A Subjective Theory of Objective Chance

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1. Introduction

There are two basic views about objective chance. Objectivists say that it is a kind of probability that is a physical quantity, and so has nothing to do with belief. (It might be a relative frequency, for example.) Subjectivists counter that such objective chances are both problematic and unnecessary. It is terribly difficult to give a satisfactory theory of them, and they are quite superfluous to the practice of science. In this paper I will develop and defend a third option, which one might call a subjective theory of objective chance. Physical chances do exist, on this view, with all of the standard properties, except that they are degrees of belief. It will be shown that this third option combines all the advantages of the other two, but has none of their disadvantages.

Immediately one might think that this third view is a contradiction in terms, that when we talk of objective chance, we mean something that is not a degree of belief. I think this would be a mistake, however. As I argue in the next section, chances are known to be objective only in the sense that they are determined by the physical facts alone, and can be measured (approximately and fallibly) by performing physical experiments.

Since concerning objective chances we now have three, rather than two, basic positions, the terms ‘objectivist’ and ‘subjectivist’ are no longer appropriate. I suggest we use the terms below.

- **Propensity view** Objective chances exist, and they are not degrees of belief.
- **Epistemic view** Objective chances exist, and they are degrees of belief.
- **Eliminativism** Objective chances do not exist.
I use ‘eliminativism’ to describe the position of de Finetti, Savage and others, even though this is usually called ‘subjectivism’. This is because my (epistemic) view is also subjectivist in its claim that all probabilities are degrees of belief. I use the term ‘propensity view’ (rather than ‘objectivism’) for the positions of Popper, Giere, Gilles, and others, since my view also affirms the existence of objective chances.

In this paper I will defend the epistemic view against the other two. Note that the “epistemic view” defended here is not specific or precise enough to be called a theory of chance. There could be many distinct theories of chance, all of which conform to the epistemic view. I’m not going to explain or defend my ‘causal theory of chance’⁴¹, even though it is an epistemic theory, as it raises too many complex issues.

2. The Objectivity of Chance

The purpose of this section is not to argue that objective chances exist, but rather to explore our concept of chance, and investigate what we mean when we say that chance is objective. My conclusion will be that this objectivity consists in the following properties, that

(i) chances are determined by the physical facts alone, and
(ii) the Principal Principle holds, so that chances can be measured experimentally.

We believe in chance through an inference to the best explanation. Consider, for example, the differences in stability between isotopes. If you buy some copper piping at a hardware store, it will be mostly composed of Copper 63 and 65. It will contain very little Copper 62, as this isotope decays so quickly. Copper 63, on the other hand, lasts indefinitely.

The difference in stability between Copper 62 and Copper 63 must be due to some physical difference between them. And we know what that difference is: while both have 29 protons per nucleus, Copper 63 has 34 neutrons, one more than Copper 62. Somehow this extra neutron settles everything down. All will surely agree that there is a physical difference between Copper 62 and Copper 63, which makes the latter far more stable than the former.

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¹ See Johns (2002).
Given that this difference exists, there is a strong impulse to describe it quantitatively. Physicists have a well-established numerical measure of isotope stability, namely the half-life. Copper 62 has a half-life of about 9.74 minutes, whereas the half-life of Copper 63 is effectively infinite. These half-lives seem to be hard facts about the physical world, like the speed of light being $3 \times 10^8$ metres per second, or the electronic charge being $1.6 \times 10^{-19}$ Coulombs. While their measurement does depend upon a theoretical framework as well as raw data, there is no serious disagreement about their values.

To regard half-lives as objective, physical quantities seems to commit us to objective probabilities, however, as the half-life of an isotope is defined as the length of time in which the probability of the nucleus decaying is one half. Thus, if half-lives are hard physical facts then there must be probabilities that are also hard physical facts. These are known as objective chances.

Physicists, like philosophers, do not know what chances are. (Unlike philosophers, however, most are them are blissfully unconcerned by this ignorance.) So how do physicists measure chances, and use them to make empirical predictions? While physicists generally not describe their procedures in this way, they effectively operate by assuming the Principal Principle (PP)\(^2\). This principle roughly says that a person is rationally required to have degrees of belief that equal the known chances. It is well known that this allows one to derive empirical predictions from a hypothesis about chance, and thus (using Bayes's theorem) experimental data can confirm or disconfirm such hypotheses.\(^3\) Without PP, on the other hand, there is no known way to derive empirical consequences from claims about chance.

In summary, chances (if they exist) are probabilities that are determined by the physical facts, and they are such that a person who knows the chances is rationally constrained to adjust his degrees of belief to match them. These properties are sufficient for the practice of science.

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\(^2\) See Lewis (1980: 86-87).
\(^3\) Ibid, 106-108.
3. Argument Strategy

The first step in my argument is to define a probability measure that is neutral, in the sense that all three views agree that it exists. This “demon probability”, as I call it, is roughly the physical chance, according to the epistemic view. Since all agree on its existence, the demon probability is a useful common ground for argument between the different views.

Second, I will introduce the Authority Principle (AP). This principle says that when you know the degrees of belief of someone whom you know is both perfectly rational and has superior knowledge to yours, rationality requires that you adjust your degrees of belief to match his. AP is absolutely central to the epistemic view, since it allows one to show that the Principal Principle (PP) holds within the epistemic view of chance. (It also entails that eliminativism is incoherent, and undermines the propensity view.)

Third, I will establish the viability of the epistemic view of chance. In particular, I will show that the demon probabilities of events are determined by the physical facts, and are empirically measurable.

Fourth, I will attack the eliminativist, using AP together with the notion of demon probability. The usual claim, that the eliminativist can make inductive inferences quite successfully, without appeal to objective chances, is refuted.

Fifth, I will attack the propensity view of chance, on the grounds that it fails utterly to capture the notion of one event having a tendency to cause another. The epistemic view, by contrast, provides a clear and straightforward account of this relation.

