, the single word ‘twenty’ has made a statement. What we have to understand here is that, in the context of the question “How old are you?”, the single word ‘twenty’ is a short form of “I am twenty years old”. If you just walked up to a stranger in the street and said “Twenty”, he or she would have no idea what you meant. You could mean “I have twenty dollars”, “I have twenty years’ experience at building condos”, or anything.

For each of the following words, imagine a context in which it would mean something by itself, and then write out a full sentence that expresses what it means.

- a. Yesterday.
- b. No.
- c. Sometimes.
- d. Not really.

If sentence meaning is the basic kind of meaning, then we should ask: What is the meaning of a sentence?

1.2 Propositions

Many philosophers would say that the meaning of a sentence is the *proposition* that it expresses. The proposition is “what it says”, or the *content* of what is expressed. To get a grip on this idea of a proposition, consider first the fact that different sentences can “say the same thing”. For example, the sentences:

Victoria is the capital of B.C.
The capital of B.C. is Victoria.

say the same thing. Also consider:

Jack loves Jill
Jill is loved by Jack
Jack aime Jill
Jack liebt Jill

which also say the same thing. The proposition is the *thing* that they all express. It is the content of what is said, or the information that is conveyed.

Exercise 1.2

1. Which pairs of sentence express the same proposition, and which ones express different propositions?

- a. Williams Lake is south of Prince George.
Prince George is north of Williams Lake.
- b. Benjamin Franklin was bald
The inventor of bifocals was bald
- c. Sally lives in the same house as Diane

Diane lives in the same house as Sally

- d. This is my first philosophy class
I have never taken a philosophy class before this one
- e. All politicians are rich
All rich people are politicians
- f. The glass is half empty
The glass is half full
- g. Some professors are boring
Some boring people are professors

2. Bill is rather agitated, and is on the telephone, talking to a travel agent. The sentences he speaks are recorded below. Which of these sentences are declarative sentences, i.e. express propositions? (With some sentences the exact meaning may seem to depend on the tone of voice. In those cases, interpret the sentence literally.)

- a. It's raining again.
- b. Blast this rain!
- c. Doesn't it ever stop raining here?
- d. If it doesn't stop raining, I'll go crazy.
- e. Tell the rain to stop!
- f. I want it to stop right now.
- g. Do you have any flights to Phoenix, leaving today?
- h. I'll pay whatever it costs.
- i. I just need to get away from the rain.
- j. Thank you.

3. An ambiguous sentence is one that can be read as expressing either of two different propositions. All the sentences below are ambiguous. For each sentence (a-f) show what the two propositions are by expressing them with different (unambiguous) sentences.

Example: I often have my friends for dinner.

This can mean: I often cook dinner for my friends.
Or: I often eat my friends, as my dinner.

- a. Visiting professors can be boring.
- b. You should recycle cans and waste paper.
- c. My students are revolting.

The problem of ambiguity is worse for newspaper headlines, which are abbreviated. Here are some juicy (and no doubt deliberate) ones:

- d. Police Can't Stop Gambling
 - e. Prostitutes Appeal to Pope
 - f. Squad Helps Dog Bite Victim
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It's not always easy to judge whether two sentences express the same proposition, or different propositions. Regarding Qu.1b, for instance, did you know that Benjamin Franklin was the inventor of bifocals? Does knowing this make a difference?

One common way to test whether two sentences express the same proposition, or different propositions, is to ask the question:

Would it be possible for a rational person to believe one sentence but not the other?

If this is possible, then they express different propositions. If it is impossible, then they must be the same proposition.

Exercise 1.3

Using this method, repeat the previous exercise. You may find, for some questions, that the *background knowledge* of the thinker is relevant.

The idea behind this method is that a proposition is really a component of a belief. When one has a belief, one believes *that* such and such. Well, the "such and such" *that* one believes is a proposition. Note that propositions may be components of other mental attitudes as well. You may have seen this joke:

The optimist believes that this is the best of all possible worlds.
The pessimist fears that he may be right.

One person may believe a particular proposition, while someone else doubts it, or wishes it were true, fears that it may be true, hopes that it is true, and so on. These are just different attitudes towards the same proposition.

