

Interest Rate and the Exchange Rate: A Non-Monotonic Tale

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Abstract

For over 30 years, the empirical international finance literature has been unable to detect a clear systematic relationship between interest rates and the nominal exchange rate. We take a fresh look at the data and uncover a new stylized fact for a cross-section of countries: the relationship between the exchange rate and short-term interest rates is *non-monotonic*. Small increases in the nominal interest rate appreciate the currency, whereas larger increases depreciate the currency. We develop a model that explains this stylized fact based on the interaction of three effects. Higher interest rates increase money demand and hence appreciate the currency but also raise the fiscal deficit and depress output, both of which tend to depreciate the currency. We provide cross-country evidence for the presence of these effects in the data.

JEL Classification: F3, F4

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1 Introduction

The relationship between interest rates and exchange rates has long been a key focus of international economics. Most standard theoretical models of nominal exchange rates predict that exchange rates are determined by economic fundamentals. One such fundamental is the interest rate differential between home and abroad. Even in textbook models, however, the relation between interest rates and the nominal exchange rate can differ greatly. In the simplest monetary model of the exchange rate, for example, an increase in the home interest rate will be associated with a fall in real money demand and, hence, a *depreciation* of the domestic currency. In contrast, in a Dornbush-type, sticky prices model, an increase in the home interest rate (in response to a reduction in the level of the money supply) will *appreciate* the currency. Perhaps reflecting this tension between flexible and sticky prices models, the data has not provided an unambiguous answer either. Indeed, a slew of studies that have examined the time series relationship between interest rates and the nominal exchange rate tend to either find conflicting results that depend on the sample of countries and/or the time period studied, or tend to find insignificant results.¹

The absence of a clear empirical relationship between interest rates and the exchange rate is even more problematic from the perspective of practitioners. A short-term interest rate is the typical policy instrument used by policymakers to affect currency values (and monetary conditions more generally).² If there is no clear relationship in the data, then why do policymakers persist in using the interest rate instrument to affect exchange rates? Are the typical time series empirical results camouflaging some key features of this relationship?

It is important to note at the very outset that this issue of the effect of interest rates on the level of the exchange rate is distinct from issues surrounding the uncovered interest parity (UIP) condition. UIP is an arbitrage relationship between the *level* of the interest rate and the rate of *change* of the exchange rate. There is a voluminous literature which attempts to document

¹Amongst studies that have directly examined the relationship between interest rates and exchange rates, Eichenbaum and Evans (1995) find that for the G7 countries interest rate innovations tend to appreciate the currency. On the other hand, Calvo and Reinhart (2002) find that for developing countries there is no systematic relationship between the two variables. A more general empirical result is that fundamentals (including interest rate differentials) are poor predictors of exchange rates. Specifically, random-walk exchange rate forecasting models usually outperform fundamentals based models. In other words, most models do not explain exchange rates movements (see Meese (1990)). This fact is highlighted in Obstfeld and Rogoff (2000) who call it the “exchange rate disconnect puzzle”.

²We should note that, in the aftermath of the Asian crisis in the late 1990s, there was a contentious debate regarding the soundness of the International Monetary Fund’s (IMF) advice to affected countries of raising interest rates to stabilize the rapidly depreciating domestic currencies. IMF critics (like Joseph Stiglitz and Jeffrey Sachs) were of the view that this policy may not even work and, even if it did, its costs would be just too high. In fact, a large empirical literature on the topic has failed to unearth a systematic effect of higher interest on the currency values during the crisis period in the affected Asian economies (see Kraay (2003)).

and/or explain the failure of this relationship (the “forward premium anomaly”).³ Instead, our main focus is on the effects of policy-induced changes in the *level* of the interest rate on the *level* of the exchange rate, which is a very different exercise and focus. Our study is thus more in the spirit of the pioneering work of Meese and Rogoff (1983) who tested various models of exchange rate determination for their statistical fit to the data.⁴

We start our paper by taking another look at the data but with a new and different perspective. In contrast to most of the empirical tests that focus on the time series dimension of the relationship between the levels of the exchange rate and interest rates, we examine the relationship along the cross-sectional dimension. Our cross-sectional approach is motivated by our interest in the long run relationship between policy-induced changes in the interest rate and the effects on the nominal exchange rate. In order to abstract from issues surrounding short run rigidities (nominal or real) which may complicate the relationship between the two, we choose instead to focus on the long run.⁵ We identify long run effects using the cross-sectional variation across countries in the long run averages of the two variables.

Using data from a cross-country sample of 80 countries during periods when their exchange rates were flexible, we show that the relationship between long run averages of the exchange rate and interest rates is non-monotonic. Specifically, we find that exchange rates tend to appreciate with higher policy-controlled interest rates but only till some point. Beyond a certain point, further increases in interest rates start depreciating the exchange rate. This is a new stylized fact which, in a sense, is a cross-sectional cousin of the mixed evidence in the time series relationship between interest rates and the exchange rate.