Finally I will discuss some of the applications of the epistemic view of chance to problems in quantum physics, and describe a possible way to test the idea experimentally.

4. A Neutral Probability Measure

There is a probability measure that all three views about chance can accept the existence of, and which is useful as a common basis for arguments about the nature of chance. This measure will be called the demon probability. It is defined in detail below, but is roughly the degree of belief of a rational demon, which is provided with some special information about the physical world.
Philosophers have enlisted the support of many different demons over the years, so I should say that I have in mind something like Laplace’s demon, not Descartes’ or Maxwell’s. This demon is an embodiment of logical truth, or inferential rationality. This demon may have degrees of belief, insofar as these are dictated by reason. Where rationality does not require an exact degree of belief, the demon will not have any exact degree of belief. In that case, his belief may be described by a sub-interval of the unit interval \([0, 1]\) of real numbers, in the manner specified later in this section.

To say that the demon is an embodiment of inferential rationality means, at least, that he believes the deductive consequences of what he believes. The set of propositions that the demon believes with certainty is deductively closed. In addition, the demon’s beliefs obey the axioms of probability. If \(PP\) is a rational constraint, then he obeys that. When the demon’s knowledge expands by his becoming certain about a proposition that was previously in doubt, then his degrees of belief are updated according to the Principle of Conditionalisation. His degrees of belief satisfy any other requirements of reason as well, such as symmetry constraints, which we may not know about.

It must be stressed that the demon’s degree of belief in a proposition may not take a precise value, but may be an interval. The notion of an interval probability is not new, and the version used here is fairly typical.\(^4\) Suppose we have a (weak) preference ordering \(\leq\) between goods. Then, for any pair of goods, there are four possible preference relations, as the weak preference may hold in both directions, neither direction, or in one direction but not the other. Now consider a contract that pays $1 if some proposition \(A\) is true, and nothing if it is false, which we can write as \([$1 \text{ if } A]\). How might this contract be related to some amount of money like $0.3? For some propositions \(A\), and for some epistemic states, it is arguable that the weak preference relation holds in \(\text{neither}\) direction. This means that the subject does not regard the two as equally good but, when asked to choose between them, is unsure which one to pick. We can say that the two goods, \([$1 \text{ if } A]\) and $0.3 are incommensurable.

If a contract \([$1 \text{ if } A]\) is incommensurable with one monetary sum, then it may be shown that there exists an interval of such sums, which will of course be a subset of the unit real interval \([0, 1]\). This interval is known as the \emph{interval probability} of \(A\).

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\(^4\) The following presentation of interval probabilities is a summary of the one given in Johns (2002: 37-38).
Chances are most easily discussed in the context of a chance set-up, so let us do the same with demon probability. A chance set-up is a system that, when left to itself, evolves stochastically, i.e. not deterministically. Concerning such a system, it is useful to distinguish its intrinsic properties from the properties of its actual motion. The intrinsic properties are ones that do not depend upon what its history is, so that the system has those properties in all of its possible histories. (For example, if the system is a pendulum, then its intrinsic properties include the mass of the bob and the length of the string. The properties of the motion include the amplitude of its oscillation.) The notion of the system’s intrinsic properties allows one to talk about repetitions of the same experiment. There are two ways to repeat an experiment. One can take the same system, as long as its intrinsic properties have not changed\textsuperscript{5}, or one can use an exactly-similar copy, i.e. another system with exactly the same intrinsic properties. In addition to using a similar system, the repeated experiment must start the system in the same state that it had in the original experiment. Let us suppose that, each time we perform the experiment, we set it up in the initial state $s$.

Now let us define demon probability. Consider some type of event $E$ that can occur in the chance set-up, after a short period of natural, undisturbed evolution from the initial state $s$. What is the demon probability of $E$? It is the demon’s degree of belief that $E$ occurs, given maximal knowledge of three things:

(i) the dynamical laws of physics,
(ii) the intrinsic properties of the chance set-up, and
(iii) the initial state of the chance set-up.

The demon knows nothing at all, apart from these three things. By ‘maximal’ knowledge, I mean that he has the fullest possible knowledge of these matters. Or, more technically, he knows the unique true proposition about these things that entails all the other true propositions about them. It is easy to see that this maximal (true) proposition is unique. For suppose there were two such (non-equivalent) maximal propositions. Since they are both true, it follows that their conjunction is also true, and it entails each conjunct, so that neither conjunct is maximal (which is contrary to the hypothesis).

\textsuperscript{5} The parts must not be worn, or stretched, or bent, the paint must not be chipped, and so on.
For brevity, I will loosely refer to factors (i)-(iii) above as the *three causes* of E. In my view these factors really are causes, but that is not important here. One can take this term ‘cause’ as a mere abbreviation. We can then say that the demon probability of E is the demon’s degree of belief that E occurs, given (only) maximal knowledge of the causes of E.

The epistemic view of chance holds that chances exist, and that they are degrees of belief. Now, the demon probability of E is a degree of belief, and it is also determined by the physical causes (i)-(iii) above. It is therefore a candidate for being the objective chance of E, on the epistemic view. It argued in section 6 below that, while the epistemic view cannot quite identify chance with demon probability, the two are closely related, and necessarily equal in value. It should be noted that the demon probability of E might not be a precise value, since degrees of belief are not always precise values. Thus, according to the epistemic view of chance, it is likely that some chances are intervals.

What do the other two views say about demon probability? If one takes the propensity view of chance, then we see that the demon probability of event E necessarily equals the objective chance of E, by the following argument. First, the demon is rational, so its beliefs will satisfy the Principal Principle. Second, if objective chances are physical properties, then they must surely reside somewhere among the initial state, the intrinsic properties of the chance set-up, and the laws of physics. It follows that the demon would know the chance, and that his degree of belief would necessarily equal the chance, according to the Principal Principle.