Another method to test whether two sentences express the same proposition is to ask:

Are they true in exactly the same set of possible circumstances?

Or is there a way in which one may be true and the other false? For example, consider question 3. If Sally lives in the same house as Diane, then clearly Diane must live in the

same house as Sally, and vice-versa. There is no way for one of these to be true and the other false. So they express the same proposition.

The idea behind this method is that a proposition is a ‘possible state of affairs’, or a ‘way the world could be’.

Exercise 1.4

Repeat Exercise 1.2, using this method. Does it give the same results as before?

You may find that there are some disagreements between these two methods. This possibility of disagreement results from the fact that the methods are based on different notions of what a proposition is. Even good philosophers often mix up these two notions, which leads to confusion. In the next section we shall investigate the difference between these notions, and find that it is similar to the difference between a map and the physical territory it represents.

Exercise 1.5

1. Suppose Zhang is quite familiar with Chinese geography, but does not speak a word of English. Would it be true to say ‘Zhang believes that Beijing is north of Shanghai’? (If you showed Zhang the sentence ‘Beijing is north of Shanghai’, he would not understand it.)

2. A tourist is in safari with an African guide, who speaks little English.

tourist: Are there poisonous snakes around here?

guide: ‘Snakes’? What is ‘snakes’?

tourist: I can’t believe it! He grew up in the jungle and doesn’t know what a snake is!

What mistake did the tourist make?

1.3 The Map is Not the Territory

We all know that the map is not the territory. For one thing, the map may not be perfectly accurate. This path, marked on the map, may not exist in fact. Another path, which does exist in reality, may not be shown on the map. (Maps are made by fallible

human beings, after all.) Moreover, even if there were a perfect map, entirely free of error, it would still be different from the territory. The map is small, and made of paper and ink, whereas the territory is big, and made of dirt, rock, water, vegetation and so on. No one could be so silly as to confuse the map with the territory, to think that they are one and the same! Could they?

Consider this story. In ancient times, astronomers distinguished between the fixed stars, which keep their positions relative to each other, and the wanderers (planets) that move slowly against the backdrop of the fixed stars. Two of these planets were called Hesperus and Phosphorus. Hesperus was visible, at certain times of the year, as a bright light above the western horizon after sunset. At other times of the year, Phosphorus was visible as a bright light over the eastern horizon, before sunrise. Hesperus and Phosphorus were never both visible on the same day. Hesperus would be visible for a few months; then it would disappear for a couple of weeks. Then Phosphorus would appear, and so on.

As science progressed, the trajectories of the planets were plotted accurately against the sphere of the fixed stars, and a curious fact emerged. The paths of Hesperus and Phosphorus were clearly segments of a *single* path through the heavens. The conclusion was hard to avoid: Hesperus and Phosphorus are not two separate planets, as was previously thought, but are really the *same* planet. Hesperus *is* Phosphorus. This view, that Hesperus and Phosphorus are identical, is still believed today. Indeed, we now refer to this planet using the single name *Venus*.

This situation has created a puzzle about meaning. Some questions are:

(1) Do the names ‘Hesperus’ and ‘Phosphorus’ have the same meaning, or different meanings?

(2) Do the sentences ‘There is life on Hesperus’ and ‘There is life on Phosphorus’ mean the same thing, or express the same proposition?

How do you answer these questions? Let’s look at (1) first. On the one hand, the names ‘Hesperus’ and ‘Phosphorus’ are talking about the same real planet. So it looks like they have the same meaning. Similarly, the sentences ‘There is life on Hesperus’ and ‘There is life on Phosphorus’ seem to describe the same possible state of affairs. Since Hesperus and Phosphorus are the same planet, it would be impossible for there to be life on Hesperus but not on Phosphorus, and vice-versa. So it appears that the two sentences are making the same claim, and thus have the same meaning.