We then develop a simple monetary model to argue that there is no theoretical reason to expect a monotonic relationship between interest rates and exchange rates. In fact, our model predicts that the relationship between interest rates and exchange rates is inherently non-monotonic. We show that it takes surprisingly small modifications of an otherwise standard optimizing monetary model to generate non-monotonicities. Hence, assuming that our model captures some essential features of the real world (as we will argue), it follows that there is nothing surprising in the fact that the empirical literature has failed to unearth a systematic monotonic relationship between

³See Engel (2011) for a recent analysis. In fact in the model that we formalize later in the paper, interest parity holds for internationally traded bonds. Hence, almost by construction, we do not shed any new light on the observed deviations from *UIP* since that is not the object of our interest.

⁴Some recent papers that also share our focus on examining the effects of policy controlled interest rates (via Taylor rules) on exchange rates are Engel and West (2006) and Molodtsova and Papell (2009), among others.

⁵Put differently, we are interested in a horizon that is long enough for short-run rigidities not to matter since we will be thinking in terms of a model with fully-flexible prices and no other rigidities.

interest rates and exchange rates, since there may be none to begin with.

In building our model, we take as a starting point the fact that most analysts seem to agree that higher interest rates affect key macroeconomic variables essentially through three channels. First, higher interest rates raise the demand for domestic-currency denominated assets thus leading, all else equal, to an appreciation of the currency. Henceforth, we shall refer to this as the “money demand effect”. Second, higher domestic interest rates induce a contraction in domestic output through a credit channel. All else equal, this effect tends to depreciate the currency. We shall refer to this channel as the “output effect”. Third, an increase in interest rates increases the debt service burden of the fiscal authority. This in turn increases inflationary expectations and, hence, weakens the currency. We shall call this the “fiscal effect”.⁶

We proceed to construct a model that captures these three effects. We incorporate the money demand effect by allowing for an independent interest rate instrument. Following Calvo and Vegh (1995), interest rate policy is essentially modeled as the central bank’s ability to pay interest on part of the money supply. In particular, we assume that households hold interest-bearing money in the form of bank deposits. Commercial banks, in turn, hold as part of their portfolio non-tradable government bonds (“domestic bonds”) issued by the central bank. In this set-up, the interest rate on these non-tradable bonds (hereafter referred to as the *policy-controlled* interest rate), is an *additional* policy instrument.⁷ Raising this policy-controlled interest rate leads, all else equal, to higher money demand (the money demand effect).

The model also embodies a credit channel à la Bernanke and Blinder (1988, 1992). The presence of this credit channel implies that higher interest rate on domestic bonds crowds out the supply of bank credit to the private sector and causes an output contraction (the output effect). Finally, the model incorporates the fiscal effect by assuming an exogenous path for government spending. This implies that higher interest rates on domestic bonds also have fiscal costs since they lead to a higher debt service burden, which requires a higher inflation rate.

For a given supply of money and a given inflation rate, the positive money demand effect of higher interest rates leads to an appreciated domestic currency. However, both the fiscal and output channels tend to depreciate the currency. The fiscal cost of higher interest rates on domestic bonds

⁶Indeed, the specific margins we introduce here reflect some of the concerns and arguments made by observers during the Asian crisis regarding the efficacy of using interest rates to defend against speculative attacks. The arguments at that time often centered around the positive effects of interest rate defense on the demand for domestic currency denominated assets (argued by Stanley Fisher representing the IMF) and the deleterious effects of higher interest rates on domestic output and inflation (argued by people like Joseph Stiglitz and Jeffrey Sachs).

⁷The nominal interest rate on tradable bonds will be referred to as the *market* interest rate.

implies that the inflation rate needs to be higher to finance the exogenous level of government spending. This tends to increase the market interest rate, thus raising the opportunity cost of holding money and depreciating the currency. In a similar vein, for a given level of deposits, the fall in bank credit associated with the output effect implies that banks will lend more to the government. This higher stock of government liabilities also requires, all else equal, a higher inflation rate. In this set-up, therefore, whether an increase in the policy-controlled interest rate appreciates the currency or not will depend on the interaction between these three effects.

Our main results indicate that the relationship between nominal interest rates and the exchange rate is inherently non-monotonic. In particular, the relation between the *policy-controlled* interest rate and the exchange rate is U-shaped. This implies that, up to a certain point, an increase in the policy-controlled interest rate will indeed appreciate the domestic currency. Beyond that point, however, further increases of the policy-controlled interest rate will actually begin to depreciate the currency.

While the theoretical possibility of non-monotonicities in the relationship between interest rates and exchange rates is interesting, could they actually arise in a realistically parameterized model? Is there any data evidence of the existence of the key channels formalized in the model? We address this issue by conducting two exercises. First, we calibrate the model to Argentinian data for the period 1983-2002 and show that the steady state response of the nominal exchange rate to increases in the policy-controlled interest rate is non-monotonic in the calibrated model, just as in the cross-sectional data. Second, we use the cross-country data to present evidence in support of the fiscal, output and money demand effects which are the three key mechanisms embedded in the model.