Eliminativists about chance will generally accept the notion of rational constraints on degrees of belief, as they typically regard the axioms of probability and the Principle of Conditionalisation as such constraints. They should therefore accept the definition of demon probability as meaningful. They cannot, however, allow the demon probability of E to take a precise value, or an interval smaller than (0,1), as this is exactly the epistemic view. Thus they must say that rationality does not constrain the demon’s degrees of belief at all, so that the demon probability is always the full (open) unit interval (0,1). This squares with the view of some eliminativists that the axioms of probability (including Conditionalisation) are the *only* constraints on rational belief, since it is clear that these axioms by themselves place no constraint at all on the probability of E.
5. The Authority Principle

As mentioned in section 3, the Authority Principle (AP) roughly says that when you know the degrees of belief of someone whom you know is both perfectly rational, and has superior knowledge to yours, then rationality requires that you adjust your degrees of belief to match hers. Thus, AP attempts to specify some conditions under which one person, say Bob, should regard another (perhaps Alice) as an epistemic authority relative to himself. It should be noted that AP tries to find sufficient conditions for such an authority relation, not necessary conditions.

Under what conditions should Bob regard Alice as an epistemic authority? An obvious condition is that she should know more than him, at least on matters relevant to the case at hand. But what does this mean? It will be helpful here to borrow a concept from belief dynamics, namely that of an expansion of belief. An expansion is a change of epistemic state that occurs when the subject learns something new that is consistent with what he believed before. Thus a new belief is added, but nothing is removed. The simplest kind of expansion occurs when some proposition A, that was previously in doubt, is now believed with certainty. In general we can say that an epistemic state K’ is an expansion of K if and only if it is possible for a perfectly-rational thinker to move from K to K’. Note that a perfectly-rational thinker does not forget anything, and never draws unwarranted conclusions, and so never retracts any belief.

Can we say that Alice knows more than Bob when her epistemic state is an expansion of his? One problem here is that one can learn falsehoods as well as truths. Perhaps some of the additional things Alice believes are false? Or, even if those things happen to be true, it might be that Alice has no grounds to believe them. In such cases, we do not think that Alice knows more than Bob, nor that Bob should regard Alice as an authority.

To fix this problem, we can add the condition that Alice’s beliefs are all warranted, to exactly the same degree as her credence, by the information she has received. Thus, in particular, any proposition she believes with certainty is fully warranted, and therefore true. In other words Alice is a perfectly rational thinker who always draws the correct conclusions, obtaining the right degrees of belief, from the information she receives.

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6 The presentation of the Authority Principle in this section is necessarily a little imprecise, as its full development requires a more sophisticated treatment of rational belief dynamics. For the full details, see Johns (2002: 18-27).

7 See, for example, Gärdenfors (1988:48-52).
We now have two conditions for Alice to “know more” than Bob. First, her epistemic state is an expansion of his; and second, she is a perfectly rational thinker. These are no doubt rather strict conditions, but remember that we merely seek sufficient conditions for epistemic authority, not necessary conditions. These conditions are, in one respect, a little too strict for the purpose of this paper, however. We shall weaken the first condition by requiring that Alice’s epistemic knowledge relevant to the matter at hand is an expansion of Bob’s knowledge on that issue. Thus, whether or not Alice is an authority for Bob will depend on the issue concerned. We therefore have the following definition:

Alice is an (epistemic) authority for Bob, concerning some issue, iff:
(i) Alice’s knowledge relevant to that issue is an expansion of Bob’s, and
(ii) Alice is a perfectly rational thinker.

Now, suppose that Alice is an authority for Bob, concerning some issue. What follows from this? Well, it may seem that if Bob knows that she is such an authority, and happens to know Alice’s degree of belief for some proposition A concerning that issue, then Bob should believe A to the same degree. For example, if Alice is an authority for Bob on knitting, and Alice is fairly confident that thicker needles are required, then surely Bob should follow her thinking?

Not necessarily. What if Charles, who happens to be an authority for Alice (and therefore for Bob) on knitting, disagrees with her judgment? In that case, Bob should follow the higher authority, and accept what Charles tells him. As soon as Charles shows up, Alice’s beliefs become irrelevant. For a more tricky case, suppose that Charles is absent, but Darlene is available. Darlene, who is an authority for Bob but not for Alice believes that thicker needles are probably a mistake. What is Bob to think in this case, when faced with competing authorities, neither of which knows more than the other? He cannot follow both of them, on pain of incoherence, and has no grounds for following one and not the other. Thus Bob is surely not required to follow either of them.

The following Authority Principle takes account of this example, by requiring that there is only one authority whose degree of belief is known.
Authority Principle

If
(i) Alice is an authority for Bob on matters related to the proposition A, and
(ii) Bob knows that Alice is such an authority, and
(iii) Bob knows that $P_{\text{Alice}}(A) = q$, and
(iv) Bob does not know anything about $P(A)$ for any other authority,

then Bob is rationally required to believe A to degree q as well.

This Authority Principle seems very plausible, and should be accepted unless a counter-example can be found.

6. The Epistemic View

The epistemic view of chance is that objective, single-case chances exist, and they are degrees of belief. One rather crude theory of chance is that chance is nothing other than demon probability. This theory is rather easily refuted, by the same argument used long ago by Plato in the *Euthyphro*. In the light of this dialogue one is bound to ask: Does the demon have this particular degree of belief because it is correct, or is it correct because the demon has it? The answer is clearly the former: the demon has this degree of belief because it is correct, i.e. it is the one dictated by reason. But then, obviously, the demon probability is not *identical to* the chance, but merely something that tracks the chance, so that it necessarily takes the same numerical value as the chance.

