What I’m calling “Subjective Logic” is a new approach to logic. Fundamentally it is a theory about what sentences mean, i.e. a theory of the proposition, but it includes an account of logical consequence, the propositional connectives, probability, and the nature of truth.

This theory, I believe, helps to clarify of a number of important philosophical issues.

1. Subjective and Objective Propositions

So what do sentences mean, according to subjective logic? They actually have two different kinds of meaning – one at the level of truth, or the real world, and another at the level of belief, or the epistemic state. I call these the objective meaning, and the subjective meaning, respectively. Philosopher in the analytic tradition have focused almost exclusively on the objective meaning of a sentence, so I am using the term ‘subjective logic’ (perhaps unwisely) for the position that recognises both kinds of meaning. In this talk I will be dealing just with subjective meaning.

Why do we need two kinds of proposition? We are forced to this position, I believe, by virtually the same reasoning that led Frege to distinguish between Sinn and Bedeutung (sense and meaning). Indeed, I see my distinction between objective and subjective propositions as a modification of Frege’s Sinn/Bedeutung distinction.

We all know the story about Hesperus, the evening star, and Phosphorus, the morning star. At one time they were believed to be two different objects, but it was later discovered that they are in fact the same planet (which we now call Venus). Now consider an ancient astronomer, Ralph, who is living prior to the discovery of the identity of Hesperus and Phosphorus. He writes down the following pair of sentences.

Hesperus has a moon
Phosphorus has a moon
Do they have the same meaning, or two different meanings? Do they express the same proposition, or two different propositions?

We now know that Hesperus and Phosphorus are the same planet (Venus), so the sentences have the same truth conditions. They represent the same possible state of affairs. But for Ralph, our ancient astronomer who (in the absence of evidence) does not believe that Hesperus is Phosphorus, they express different beliefs. He might say, for example, that he thinks it unlikely there’s any moon orbiting Hesperus (having failed to see one). Yet he wouldn’t know about Phosphorus, as he never gets up early enough to see it.

Most discussions of this problem attempt to make sense of it without introducing subjective propositions. Russell’s theory that names abbreviate definite descriptions is the most obvious example. None is as compelling as the observation that:

*For the ancient astronomer (in his epistemic state) Hesperus and Phosphorus are two different planets.*

Taking this idea seriously requires that we admit two kinds of sentence meaning: the subjective meaning (in the epistemic state) and the objective meaning (in the real world). Since Hesperus and Phosphorus are distinct planets, in Ralph’s epistemic state, the two sentences have different subjective meanings. Since Hesperus and Phosphorus are the same real planet, the two sentences have the same objective meaning.

Another way to think about this problem is in terms of what it is for two sentences to be synonymous, or express the same proposition. Two different criteria have been proposed, as shown in the table below.

<table>
<thead>
<tr>
<th>Objective Criterion</th>
<th>Subjective Criterion</th>
</tr>
</thead>
<tbody>
<tr>
<td>Truth conditions</td>
<td>Belief conditions</td>
</tr>
<tr>
<td>Two sentences express the same proposition iff it is not possible for one to be true and the other false.</td>
<td>Two sentences express the same proposition iff it is not possible for one to be believed more strongly than the other.</td>
</tr>
</tbody>
</table>

For the two sentences above, we see that they have the same truth conditions, but different belief conditions (for Ralph), so that these two criteria appear to be inconsistent. Rather than reject one of them, however, we should regard them as talking about two different kinds of proposition.

### 2. Other Logical Notions

The distinction in logic between the objective and the subjective applies not just to propositions, but to all logical concepts. For example, the relation of logical consequence or entailment between propositions also has objective and subjective flavours. What is logical consequence?
It is generally held to be a matter of necessary *truth* preservation. This objective concept is actually of limited application, however. In most contexts, we really should be talking about rational *belief* preservation.

The idea of an *object* as the meaning of a singular term has objective and subjective flavours. So does the concept of *probability*. There isn’t time today to discuss each of these, but this table provides a summary.