The paper proceeds as follows: the next section presents the empirical evidence regarding the non-monotonic relationship between interest rates and the exchange rate. Sections 3 and 4 develop the model and derive the main results. Section 5 shows that the simulated version of the model calibrated to Argentinian data produces a non-monotonic relationship between interest rates and the exchange rate. Section 6 provides empirical evidence from a cross-section of countries in support of the key mechanisms underlying the model. Section 7 contains concluding remarks.

2 Some new facts

We start by presenting some new evidence on the relationship between interest rates and nominal exchange rates. In a departure from the standard time series orientation of empirical tests of this

relationship, we focus on the cross-sectional aspect by examining the averages of interest rates and exchange rates in a cross-section of countries during periods when their exchange rates were flexible.

We use annual data during 1974-2009 for a large cross-section of countries. Most of the data series are from International Financial Statistics (IFS) compiled by the International Monetary Fund (IMF) and from the World Development Indicators (WDI) compiled by the World Bank. Details about the data and sources are provided in the Appendix A. Exchange rate series are period average official exchange rates. If the official rate was not available for a given country, then we used period average market rate. If that series was not available either, we used the period average principal rate or the commercial rate.

To construct the policy-controlled interest rate series we used the period average T-bill rate. If the T-bill rate was not available, we used the discount rate. In case the latter was missing, we used the money market rate for the country. We should point out that for most countries the rate used is the T-Bill rate. We use this as a proxy for the policy rate since overnight interbank lending rates (which are perhaps closer to the policy rate) are not available for a number of countries.⁸

Given our interest in determining the response of exchange rates to policy shocks, we restrict the sample to only those countries and time periods that are characterized by a flexible exchange rate regime. To perform the selection, we rely on the Reinhart and Rogoff (2004) classification of historical exchange rate regimes. We classify a country as having a flexible exchange rate regime if, in a given year, its exchange rate was either (i) within a moving band that is narrower than or equal to $\pm 2\%$ (i.e., allows for both appreciation and depreciation over time); or (ii) was classified as managed floating; or (iii) was classified as freely floating; or (iv) was classified as freely falling according to Reinhart and Rogoff (2004). These correspond to their fine classification indices of 11, 12, 13, and 14, respectively.⁹ We only focus on the post-Bretton Woods period for all countries. High income OECD countries are included in our sample, irrespective of their

⁸The T-bill rate is the closest to the policy-controlled rate that we can get for a wide sample of countries. Since our interest is in determining the effects of policy-induced changes in interest rates on the exchange rate, it is important that the T-bill rate move very closely with the policy rate. For most countries that we could get the policy rate explicitly, the correlation between the policy rate and the T-bill rate is very high. For example, for the US the correlation between the Federal Funds rate and the 3-month T-bill rate is 0.984; for Mexico the correlation between the Cetes 91 days rate and the Interbank Equilibrium Interest Rate (TIIE) is 0.992; for South Africa the correlation between the 91 day T-bill rate and the repo rate is 0.9917.

⁹We also considered the coarse exchange rate classification of Reinhart and Rogoff (2004) to select countries and episodes into the sample. We found the results to be robust with respect to the classification. The coarse classification included countries that were on (i) pre announced crawling band that is wider than or equal to $\pm 2\%$; (ii) de facto crawling band that is narrower than or equal to $\pm 5\%$; (iii) moving band that is narrower than or equal to $\pm 2\%$; (iv) managed floating; (v) freely floating; (vi) freely falling. These correspond to indices 3, 4, and 5 in Reinhart and Rogoff (2004).

exchange rate classification.¹⁰ The selection leaves us with a sample of 80 countries, and 88 country-episode pairs. These country-episode pairs are listed in the Appendix A. The average duration of an episode in our sample is 11 years, although this differs significantly across developed and developing countries. Thus, for developed countries the average duration of an episode is 23 years, while for developing countries it is 6 years.

The *levels* of exchange rates and interest rates differ dramatically across countries. To examine the relationship between the levels of the two variables across countries, we need to normalize them to a common base. To do so, for every country we compute the averages for the interest rate and the exchange rate during each flexible regime episode. We then scale these averages by the initial level of the corresponding variable.¹¹

The scatter plot of the normalized (log) exchange rate and interest rate for our sample of countries is presented in Figure 1. The solid line in Panel (a) presents the smoothed value of log exchange rate for every value of the interest rate. To obtain the smoothed values we relied on locally weighted scatterplot smoothing (lowess) technique proposed by Cleveland (1979). This method is robust to the presence of large outliers as it downweights their effect on the smoother.¹²

Briefly, each point of the solid line is obtained by fitting a weighted least squares regression for each value of x_i (interest rate) using the observations in the neighborhood of that point. We chose about 37.5% of the data on each side of the x_i to be used in the estimation of the fit around that point. The fitted model has a constant and a linear term (running line) and it assigns larger weight to the points close to x_i , while those farther away receive less weight. Then the robust smooth is obtained by applying Cleveland’s tricube weighting function.¹³ The shaded area around the smoothed line is a 90% pointwise confidence interval.