To remedy this problem the epistemic view must define chance using rationality itself, rather than using some fictitious being that merely happens to be rational. The details of this are rather lengthy, however, and cannot be given here.\(^8\) Moreover, this Euthyphro problem is irrelevant to the main properties of the epistemic view that I wish to demonstrate, that a suitable epistemic probability can be determined by the physical facts, and can be empirically

\[^8\text{See John} (2002: 13-29).\]
measurable. Thus we will use the crude epistemic theory of chance, and pretend that chance is just demon probability.

The first task is to show that the demon probability of an event-type E is determined by the physical facts alone. More precisely, the demon probability is determined by the three physical factors, or “causes”, identified in section 4, namely the dynamical laws of physics, the intrinsic properties of the chance set-up in which E can occur, and the initial state of the chance set-up.

The first important point is that the demon is always provided with maximal knowledge of these three causes, and we have seen that there is precisely one maximal (true) proposition describing these causes. Thus, even though in general the physical facts do not determine a person’s knowledge of them, they do determine this demon’s knowledge.

The second point is that the demon is constrained to think rationally. Thus the demon’s degree of belief in E depends only on the nature of rationality (as well as the physical facts). But the truths of reason and logic are fixed and necessary, so that dependence on them is trivial. For example, if I have two sources of income, then I might say that my total income is determined by my incomes from these sources. There is no need to continue “…and by the laws of addition”.

Thus demon probability is determined by the physical facts alone, so it has one property that is required for objectivity. What about the Principal Principle? Consider a subject Bob, relative to whose knowledge the demon’s epistemic state is an expansion, concerning the occurrence of E within the chance set-up. Since the demon is also a perfectly rational thinker, it follows that the demon is an authority for Bob, concerning the occurrence of E. If Bob knows this, and also knows the demon probability for E, then by the Authority Principle Bob’s degree of belief that E occurs should equal the demon probability.

This looks very much like the Principal Principle, but one thing needs to be checked, concerning the condition that the demon’s epistemic state is an expansion of Bob’s. We must verify that the cases where this condition holds coincide with the cases where PP holds. In other words, we need to make sure that any information relevant to E that the demon does not have is inadmissible, in Lewis’s sense.  

What kind of information does one typically have about a chance set-up? Bob might know something about its intrinsic properties, and its initial state. He might know something

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about the laws of physics. In these matters the demon’s knowledge is maximal, however, and so will be an expansion of Bob’s. One thing that Bob might know, outside of the demon’s knowledge, is the past history of outcomes for that chance set-up. This information is admissible, in Lewis’s sense, so that it is screened off by knowledge of the chance of E. The question then is whether this information is relevant to E, for the demon. That is, if the demon were told about the past outcomes of the chance set-up (in addition to maximal knowledge of the three causes) could this affect his degree of belief that E?

On the face of it, it seems possible for the past outcomes to be relevant to E. After all, we generally consider that the past outcomes of a chance set-up are a good guide to its future outcomes. Contrary to this, however, I shall argue that the past outcomes are irrelevant to E for the demon. This follows from a certain view about how inductive inferences work, in the case of repeated experiments using the same chance set-up.

This view about inductive inference is that the inference from knowledge of past outcomes to predictions about future outcomes proceeds in two steps. First, from the past outcomes one infers general facts about chance set-up (i.e. about the three causes). Then, from that information about the three causes, one makes a prediction about future outcomes. There is, in other words, no direct inference from the past to the future. This view assumes, of course, that the causes are the same for each experiment, so it is a version of the common view that induction presupposes that nature is uniform. In this case one must assume that the three causes are invariant, i.e. that the intrinsic properties of the system and the laws of physics do not change, and that the initial state is the same for each trial.10

This view about inductive inference entails that information about any past outcomes of the chance set-up is irrelevant to E, from the demon’s point of view. For the demon’s knowledge of the three causes is already maximal, and so cannot be expanded. Therefore the past outcomes tell the demon nothing about the three causes for the chance set-up, and thus nothing about E.

I take this view about induction to be obviously true. For suppose one carried out a large number of trials using a chance set-up, recording the outcomes, and then used those outcomes to predict the next outcome of a different chance set-up, i.e. one with quite different intrinsic properties, or using a different initial state. It is clear that the past outcomes of one chance set-up

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10 In stating this view about inductive inference I am not, of course, claiming to refute the inductive sceptic. For there remains the question of how we can know (or reasonably believe) that the three causes remain constant.
are irrelevant to the future outcomes of another, so that any such inductive inference is incorrect. For the past outcomes to be logically connected to the future ones, they must at least share a common (type of) cause.

Returning to the question of whether PP holds for demon probability, we see that the demon is an authority for Bob, even if Bob has knowledge of past outcomes of the chance set-up, as this extra knowledge is not relevant to E, from the demon’s point of view.\textsuperscript{11} The only knowledge relevant to E (from the demon’s point of view) that Bob might have, and the demon does not, is knowledge derived from observing the behaviour of the chance set-up, and seeing whether E occurs or not. Thus, for example, if Bob observes that E occurs, then the demon is no longer an authority for Bob on this matter. In this case, however, Bob has knowledge that is inadmissible.

It does seem that any information that is relevant to E, from the demon’s point of view, is inadmissible, so that the cases where the demon is an authority for Bob are exactly the cases where PP holds.

Now let us verify that demon probability is empirically measurable. This is fairly trivial, as it has already been shown that knowledge of the demon probability authorises a numerically-equal degree of belief.