<table>
<thead>
<tr>
<th>Objective Flavour</th>
<th>Subjective Flavour</th>
</tr>
</thead>
<tbody>
<tr>
<td>Real world</td>
<td>Epistemic state</td>
</tr>
<tr>
<td>Truth</td>
<td>Belief</td>
</tr>
<tr>
<td>Possible state of affairs</td>
<td>Subjective proposition (thought)</td>
</tr>
<tr>
<td>Metaphysically-necessary implication</td>
<td>Logical entailment</td>
</tr>
<tr>
<td>Real object</td>
<td>Epistemic object</td>
</tr>
<tr>
<td>Physical chance</td>
<td>Epistemic probability</td>
</tr>
</tbody>
</table>

3. **Subjective Propositions as Epistemic Expansions**

What exactly are subjective propositions? Let us be guided by H. P. Grice here. Grice held that, in uttering a sentence S, the speaker intends that the audience come to believe some proposition. This proposition is said to be the meaning of S.

We can use this idea, although it will be convenient to change the terminology a little, borrowing a term from belief dynamics. An *expansion* is a change of epistemic state that occurs when one learns something new, or “acquires a new belief”. Beliefs that are *certain*, having subjective probability one, are preserved under expansion. Expansion is the one kind of belief change that is possible for perfectly rational thinkers – the ideal of “cumulative” knowledge gain. Other kinds of belief change involve loss of information, mistakes, or correction of earlier mistakes.

A sentence such as “the cat is on the mat” is usually uttered with the intention to cause the audience’s epistemic state to undergo an expansion. We may (rather crudely) picture this expansion as below.
In the initial epistemic state the position of the cat is indeterminate. It could be on the mat, or on the chair, or hiding under the table. (These degrees of belief are shown in the picture by the degrees of opacity in the cat.) In the expanded epistemic state, by contrast, the cat is definitely on the mat and nowhere else.

An epistemic state is sometimes viewed as a set of propositions, so that an expansion is simply the addition of an extra proposition to the set. This view has difficulty with sentences like “Vulcan does not exist”. (Vulcan was a planet once postulated to account for the perihelion shift in Mercury’s orbit. It proved hard to observe, and the perihelion shift was later explained by General Relativity.) The intended expansion for “Vulcan does not exist” presumably is something like this:
The speaker intends the listener to expunge the notional object Vulcan from his epistemic state. It makes little sense to see the expanded epistemic state as a set that includes ‘Vulcan does not exist’ among its members, since the name ‘Vulcan’ in this state is meaningless. (The sentence becomes meaningless during its acceptance.) Rather, the meaning of the sentence seems to be the expansion itself. Propositions, on this alternative view, are “information vectors” from one epistemic state to another.

(I realise that my treatment of this Vulcan example departs significantly from that proposed by Russell and Quine, but I think this one is better as it allows natural languages to contain singular terms. It’s a virtue of subjective logic that this treatment becomes possible.)

It is a mistake, in my view, to regard an epistemic state as a set of propositions, or as a probability function over a set of propositions, or indeed as any kind of construction using propositions as raw material. An epistemic state is not like that. It is more like an internal, cognitive map of the world. It is the world according to some person, at some time.

Let us pursue this map analogy, and suppose someone is trying to draw a map based on your verbal description of the territory. It will be very hard to do this if they start with a blank sheet of paper, so let’s suppose that a few basic landmarks (such as the coastline) are already in place. One’s descriptions will then be sentences like: “Hope is 150 km east of Vancouver along the
Highway 1”. Each description will enable more features to be drawn, based on what’s already there. After the map is complete, it is impossible to derive from it the set of descriptions used to draw it. The map shows Kamloops as being 355km east of Vancouver, for example, but this might not have been among the descriptions. Instead, Hope’s position might have been given as 150 km east of Vancouver, and Kamloops’s as 205 km east of Hope.

In a similar way, while epistemic states “contain” propositions in some sense, it is not in the crude sense of set membership.

It is worth mentioning that this account of subjective propositions, as epistemic expansions, allows sentences of fiction to have subjective meaning. (Such sentences clearly have no truth conditions, and do not describe possible states of affairs.) We humans have the capacity of imagination, that allows us to build quite complex epistemic states that we do not actually inhabit. When reading a novel, for example, the author guides us in a series of expansions that gradually construct such a fictional epistemic state. The later sentences of the novel have meaning in that epistemic state. Fictional and serious epistemic states are often mixed together. Sherlock Holmes, for example, is a fictional character who lives in a real city.