Panel (a) of Figure 1 reveals a U-shaped relationship between the average long run levels of interest rates and exchange rates. In particular, the figure shows that the correlation between

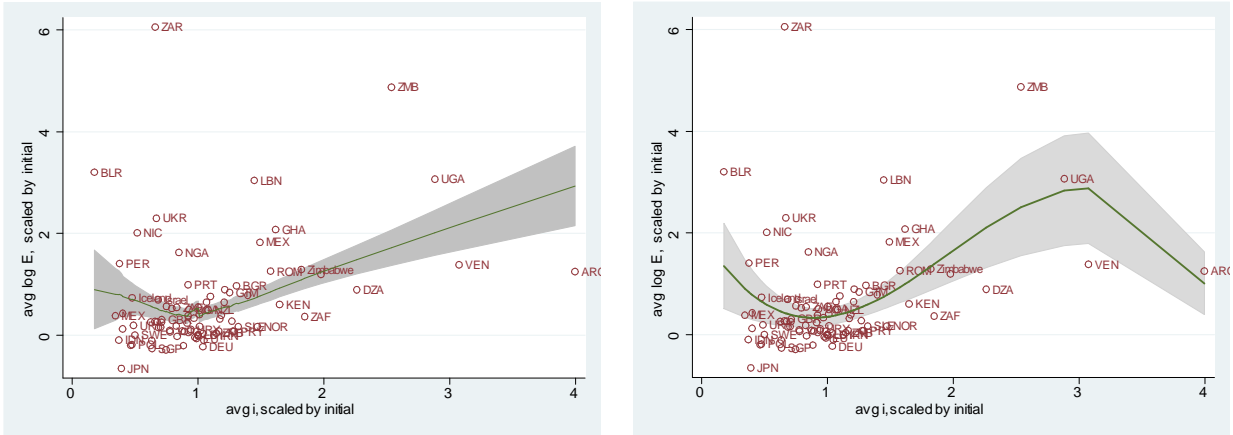
¹⁰The current Eurozone countries were included as separate episodes before joining the Euro. From 1999 Eurozone is included as a separate episode.

¹¹One might suspect that since we normalize all the exchange rate levels by the initial level of the exchange rate during that flexible rate episode, the average level and the rate of change of the exchange rate reveal the same information. That is not so. The rate of depreciation of the exchange rate is distinct from the average level of the exchange rate in our specification above. To see it, let $e = \ln E$ where E denotes the nominal exchange rate. Then, the average normalized level during a flexible rate episode lasting T periods is $\bar{e} = \left(\frac{1}{T} \sum_{t=1}^T e_t\right) - e_0$ while the average rate of depreciation during the episode is $\bar{\varepsilon} = \frac{1}{T} (e_T - e_0)$.

¹²The advantage of using the scatterplot smoothing technique to fit the data lies in its non-parametric treatment of the data. That is, it does not impose any parametric restrictions on the relationship between interest rates and the exchange rate. In this way, it allows the data to “speak for itself”.

¹³See Cleveland and McGill (1984) for detailed discussion of the method, and its advantages relative to alternative methods.

Figure 1: Exchange Rate-Interest Rate Relationship, 1974-2009



Panel (a). Non-parametric (lowess) fit

Panel (b). Cubic regression fit

Notes: The figures plot average interest rate against average log exchange rate for a sample of 88 country-episodes during 1974-2009 period, with each point obtained over a flexible exchange rate period. For each episode, both interest rate and exchange rate are scaled by their respective initial levels in the episode. Panel (a) presents the non-parametric fit of the data using lowess technique; Panel (b) presents the results from the fit of the 3rd order polynomial in interest rate to the (log) exchange rate.

interest rates and exchange rates is negative on the left arm of the solid line, while it is positive on the right arm of the solid line. Consider, for instance, Iceland whose interest rate-exchange rate combination is located on the left arm of the solid line. The figure suggests that if Iceland raised its interest rates by a small amount, it would look like Israel, which has lower (more appreciated) exchange rate relative to Iceland. However, if Iceland raised its interest rate by a large amount, it would start looking more like Zimbabwe – with higher (depreciated) exchange rate.

We assess the robustness of our findings above using a parametric approach where we regress average (log) exchange rate on a high-order polynomial of the average interest rate. Both variables are normalized using their initial values, as before. To allow sufficient flexibility in the relationship between interest rate and the exchange rate, we begin our analysis with the polynomial of order 5. In particular, we estimate the following cross-sectional regression:

$$y_k = \beta_0 + \beta_1(i/i_0)_k + \beta_2(i/i_0)_k^2 + \beta_3(i/i_0)_k^3 + \beta_4(i/i_0)_k^4 + \beta_5(i/i_0)_k^5 + \varepsilon_k, \quad (2.1)$$

where ε_k is the regression error term and y_k is the average (log) exchange rate, scaled by its initial level. We used i to denote average nominal interest rate and i_0 – its initial level for the country-episode pair k .