Suppose the chance set-up is run a large number of times, and that the relative frequency of the event E is recorded. Each possible value of this relative frequency has a demon probability, which may be calculated from the demon probability of E in a single trial using the axioms of probability. In fact, if $P_d(E) = q$, say, then there is a high demon probability that the relative frequency of E will lie close to $q$. Now consider a person who does not know the demon probability of E, but performs a large number of experiments using the chance set-up. Provided that person has a prior subjective probability function over the possible values of the demon probability, he can apply Bayes’s theorem to update his priors using the experimental results. Indeed, in cases where his prior distribution is reasonably flat, and there is a large number of experiments, his posterior distribution will be peaked fairly sharply near the observed relative frequency of E. One can take the expected value of this distribution, plus or minus a standard deviation or two, as an empirical estimate of the demon probability.

\textsuperscript{11} More precisely, the authority for Bob here is another demon who knows not just the three causes but also all the past outcomes. This extra demon has the same degrees of belief as the standard demon, however.
Is this estimate likely to contain the true demon probability, however? In other words, is it reliable? All one needs to say here is that the posterior distribution is warranted, as long as the prior distribution was warranted. Of course the use of prior probabilities in scientific reasoning is controversial, but it is not a special problem for the epistemic view of chance. Moreover, if one wishes one can also use orthodox methods to estimate demon probabilities from experimental data, calculating a 95% confidence interval in the usual way. In this case, the value 95% is the demon probability that the observed relative frequency of E will lie within 1.96 sampling standard deviations of the demon probability of E. Thus, from the demon's point of view, the experimental measurement of $P_d(E)$ has a probability 0.95 of being correct.

In general, when we observe a phenomenon, we want to infer the cause of that phenomenon. Thus, in the case of a chance set-up, we would like to infer the three causes. But, unfortunately, it is easy to show that only demon probability is measurable.

Consider all the possible hypotheses about the causes, $C_1$, $C_2$, … Suppose we were able to write these down, and calculate empirical predictions from them. These predictions would be in the form of likelihoods $P(E | C_i)$. The hypotheses are empirically distinguishable to the extent (and only to the extent) that these likelihoods differ from each other. The hypotheses $C_1$ and $C_2$, for example, where $P(E | C_1) = P(E | C_2) = q$, say, are empirically indistinguishable. (Of course $C_1$ might be more plausible than $C_2$ on other grounds, such as simplicity or elegance, but it has no empirical advantage.) Now consider a third hypothesis $D_q$, that the demon probability of E takes the value $q$. This hypothesis is much weaker than $C_1$ and $C_2$, as it is entailed by each of them, but is also empirically indistinguishable from them, since $P(E | D_q) = q$. Thus measuring the relative frequency of E-type events in a large set of trials only tells us the demon probability of E.

In the remainder of this section we shall explore some of the metaphysical aspects of the epistemic view of chance, and consider some possible objections to it.

It is clear that, according to the epistemic view, the three causes do not determine the behaviour of the system. In a deterministic system, the demon would be able to predict the occurrence (or non-occurrence) of E with certainty, so that the demon probability of E would be either 0 or 1. It then appears that demon probabilities are a natural extension of the
determination relation, assigning *degrees* of determination to events that are neither determined to occur, nor determined not to occur.\textsuperscript{12}

It is worth noting that the determination relation has its own credence principle, which appears to be a special case of the Principal Principle. This credence principle says that, if you know that \(C\) occurs, and that \(C\) determines \(E\), then you should believe with certainty that \(E\) occurs. It can be viewed as a special case of PP, namely the case where the chance of \(E\) is one, after \(C\) has occurred.\textsuperscript{13}

I will now consider some possible objections to the epistemic view, in order of increasing severity. First, despite the apparent soundness of the calculations, one might think it impossible for physical experiments to give information about the degrees of belief of a spectator. This is not as strange as it might seem, however, as one can think of an authority’s superior knowledge as putting him ‘closer than us to the truth’. So, observations caused by the truth give us information about his state of knowledge. For a clear example of this, suppose that some person looks at the outcome of the chance set-up before we do. (Let their epistemic state be \(K\), after they make the observation.) Assuming this observer is of sound mind and body, we can empirically measure their degrees of belief, by observing the outcome ourselves. If we observe the outcome \(E\), for example, then we infer that \(P_K(E) = 1\).

A second possible objection is that, since the chance set-up evolves stochastically, it must be governed by a probabilistic law, i.e. a law that specifies transition chances. These probabilities, being part of the information that the demon is given, obviously cannot be the demon’s own degrees of belief, on pain of circularity.

This objection highlights an important aspect of the epistemic view. According to this view it must be possible, at least for the demon, to describe the dynamical properties of a system without using probabilities of any kind. One way to think about this is to suppose that every system has an Aristotelian ‘dynamical nature’, i.e. a set of dispositions to undergo certain kinds of change. It is the system’s dynamical nature that causes it to evolve as it does, from its initial state. On this view, the ‘three causes’ reduce to two, namely the dynamical nature of the chance

\textsuperscript{12} If one takes an epistemic view of chance, one will no doubt take an epistemic view of determination as well, and regard an event as physically determined just in case it is predictable with certainty by the demon, from maximal knowledge of the three causes.

\textsuperscript{13} Strictly speaking, we should say that the chance of \(E\) is the rational (quotient) number 1. Since real-number probabilities are defined as Dedekind cuts they are somewhat imprecise, so that events that are not quite certain can have (real-number) probability one. For more details see Johns (2002: 32).
set-up and the initial state. The transition chances that appear in stochastic laws of motion are then considered to be demon probabilities.

It is true that we presently have no way to describe the dynamical nature of a system without using transition chances. But, as we have seen, experiments cannot reveal the three causes (including the dynamical nature) to us, but just tell us the demon probabilities. It may be that the only way we can describe the dynamics of a stochastic system is through its transition chances.

A third objection to the epistemic view, which many will consider to be the most serious, arises from its need for very tight rational constraints on the demon's degrees of belief. It is widely held that the axioms of probability are the only such rational constraints, and these are nowhere near sufficient for demon probabilities to take precise values. If this is so, then epistemic probabilities are only derivable from other probabilities. If one's epistemic state is “pure”, in the sense that it consists of a set of propositions, each believed with certainty, then no proposition has a precise-valued probability other than 0 or 1.