Suppose, however, we think of an ancient astronomer (call him Ralph) who considers these questions. Ralph does not think that the sentences ‘There is life on Hesperus’ and ‘There is life on Phosphorus’ have the same meaning. After all, he says, one sentence is about Hesperus, and the other is about Phosphorus, and these are different planets. Though they are the same planet *in fact*, they are not the same *in his mind*. For *him*, we are inclined to say, the two sentences have different meanings. In a similar way, there

seems to be some sense in which ‘Hesperus’ and ‘Phosphorus’ have different meanings for Ralph. In his “subjective world”, i.e. in the world as he sees it, there are two different planets Hesperus and Phosphorus, so that the names ‘Hesperus’ and ‘Phosphorus’ are not interchangeable for him.

We are forced to say that there are two kinds of meaning. First there is the objective meaning, or meaning in terms of the real world. In this sense, ‘Hesperus’ and ‘Phosphorus’ have the same meaning, as they refer to the same planet in the real world. Second there is the subjective meaning, or the meaning in terms of one’s beliefs. In this sense, ‘Hesperus’ and ‘Phosphorus’ have different meanings for Ralph, since in the world of his belief Hesperus and Phosphorus are different planets.

This distinction between objective and subjective meaning must also apply to sentences. For Ralph, the sentences ‘There is life on Hesperus’ and ‘There is life on Phosphorus’ are saying quite different things. They are not equivalent, or interchangeable. Thus their subjective meanings are different. They do have the same objective meaning, however, as in the real world there is no difference between there being life on Hesperus and there being life on Phosphorus.

To mark the difference between objective and subjective meanings of sentences, let us call the subjective meaning of a sentence a *thought*, and the objective meaning a *possible state of affairs*. Thus ‘There is life on Hesperus’ and ‘There is life on Phosphorus’ express different thoughts (for Ralph), but express the same possible state of affairs. The term ‘proposition’ seems to be ambiguous between thoughts and possible states of affairs. A thought exists only in the mind of the person who has that thought (although someone else may have a similar thought). A state of affairs is a feature of the real world. A thought is a *representation* of a possible state of affairs. Thus a thought is like a map, and a possible state of affairs is like the territory that the thought represents.

Maps and territories have the same basic structure. A territory, for instance, has roads, towns, rivers and mountains, and so does the map. In a similar way, since a thought is a representation of a state of affairs, thoughts and possible states of affairs have the same basic structure. In the context of map reading, the similarity of structure allows one to use the same terminology for both maps and territories. The word ‘path’, for instance, can mean either something on the map or in the territory. E.g. ‘This path (pointing to the ground) is not shown on the map’, or ‘This path (pointing to the map) does not really exist’. In a similar way, we can say that thoughts and states of affairs are both kinds of proposition.

Logic is the study of propositions.

Thus logic studies human cognition and reasoning (in studying thoughts), and also studies the most general structure of the world (in studying states of affairs). Further, since a declarative sentence expresses a proposition, logic studies those as well.

1.4 Truth and Belief

We have shown that there are two kinds of meaning, objective meaning (in terms of the real world) and subjective meaning (in terms of beliefs about the world). This contrast between objective and subjective meaning is very closely related to the familiar difference between *truth* and *belief*.

Suppose that, about two hours ago, you put a six-pack of beer in the fridge. Thus you believe that there is now some beer in the fridge; and, being thirsty, you go to get one. Unfortunately, without your knowledge, your room-mate grabbed them an hour ago and took them to a party. Thus, while you *believe* that there is beer in the fridge, in *fact* there is none. The awful *truth* of the matter is that the fridge is quite devoid of beer.

This is a case where truth and belief fail to coincide, so that the difference between truth and belief is quite obvious. Even when truth and belief match perfectly, however, they are still quite different things. If your room-mate had not touched the beer, so that it remained in the fridge, your belief that there is beer in the fridge would have agreed with the fact of there being beer in the fridge. The belief and the fact are not the same thing, though, since the belief is just in your mind, whereas the fact depends on how things really stand. The belief and the fact are like a map and the territory it represents.

I find it helpful to talk about two different “worlds”: the world of truth, and the world of belief. Note that there is only one world of truth, or real world, but many different worlds of belief. Each person has different beliefs, so each person has his or her own world of belief.