Table 1 presents results from estimating equation (2.1) with average log exchange rate normal-

ized by its initial level as the dependent variable using a cross-section of 88 country-episode pairs comprising our sample. We start with the regression specification in equation (2.1), but sequentially drop higher-order terms in the interest rate if they are insignificant. This approach left us with a 3rd-order polynomial.

Table 1: Regressions underlying polynomial fit results

	$\ln(E/E_0)$
i/i_0	-4.1149*** (1.5531)
$(i/i_0)^2$	2.9820*** (0.9853)
$(i/i_0)^3$	-0.5040*** (0.1639)
<i>constant</i>	1.9878*** (0.7414)
N	88
R^2	0.235

Notes: This Table presents the results from a cross-section regression of the average (log) exchange rate on average interest rate and its quadratic and cubic transformation. Interest rate and exchange rate variables are normalized by their initial levels. Robust standard errors are in parenthesis. * p-value \leq 0.10, ** p-value \leq 0.05, *** p-value \leq 0.01.

Panel (b) of Figure 1 summarizes our findings by plotting the predicted values from this regression together with the 90% confidence interval. It shows that the relationship between interest rate and the exchange rate is U-shaped for most of the interest rate range, thus confirming the non-monotonicity results from our non-parametric analysis.¹⁴

To the best of our knowledge, ours is the first paper to document empirically these non-monotonicities in the relationship between the nominal exchange rate and interest rates. The results suggest that there is no simple linear relationship between the two variables in the data. The natural question that it raises is “why”? We turn to this issue next.

3 The model

We would like to understand the reasons behind the non-monotonic relationship between interest rates and the nominal exchange rate that we found in the data. In order to do so, we now develop a relatively standard, open economy monetary model where we can study the relationship between the two variables. In constructing our model, we incorporate three features of interest rate innovations

¹⁴The right tail is somewhat noisy due to a very small number of observations there.

that are often considered important. In particular, we consider an environment in which higher policy interest rates raise the fiscal burden on the government (the fiscal effect), have a negative effect on output (the output effect) and raise the demand for domestic currency assets (the money demand effect). We will show that the interaction between these three effects is key to understanding the empirical facts.

Consider a representative household model of a small open economy that is perfectly integrated with the rest of the world in both goods and capital markets. The infinitely-lived household receives utility from consuming a (non-storable) good and disutility from supplying labor. The world price of the good in terms of foreign currency is fixed and normalized to unity. Free goods mobility across borders implies that the law of one price applies. The consumer can also trade freely in perfectly competitive world capital markets by buying and selling real bonds which are denominated in terms of the good and pay r units of the good as interest at every point in time. We assume that there is no uncertainty so that everything is known and deterministic.

3.1 Households

Household's lifetime welfare is given by

$$V = \sum_{t=0}^{\infty} \beta^t U(c_t, x_t), \quad (3.2)$$

where c denotes consumption, x denotes labor supply, and $\beta (> 0)$ is the exogenous and constant rate of time preference. We assume that the period utility function of the representative household is given by

$$U(c, x) = \frac{1}{1 - \sigma} (c - \zeta x^\nu)^{1 - \sigma}, \quad \zeta > 0, \quad \nu > 1.$$

Here σ is the intertemporal elasticity of substitution, $\nu - 1$ is the inverse of the elasticity of labor supply with respect to the real wage. These preferences are well-known from the work of Greenwood, Hercowitz, and Huffman (1988), which we will refer to as GHH.

Households use cash, H , and nominal demand deposits, D , for reducing transactions costs. Specifically, the transactions costs technology is given by

$$s_t = v \left(\frac{H_t}{P_t} \right) + \psi \left(\frac{D_t}{P_t} \right), \quad (3.3)$$

where P is the nominal price of goods in the economy, and s denotes the non-negative transactions

costs incurred by the consumer. Let h ($= H/P$) denote cash and let d ($= D/P$) denote interest-bearing demand deposits in real terms. We assume that the transactions technology is strictly convex. In particular, the functions $v(h)$ and $\psi(d)$, defined for $h \in [0, \bar{h}]$, $\bar{h} > 0$, and $d \in [0, \bar{d}]$, $\bar{d} > 0$, respectively, satisfy the following properties:

$$\begin{aligned} v &\geq 0, & v' &\leq 0, & v'' &> 0, & v'(\bar{h}) &= v(\bar{h}) = 0, \\ \psi &\geq 0, & \psi' &\leq 0, & \psi'' &> 0, & \psi'(\bar{d}) &= \psi(\bar{d}) = 0. \end{aligned}$$

Thus, additional cash and demand deposits lower transactions costs but at a decreasing rate. The assumption that $v'(\bar{h}) = \psi'(\bar{d}) = 0$ ensures that the consumer can be satiated with real money balances.