This view, which is sometimes expressed by saying that there are no logical probabilities, is certainly false, however, for it is easy enough to construct a pure epistemic state in which non-trivial probabilities are fixed by symmetry. Consider the so-called “statistical syllogism”, for example, where all one knows is that (e.g.) this person is a Vancouverite, and 71% of Vancouverites own skis. In this epistemic state, the probability that this person owns skis is 0.71. Someone who assigned a different probability, such as 0.4, would be irrational, even though their beliefs might well be ‘strictly coherent’. Franklin (2001) has argued this point at some length, showing that even the avowed opponents of symmetry constraints on probability are forced to make use of them.

The usual objection to such symmetry arguments is that, even though some are apparently valid, others are definitely wrong, and there are no clear criteria to distinguish the good from the bad. All attempts to formulate a general principle of symmetry, or indifference, have been destroyed by counter examples. This objection is weak, however, since the failure to unify the plausible symmetry arguments under a general principle does not show these arguments to be invalid. Moreover, a general symmetry principle has recently been formulated.

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14 For more information about dynamical natures, see Johns (2002: 66-69).
15 That is, their degrees of belief obey the axioms of probability, and they are not too dogmatic.
that is apparently free of counter examples. The only drawback of this principle is that the criterion for symmetry is very strict, and so will hardly ever be satisfied for real epistemic states.

This objection, that there are no logical probabilities, fails therefore. At best, it creates doubt about whether the demon’s knowledge of the three causes will support exact degrees of belief in events. Unfortunately, since we do not know what the demon knows, it is hard to say whether symmetry arguments can be used in his epistemic state.

It should be noted that, even for the epistemic view, there is no a priori reason to expect demon probabilities to be precise values, or even narrow intervals. The epistemic theorist concedes that the cases where rationality determines precise probabilities are very rare, so that the only evidence that demon probabilities exist as narrow intervals or precise points is empirical. This raises an interesting question: What experimental results would one expect to see if demon probabilities were not precise values, but (relatively) wide intervals?

The short answer is that one would not expect to see the tight convergence in the relative frequencies that is observed in actual experiments. Mathematically, not every binary sequence displays such behaviour, and if the demon probability of E were the full unit interval (0, 1), then one would not expect to see any convergence at all. It is even quite likely, on the epistemic view, that some chances truly are intervals, and that this might be shown by experiments. This possibility will be discussed further in section 9.

7. Eliminativism

In our example of the isotopes of copper, eliminativism about chance seems untenable. But the eliminativist uses an instrumentalist strategy, arguing that he can calculate half-lives like everyone else, and make the usual predictions on the basis of his calculations. Thus, in short, the eliminativist claims that belief in objective chances is superfluous to the practice of science. This position has not been attractive to many, but has been surprisingly difficult to refute convincingly. We will see that it is inconsistent with the Authority Principle, however, and is thus untenable.

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17 De Finetti and Savage have argued for eliminativism along these lines.
The usual calculations, employed by those who believe in objective chances, involve a starting distribution $\Phi(p)$ over the possible values of the objective chance $p$, which is progressively refined by conditioning on the empirical data. After a long run of experiments, the distribution usually becomes strongly concentrated around a particular value of $p$. That value can then be used to predict future outcomes.

On the face of it, this calculation is unavailable to the eliminativist, as for him the distribution $\Phi(p)$ seems to have meaning. But, contrary to this, the eliminativist argues that the de Finetti theorem shows that his personal probability function can be expressed in exactly the same form as the objectivist’s. That is, there probably exists a function $\Phi(p)$, having the form of a probability distribution, and the eliminativist’s subjective probability that the next outcome will be $E$ is simply the definite integral of $p \times \Phi(p)$. Then, since his subjective probability is indistinguishable from the objectivist’s, it will change in just the same way when conditioned upon the experimental data. Thus the eliminativist will be able to ‘measure chances’, and also to make predictions on the basis of his observations.

The eliminativist’s argument seems to work by magic. How does a mere mathematical theorem allow one to dispense with all appeals to objective tendencies in the system? How can one predict, for example, that the Copper 62 will decay before the Copper 63 does, without ever making a claim about the physical nature of either isotope? We should examine the content of the de Finetti theorem very closely, to make sure we are not the victims of some sleight of hand.

The theorem assumes that the subject has a probability function that is exchangeable. I will not take the trouble to describe this assumption, or attack it, even though this is usually taken to be the weak point of the eliminativist’s argument. In my view, this line of attack is wholly misguided, since the assumption of exchangeability is perfectly sound. It is far more problematic to assume that the subject’s probability function takes precise values.

Recall that, according to the eliminativist, the demon probability of $E$ is the entire unit interval $(0,1)$. Thus, by the Authority Principle (AP), the eliminativist’s initial subjective probability of $E$ should also be $(0,1)$. Contrary to this, however, the de Finetti assumes that there is some precise degree of belief in $E$. Thus the theorem cannot be used, as its assumptions violate AP. In order to have a precise subjective probability for $E$, the eliminativist would have to know more than the demon.

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18 See, for example, Howson and Urbach (1993: 349-51).
The eliminativist might respond by saying that the precise-valued subjective probability function is tentative, and not to be taken too seriously. Instead, one might represent the subject’s epistemic state by a large class of such probability functions, so that the initial probability of E takes a wide range of different values. In this way, the eliminativist initially satisfies AP. The question then is whether inductive inference is possible, from such a class of initial probabilities. Here, unfortunately for the eliminativist, any success entails violation of AP. For suppose that, after 1000 trials are observed, and the prior probabilities are all conditioned in these outcomes, the posterior probabilities for E range between 0.2 and 0.7. Now, since past outcomes of the chance set-up are always irrelevant from the demon’s point of view, the demon probability of E is still (0,1), and so the subject is again in violation of AP.