I said above that the objective meaning of a sentence is the possible state of affairs that it expresses. Now, since we are talking about truth, we should note that there are two basic kinds of possible states of affairs. Here are some examples of each:

A	B
Jean Chrétien’s being from Shawinigan	Jean Chrétien’s being from Vancouver
Venus’s having no moon	Venus’s having 6 moons
B.C.’s having a border with Alberta	B.C.’s having a border with Texas

The possible states of affairs on the A-list are all ones that are “real”, or “actual”, whereas the ones in the B-list are not. While Jean Chrétien could (conceivably) have been from Vancouver, had the Chrétien family moved west before his birth, in fact he grew up in Shawinigan, Quebec. Thus *Jean Chrétien’s being from Vancouver*, while a possible state of affairs, is not an *actual* state of affairs.

We often talk about a sentence being true or false. What do we mean by this? A sentence is true just in case the thought it expresses represents an *actual* state of affairs, a state of affairs that is *real*. A false sentence is one that describes a state of affairs that is not real, one that is not actual.

I said above that a belief is a certain attitude toward a thought. It is hard to say what this attitude is, exactly, but we should note the following. If p is some thought, such as the thought that there is beer in the fridge, then there is no difference between believing p and believing that p is true, or that p represents an actual state of affairs. Thus, it seems, belief is inherently truth-directed, or aimed at real states of affairs. There is no need to say that your belief is directed at the truth, as this is essential to the nature of belief. The aim of belief is truth.

Indeed, we might go even further than this, in linking belief with truth. We might say that belief is merely the subjective counterpart of truth, and that truth is the objective counterpart of belief. Roughly speaking, beliefs are subjective truths, and truths are objective beliefs. I'm not sure what to make of this idea, but it does seem to me that thoughts (subjective propositions) and possible states of affairs (objective propositions) have the same logical structure. This is a difficult (and contentious) area of philosophy.

Exercise 1.6

Think about the following sentences, and try to decide whether (or to what extent) you find them acceptable. What criticisms (if any) would you make of them?

1. Lois Lane believes that Superman can fly faster than a speeding bullet. Now Superman and Clark Kent are one and the same person, so Lois believes that Clark can fly faster than a speeding bullet.
2. Many people have seen flying saucers.
3. In the Middle Ages, the earth was stationary, at the centre of the universe. After the work of Copernicus, Galileo and Kepler it became a planet, orbiting the sun.
4. "Consider, for another example, the men who called Copernicus mad because he proclaimed that the earth moved. They were not either just wrong or quite wrong. Part of what they meant by 'earth' was fixed position. Their earth, at least, could not be moved." (from Thomas Kuhn, *The Structure of Scientific Revolutions*, p.149)
5. He's an orthodox Jew, so the Exodus really happened, for him. I wouldn't offend him by questioning it. But for me, no such event ever took place. It's true for him, but not true for me.

6. It is impossible for us to establish an exact date for the death of Caesar Augustus. So it's silly and pointless to hold that such a date exists.
7. When people talk of 'truth', whose truth do they mean? The European male truth? Jewish truth? Gay truth? African truth? Feminist truth?
8. The word 'truth' is oppressive, as it's mostly used to silence other voices. When you say "This is the truth of the matter", you are effectively saying that no other opinion is valid. You are shutting down the discussion.
9. The concept of objective truth is essential for any healthy discussion. Surely the point of intelligent discussion is to discover the truth? If there is no truth to be found, then what can it mean to persuade someone to accept your view?
10. If there's no objective truth, then I don't know what it means to have a belief, or opinion. If there's no territory, then there are no maps either. (A map must represent a territory – otherwise it's just an arbitrary sheet of paper.) If there's no truth, then there are no beliefs either, just odd psychological states.
11. How can someone say there's no objective truth, or that truth is just an illusion? Are they saying that's the *truth* of the matter? *In fact*, there are no facts?
12. Elements of what we call 'language' or 'mind' penetrate so deeply into what we call 'reality' that the very project of representing ourselves as being 'mappers' of something 'language-independent' is fatally compromised from the start. Realism is an impossible attempt to view the world from Nowhere.

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