In addition to the two liquid assets, households also hold a real internationally-traded bond, b . The households flow budget constraint in nominal terms is

$$\begin{aligned} &P_t b_{t+1} + D_t + H_t + P_t (c_t + s_t) \\ = &P_t \left(Rb_t + w_t x_t + \tau_t + \Omega_t^f + \Omega_t^b \right) + \left(1 + i_t^d \right) D_{t-1} + H_{t-1}. \end{aligned}$$

Foreign bonds are denominated in terms of the good and pay the gross interest factor $R(= 1 + r)$, which is constant over time.¹⁵ i_t^d denotes the deposit rate contracted in period $t - 1$ and paid in period t . w denotes the wage rate. τ denotes lump-sum transfers received from the government. Ω^f and Ω^b represent dividends from firms and banks respectively. In real terms the flow budget constraint facing the representative household is thus given by

$$\begin{aligned} &b_{t+1} + h_t + d_t + c_t + s_t \\ = &Rb_t + w_t x_t + \tau_t + \frac{h_{t-1}}{1 + \pi_t} + \left(\frac{1 + i_t^d}{1 + \pi_t} \right) d_{t-1} + \Omega_t^f + \Omega_t^b, \end{aligned} \tag{3.4}$$

where $1 + \pi_t = \frac{P_t}{P_{t-1}}$ denotes the gross rate of inflation between periods $t - 1$ and t . It will often prove convenient to work in terms of the nominal interest which we define as

$$1 + i_{t+1} = R(1 + \pi_{t+1}). \tag{3.5}$$

¹⁵Note that denominating foreign bonds in terms of foreign currency or goods is equivalent here since the law of one price holds and the world foreign currency price of the good is constant by assumption.

Households maximize their lifetime welfare equation (3.2) subject to equations (3.3) and (3.4).

3.2 Firms

The representative firm in this economy produces the perishable good using labor

$$y_t = Al_t, \quad (3.6)$$

where l denotes labor demand and A is a constant. At the beginning of the period, firms rent labor. However, a fraction ϕ of the total wage bill needs to be paid upfront to workers. Since output is only realized at the end of the period, firms finance this payment through loans from banks. The loan amount along with the interest is paid back to banks next period.¹⁶ Formally, this constraint is given by

$$N_t = \phi P_t w_t l_t, \quad \phi > 0, \quad (3.7)$$

where N denotes the nominal value of bank loans. The assumption that firms must use bank credit to pay the wage bill is needed to generate a demand for bank loans.

The firm's flow constraint in nominal terms is given by

$$P_t b_{t+1}^f - N_t = P_t \left(R b_t^f + y_t - w_t l_t - \Omega_t^f \right) - \left(1 + i_t^l \right) N_{t-1},$$

where i^l is the lending rate charged by bank for their loans and Ω^f denotes dividends paid out by the firms to their shareholders. b^f denotes foreign bonds held by firms which pay the going world interest factor R . In real terms the flow constraint reduces to

$$b_{t+1}^f - n_t = R b_t^f - \left(\frac{1 + i_t^l}{1 + \pi_t} \right) n_{t-1} + y_t - w_t l_t - \Omega_t^f.$$

Define

$$a_{t+1}^f \equiv b_{t+1}^f - \frac{(1 + i_{t+1}^l)}{R(1 + \pi_{t+1})} n_t.$$

Substituting this expression together with the credit-in-advance constraint into the firm's flow

¹⁶Alternatively, we could assume that bank credit is an input in the production function, in which case the derived demand for credit would be interest rate elastic. This would considerably complicate the model without adding any additional insights.

constraint in real terms gives

$$a_{t+1}^f + \Omega_t^f = Ra_t^f + y_t - w_t l_t \left[1 + \phi \left\{ \frac{1 + i_{t+1}^l - R(1 + \pi_{t+1})}{R(1 + \pi_{t+1})} \right\} \right]. \quad (3.8)$$

Note that $\phi \left\{ \frac{1 + i_{t+1}^l - R(1 + \pi_{t+1})}{R(1 + \pi_{t+1})} \right\} w_t l_t = \left\{ \frac{1 + i_{t+1}^l - R(1 + \pi_{t+1})}{R(1 + \pi_{t+1})} \right\} n_t$ is the additional resource cost that is incurred by firms due to the credit-in-advance constraint.¹⁷

The firm chooses a path of l to maximize the present discounted value of dividends subject to equations (3.6), (3.7) and (3.8). Given that households own the firms, this formulation is equivalent to the firm using the household's discount factor to optimize. The first order conditions for this problem are given by a static condition equating the marginal product of labor to the wage rate which is adjusted by the additional cost of credit. This is proportional to the difference between the nominal lending rate and the nominal interest rate. Firms also have an Euler equation for assets which is identical to the household's Euler equation.