We see that, for the eliminativist, the probability of E must always be (0,1), now matter how plentiful the empirical data about past outcomes may be. This is not a viable option for practising scientists.

There may seem to be a possible compromise position, that the demon probability is an interval, but one that is smaller than (0,1). This, however, is exactly the epistemic view. According to the epistemic view, only experiments can tell us how narrow an interval the chance is, for a given type of event.

8. The Propensity View

A propensity theory of chance, as I am using the term, is a theory according to which chances exist and are not degrees of belief. They are understood to be physical tendencies, or dispositions, for a physical system to produce certain events. Some such notion of a causal tendency or disposition is surely unavoidable when talking about stochastic systems, as we found it irresistible to say that Copper 62 has a greater tendency to decay than Copper 63. Also, I appealed to such tendencies when explaining the notion of a dynamical nature. The distinctive claim here is that these tendencies can be represented by precise numbers, which are not degrees of belief.

This view faces two main challenges: first, to explain how ‘tendencies’ can be measured numerically, and second to show that the Principal Principle holds of these numbers. I shall
argue that there is just one satisfactory numerical measure of causal tendency, namely the degree to which a set of causes determines a possible effect. This is precisely what an objective chance is, according to the epistemic view, but the propensity view cannot agree as it lacks the resources to define degrees of determination. A causal tendency, therefore, can only be understood as a degree of belief, and so cannot be used to explain the notion of propensity.

We saw in section 6 that demon probabilities are naturally viewed as degrees of determination by the three causes. This is surely a satisfactory way to define numerical ‘causal tendencies’, but is there no other?

The first attempt to define such objective, single-case propensities was due to Popper (1934). He defined the propensity of an event E within a chance set-up as the limiting relative frequency of E that would result, if the experiment were performed infinitely many times. This approach is beset with difficulties that are well known and quite fatal, however. First, there is no relative frequency that would (necessarily) occur, were the experiment repeated infinitely often. Second, even if such a relative frequency is known to exist, knowledge of it does not constrain one’s degree of belief in any way. Thus the propensity as defined does not exist; and even if it did exist, we would not be able to measure it experimentally. There are other frequency theories of propensity, but they all face essentially the same difficulties.

An alternative approach is to view these tendencies as degrees of causation. There are in fact two clear senses of ‘partial causation’, but neither of these is appropriate here. First, in a case where C and C’ together produced E, we can say that that C was part of the cause for E. This has nothing to do with causal tendency, however, as even the full cause of E might only have had a slight tendency to produce E. Second, we might say that C partially caused E when C caused E to occur partially. For example, an explosion might cause a building to collapse partially, or a dire warning might cause part of the class to study for the exam. Again, this has nothing to do with tendencies to cause, as the latter applies also to all-or-nothing events, like nuclear decay.

There is a third, rather murky, sense of partial causation. Suppose, for the sake of discussion, that there are uncaused events, i.e. events that are not produced by anything but just appear ‘from nowhere’. Those who believe in such events sometimes call them ‘spontaneous’. It is fairly clear that nuclear decays are not spontaneous, as in that case it would be hard to

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19 For a fuller presentation of these problems, see Johns (2002) 96-102.
explain why some isotopes decay more readily than others! The physical properties of the isotope, such as the numbers of protons and neutrons, surely play some role in producing the ejected particle. So chancy events are not purely spontaneous, but perhaps they are partially spontaneous? A chancy event might, perhaps, be 20% caused, and 80% spontaneous.

So some have suggested, but I must confess that I have not the slightest idea what this could mean. In such a case of 20% causation, we have some cause that is entirely responsible for producing the effect, as nothing else was involved. It was enough, by itself, to bring about the effect. Thus, I can see no difference between this case and one of 100% causation. Moreover, even if it is meaningful, such partial causation does not seem to be a candidate for analyzing causal tendencies. For suppose that C was 20% responsible for E, and nothing else helped to cause E at all. I see no connection between this and the idea that, once C occurs, there’s a 20% tendency for it to cause E.

To sum up, I cannot even find a suitable relation of ‘partial causation’, much less define numerical degrees thereof and show that they should constrain degrees of belief. This idea is completely unworkable.

Having exhausted all other known ways to define propensity, let us now turn to the idea of partial determination by the three causes. We have seen that the epistemic view regards chances as such degrees of determination, so the question now is whether the propensity view can do the same. Is it possible to make sense of degrees of determination, other than as degrees of belief?

To answer this question, we should first look at the meaning of the determination relation, from a non-epistemic viewpoint. A standard definition is that C determines E just in case in every dynamically-possible world where C occurs, E also occurs. There is no difficulty in generalizing this relation to include partial determination. One can say that C partially determines E just in case E occurs in some (but not all) dynamically-possible worlds where C occurs.

The problem here is to define numerical degrees of determination within this framework, and then show that these numbers are empirically measurable. One has to find a meaning for claims like ‘E occurs in 27.769% of the worlds in which C occurs’, which obviously requires the construction of a natural measure on the space of dynamically-possible worlds. What this measure might mean is anybody’s guess, however, and the possibility of deriving PP from such a
measure seems very remote indeed. Moreover, its construction would likely involve the use of symmetry arguments, which are the very thing that propensity theorists seem determined to avoid.

A further problem for the propensity view of chance is its difficulty in interpreting conditional chances. While one can, of course, calculate conditional propensities using the standard conditioning rule, one also has to say what these numbers mean. In doing so, the propensity view cannot appeal to any epistemic process, such as expansion of knowledge, but must find something purely physical.