3. Presuppositions

The view that (subjective) propositions are epistemic expansions helps us to see that most sentences presuppose a certain amount of knowledge. “The Loch Ness Monster weighs over 40 tons”, for example, presupposes that there is some sort of pre-historic creature living in Loch Ness. An epistemic expansion, of course, is an expansion from some state of knowledge to another, and generally the initial state is not empty. A proposition, in general, merely “tops up” the knowledge we already have. I sometimes call these relative propositions.

The view that a proposition is picked out by the class of possible worlds where it is true only applies to propositions with no presuppositions. The initial epistemic state, in this case, is completely empty of knowledge. [Like a “position vector”.]

4. Logical consequence

Logical consequence is usually defined in terms of preserving truth from premiss to conclusion. This account does not work well outside of mathematics, however, as the following example shows.

- Hesperus has a moon

∴ Phosphorus has a moon
The argument is invalid, as conclusion is not a logical consequence of the premiss, yet the argument is necessarily truth preserving. (I assume, following Kripke, that true identities are metaphysically necessary.)

Also, in some valid arguments the premisses and conclusion do not all have truth values, as they concern fictional characters. Consider, for example, the following argument involving Tolkien’s fictional character Aragorn. This argument is valid, even though the second premiss and the conclusion lack truth values.

\[
\begin{align*}
\text{All humans are mortal} \\
\text{Aragorn is human} \\
\therefore \text{Aragorn is mortal}
\end{align*}
\]

The following definition deals with such examples.

**Definition** The expansion of \( K \) by \( A \) is written \( K + A \).

**Definition** \( A \) entails \( B \ relative to \( K \) iff \( K + A \) is an expansion of \( K + B \).

**Definition** \( A \) entails \( B \ simpliciter \) if and only if, for all \( K \) such that \( K + A \) and \( K + B \) exist, \( K + A \) is an expansion of \( K + B \).

The rough idea here is that a valid inference is one that is made by a rational thinker. It can be shown that entailment in this sense is always truth preserving, but the converse does not hold. (A truth-preserving argument may be invalid, as shown by the example above.)

Consistency is also easy to define in terms of expansion. \( A \) is consistent with \( B \) if and only if there is some proposition that entails them both.

**5. The Connectives**

The truth-functional connectives are usually defined using truth tables, but these are theoretically burdened (requiring possible worlds) and of limited scope (applying only to propositions without presuppositions). Better definitions are possible within subjective logic, namely:

*Conjunction* of \( A, B \): the weakest proposition that entails \( A \) and entails \( B \) (total information).

*Disjunction* of \( A, B \): the strongest proposition that is entailed by \( A \) and entailed by \( B \) (information overlap).

*Negation* of \( A \): the weakest proposition that is inconsistent with \( A \). (minimum denial)
These can be applied to propositions with presuppositions as well as those without. Note that the negation of A, for example, shares all the presuppositions of A. The negation of:

The Loch Ness Monster weighs over 40 tons

is

The Loch Ness Monster weighs no more than 40 tons.

Thus the law of excluded middle holds only for propositions that make no assumptions.

\textbf{If A then B}: This sentence does not express a proposition. It is believed, in an epistemic state \( K \), to the extent that B is believed in \( K+A \). (This is Frank Ramsey’s theory of the conditional.) One consequence of this reading is Ernest Adams’ claim that \( \Pr_K(\text{If } A \text{ then } B) = \Pr_K(B \mid A) \), where \( \Pr_K \) gives the degrees of belief for propositions in the epistemic state \( K \).

\section*{6. Philosophical Uses of Subjective Logic}

I have barely begun to work out the uses of epistemic logic in philosophy, but here are a few things.

1. We can understand truth as correspondence between the epistemic state and the real world. We can define a true proposition as one that is entailed by (or consistent with) the actual world, and prove that logical entailment is truth preserving.

2. The debate about whether meanings are “in the head” reveals itself as being rather silly. Subjective meanings are in the head, but objective meanings are not.

3. Epistemic probability theory becomes part of logic.

4. We can understand the analytic/synthetic distinction. A proposition is analytic just in case it is believed with certainty in every (rational) epistemic state, and synthetic otherwise.

5. We can understand what it means to deny the existence of a thing, using a proper name for it, e.g. “The planet Vulcan does not exist.” The sentence is uttered with the intention that the listener expunge the notional object Vulcan from his epistemic state.