3.3 Banks

The banking sector is assumed to be perfectly competitive. The representative bank holds foreign real debt, d^b , accepts deposits from consumers and lends to both firms, N , and the government in the form of domestic government bonds, Z .¹⁸ It also holds required cash reserves, θD , where $\theta > 0$ is the reserve-requirement ratio imposed on the representative bank by the central bank. Banks face a cost q (in real terms) of managing their portfolio of foreign assets. Moreover, we assume that banks also face a constant proportional cost ϕ^n per unit of loans to firms. This is intended to capture the fact that domestic loans to private firms are potentially special as banks need to spend additional resources in monitoring loans to private firms.¹⁹ The nominal flow constraint for

¹⁷We should note that the credit-in-advance constraint given by equation (3.7) holds as an equality only along paths where the lending spread $1 + i^l - R(1 + \pi)$ is strictly positive. We will assume that if the lending spread is zero, this constraint also holds with equality.

¹⁸Commercial bank lending to governments is particularly common in developing countries. Government debt is held not only as compulsory (and remunerated) reserve requirements but also voluntarily due to the lack of profitable investment opportunities in crisis-prone countries. This phenomenon was so pervasive in some Latin American countries during the 1980's that Rodriguez (1991) aptly refers to such governments as "borrowers of first resort". For evidence, see Rodriguez (1991) and Druck and Garibaldi (2000).

¹⁹We should note that this cost ϕ^n is needed solely for numerical reasons since, as will become clear below, it gives us a bigger range of policy-controlled interest rates to experiment with. Qualitatively, all our results would go through with $\phi^n = 0$.

the bank is

$$N_t + Z_t - (1 - \theta) D_t + P_t q_t - P_t d_{t+1}^b = \left(1 + i_t^l - \phi^n\right) N_{t-1} + (1 + i_t^g) Z_{t-1} - \left(1 + i_t^d\right) D_{t-1} + \theta D_{t-1} - P_t R d_t^b - P_t \Omega_t^b, \quad (3.9)$$

where i^g is the interest rate on government bonds. We assume that banking costs are a convex function of the foreign debt held by the bank:

$$q_t = q\left(d_{t+1}^b\right), \quad q' > 0, \quad q'' > 0,$$

where q' denotes the derivative of the function q with respect to its argument, while q'' denotes the second derivative. The costly banking assumption is needed to break the interest parity condition between domestic and foreign bonds. Throughout the paper we assume that the banking cost technology is given by the quadratic function:

$$q_t = \frac{\gamma}{2} \left(d_{t+1}^b - \bar{d}^b\right)^2, \quad (3.10)$$

where $\gamma > 0$ and \bar{d}^b are constant parameters.²⁰

Deflating the nominal flow constraint by the price level gives the bank's flow constraint in real terms:

$$\Omega_t^b = \left[\frac{R(1 + \pi_t) - 1}{1 + \pi_t}\right] [(1 - \theta) d_{t-1} - n_{t-1} - z_{t-1}] + \frac{(i_t^l - \phi^n)}{1 + \pi_t} n_{t-1} + \frac{i_t^g}{1 + \pi_t} z_{t-1} - \frac{i_t^d}{1 + \pi_t} d_{t-1} - q_t, \quad (3.11)$$

where we have used the bank's balance sheet identity: $P_t d_{t+1}^b = N_t + Z_t - (1 - \theta) D_t$. Note that this is equivalent to setting the bank's net worth to zero at all times. Also, the quadratic specification for banking costs along with the zero net worth assumption implies that these banking costs can also be reinterpreted as a cost of managing the portfolio of net domestic assets since $d_{t+1}^b = \frac{N_t + Z_t - (1 - \theta) D_t}{P_t}$.

The representative bank chooses sequences of N , Z , and D to maximize the present discounted value of profits subject to equations (3.9) taking as given the paths for interest rates i^l , i^d , i^g , i , and the value of θ and ϕ^n . We assume that the bank uses the household's stochastic discount factor to value its profits. Note that i_{t+1}^g , i_{t+1}^l and i_{t+1}^d are all part of the information set of the household

²⁰Similar treatment of banking costs of managing assets and liabilities can be found in Diaz-Gimenez, Prescott, Fitzgerald, and Alvarez (1992) and Edwards and Vegh (1997). This approach to breaking the interest parity condition is similar in spirit to Calvo and Vegh (1995).

at time t .

The bank optimality conditions imply that we must have

$$i_{t+1}^l = i_{t+1}^g + \phi^n, \quad (3.12)$$

$$i_{t+1}^d = (1 - \theta) i_{t+1}^g. \quad (3.13)$$

These conditions are intuitive. Loans to firms and loans to the government are perfect substitutes from the perspective of commercial banks up to the constant extra marginal cost ϕ^n of monitoring loans to private firms. Hence, equation (3.12) says that the interest rate charged by banks on private loans should equal the rate on loans to the government plus ϕ^n . For every unit of deposits held the representative bank has to pay i^d as interest. The bank can earn i^g by lending out the deposit. However, it has to retain a fraction θ of deposits as required reserves. Hence, equation (3.13) shows that at an optimum the deposit rate must equal the interest on government bonds net of the resource cost of holding required reserves. We should note that the parameter ϕ^n plays no role in the theoretical results that we derive below. Hence, in our main propositions we set $\phi^n = 0$. This parameter is useful in the quantitative sections later where it allows us to calibrate some key interest rate spreads.