To sum up, while we cannot say for certain that a non-epistemic account of degrees of determination is impossible, we can say that the prospects for such a theory are exceedingly dim. Also, we are bound to ask why such a view is needed, given that the epistemic account of them does everything that we need. As it stands, the propensity view of chance is rather like a vacant lot, an empty space where a theory may one day be constructed. Given the swampy, unstable nature of the ground, however, and the availability of other housing, one might ask why anything should be built there.

9. Empirical Consequences of the Epistemic View

We have seen that, on purely philosophical considerations, the epistemic view of chance is easily the strongest position of the three. In this section I will argue that the epistemic view has the additional virtue of being relevant to physics. The view suggests a new type of experiment to investigate the nature of chance, and offers a new framework for making sense of quantum mechanics.

As noted above, the epistemic view of chance gives us little *a priori* reason to expect that chances will be precise values, or even narrow intervals. While the epistemic view allows the possibility of such precise chances, it is only the empirical evidence that reveals their existence. Now, while chances are generally thought to take precise values, on the basis of experimental data, it is clear that these data are also consistent with the chance being some small interval. Since the measurement of chances is always imprecise and fallible in any case, a failure to achieve the expected degree of convergence could easily be attributed to other causes, such as
uncontrolled variations in the initial state, or interaction with the environment. Moreover, since at present no one is expecting to see wide-interval chances, an experiment that revealed them might well be dismissed as a failure and ignored.

If the notion of interval chances is taken seriously, however, then one can design an experiment specifically to look for them. Under the hypothesis that a precise chance exists, one can predict not only that convergence to the chance will occur in the limit, but also (fallibly) predict that it will occur at a certain rate. After 10,000 trials, for example, one may predict that fluctuations in the observed relative frequency will be less than (say) 0.01 from then on. If an interval chance exists, then one can predict that the relative frequency will probably be inside that interval, after a certain number of trials, but cannot say anything more than that. While convergence to a precise value within the interval is not ruled out, it is not predicted either. Therefore, if one observes fluctuations in the relative frequency that stubbornly refuse to drop below a certain magnitude, regardless of how many trials are conducted, then this should be seen as evidence for an interval chance in that experiment.

If such an experiment were performed, and sloppy convergence were observed, would it be conclusive proof of the epistemic view? Of course it would not. Apart from the possibility of experimental error, it is always conceivable that a propensity theory of chance might predict the same result. At present there is no propensity theory of chance, so it is not at all clear what such a theory might predict. Yet, since interval probabilities have a natural epistemic interpretation, the epistemic view of chance would be significantly strengthened.

If no such sloppy convergence were ever observed, across a wide variety of experiments, then this would count against the epistemic view of chance. It would not refute the view, however, since the epistemic view is consistent with such evidence.

The second empirical virtue of the epistemic view is that it defines a new framework for understanding quantum mechanics. The task of tracing out the consequences of the epistemic view for quantum theory is very large and difficult, but the upshot seems to be something similar to the Copenhagen interpretation, though a good deal more precise. This interpretation, developed mainly by Niels Bohr, was dubbed the ‘epistemological viewpoint’ by Schrödinger, due to the quantum state vector being regarded as representing what can be predicted about the

20 For a first attempt at this, see Johns (2002: 148-232).
results of measurements on the system, from maximal information about it. At the same time, however, the state vector has physical characteristics, as it has a deterministic equation of motion, and leads to empirical predictions.

On the epistemic view, chance is itself a curious physical/epistemic hybrid, being a rational degree of belief based on maximal knowledge of certain physical facts. Now, in quantum theory, physical chance is closely related to the state vector, so one might well expect the state vector also to be such a hybrid. Roughly speaking, one can see quantum mechanics as a description of the physical world through chances, in something like the way geometry describes the world through distances.

As an illustration of this epistemic interpretation of quantum mechanics, let us briefly see what it says about quantum correlation, or entanglement. It is easy to show that, within the epistemic view of chance, a pair of systems is entangled just in case maximal knowledge of them fails to factorise. That is, the maximal proposition describing the present state of the joint system cannot be expressed as a conjunction of propositions, one describing each separate system. It seems quite possible that the maximal information might fail to factorise in this way, even if the systems are not presently interacting. In that case, if separate experiments are performed involving the each system, the outcome of one experiment can allow a prediction of the other. The experimental result on one system does not physically alter the other system, but merely allows a more definite prediction of its experimental outcome. Thus, the epistemic view can account for the observed statistics in EPR-type experiments, even in the absence of non-local interactions.

It seems very unlikely that any propensity theory of chance could match this achievement. For, in EPR experiments, conditioning the chance function on one experimental outcome alters the chance of the other outcome, sometimes even changing it to zero or one. The whole idea of conditioning a propensity function is fraught with difficulty, as one cannot think of it as representing an expansion of knowledge. If, on the other hand, conditioning represents some sort of physical development, then the induced change in the propensities of events in the (possibly distant) other system constitutes non-local causation.
10. Conclusion

The proposed experimental test of the epistemic view illustrates, rather vividly, how vague and insubstantial the propensity view is. After all, while at first it might seem that physical propensities could not be intervals, that such intervals are clearly epistemic, one soon realises that the same could easily be said about precise-value chances. Since a propensity is just a something-I-know-not-what, which seems to be authoritative for belief, why could it not be an interval? (The Principal Principle could easily be generalized to include this case.) The propensity view thus reveals itself to be an empty as if theory. Chances behave exactly as if they were rational, authoritative degrees of belief, but they are not. We cannot say anything at all about chances, except that they seem to be degrees of belief, but in fact are not.

The eliminativist view of chance, on the other hand, is even less promising. It violates the Authority Principle.

We are left, therefore, with the epistemic view of chance, which commits us to some logical probabilities, and to some sort of Aristotelian ‘dynamical nature’, but nothing worse than that. This view allows us to make sense of all the standard properties of chance, and may help us to solve some very obstinate problems in theoretical physics.
Bibliography