It is instructive to note that as the marginal banking costs becomes larger the bank will choose to lower its holdings of foreign assets. This can be checked from the bank first order conditions; all of them imply that $\lim_{\gamma \rightarrow \infty} d_{t+1}^b = 0$. Hence, in the limit as banking costs becomes prohibitively large, the bank will choose to economize by shifting to a closed banking sector with no external assets or liabilities.

3.4 Government

The government issues high powered money, M , and domestic bonds, Z , makes lump-sum transfers, τ , to the public, and sets the reserve requirement ratio, θ , on deposits. Domestic bonds are interest bearing and pay i^g per unit. Since we are focusing on flexible exchange rates, we assume with no loss of generality that the central bank's holdings of international reserves are zero. We assume that the government's transfers to the private sector are fixed exogenously at $\bar{\tau}$ for all t . Hence, the consolidated government's nominal flow constraint is

$$P_t \bar{\tau} + (1 + i_t^g) Z_{t-1} = M_t - M_{t-1} + Z_t.$$

As indicated by the left-hand-side of this expression, total expenditures consist of lump-sum transfers, debt redemption and debt service. These expenditures may be financed by issuing either high powered money or bonds. In real terms the government's flow constraint reduces to

$$\bar{\tau} + \left(\frac{1 + i_t^g}{1 + \pi_t} \right) z_{t-1} = m_t + z_t - \frac{m_{t-1}}{1 + \pi_t}. \quad (3.14)$$

Lastly, the rate of growth of the nominal money supply is given by:

$$\frac{M_t}{M_{t-1}} = 1 + \mu_t, \quad M_0 \text{ given.} \quad (3.15)$$

It is worth noting that from the central bank's balance sheet the money base in the economy is given by

$$M_t = H_t + \theta D_t.$$

Hence, M can also be interpreted as the level of nominal domestic credit in the economy.

The consolidated government (both the fiscal and monetary authorities) has three policy instruments: (a) monetary policy which entails setting the rate of growth of nominal money supply, μ ; (b) interest rate policy which involves setting i^g (or alternatively, setting the composition of m and z and letting i^g be market determined); and (c) the level of lump sum transfers to the private sector τ . Given that lump-sum transfers are exogenously-given, only one of the other two instruments can be chosen freely while the second gets determined through the government's flow constraint (equation (3.14)). Since the focus of this paper is on the effects of interest rate policy, we shall assume throughout that i^g is an actively chosen policy instrument. This implies that the rate of money growth μ adjusts endogenously so that equation (3.14) is satisfied.

3.5 Resource constraint

By combining the flow constraints for the consumer, the firm, the bank, and the government (equations (3.4), (3.8), (3.11) and (3.14)) and using equations (3.6) and (3.7), we get the economy's flow resource constraint:

$$a_{t+1} = Ra_t + y_t - c_t - s_t - q_t, \quad (3.16)$$

where $a = b + b^f - d^b$. Note that the right hand side of equation (3.16) is simply the current account.²¹

3.6 Equilibrium relations

We start by defining an equilibrium for this model economy. The three exogenous variables in the economy are the productivity process A and the two policy variables \bar{r} and i^g . We denote the entire state history of the economy till date t by $s^t = (s_0, s_1, s_2, \dots, s_t)$. An equilibrium for this economy is defined as:

Given a sequence of realizations $A(s^t)$, $i^g(s^t)$, r and \bar{r} , an equilibrium is a sequence of state contingent allocations $\{c(s^t), x(s^t), l(s^t), h(s^t), d(s^t), b(s^t), b^f(s^t), d^b(s^t), n(s^t), z(s^t)\}$ and prices $\{P(s^t), \pi(s^t), i(s^t), i^d(s^t), i^l(s^t), w(s^t)\}$ such that (a) at the prices the allocations solve the problems faced by households, firms and banks; (b) factor markets clear; and (c) the government budget constraint (equation (3.14)) is satisfied.

3.7 Exchange rate determination

Before proceeding further it is useful to illustrate at this point how the exchange rate gets determined in the model. There are two sources of liquidity in the model— cash and demand deposits. Hence, the money base is $M = H + \theta D$. Since the demand for real cash balances is a function of the nominal interest rate i and the demand for real deposits is a function of both i and i^d , we can define a notional aggregate real demand for money balances to be $m_t = L(i_t, i_t^d)$. From the bank optimality conditions we know that $i_t^d = (1 - \theta) i_t^g$. Clearly, if we knew i_t and i_t^g then we would know $m_t = \frac{M_t}{E_t}$. If further we knew M_t then we would also know the equilibrium E_t . But how does one determine i_t ? For that we go to the government budget constraint. Combining the government flow constraint with the central and commercial bank balance sheets yields the combined government flow constraint:

$$\bar{r} = h_t - \left(\frac{1}{1 + \pi_t} \right) h_{t-1} + \theta \left(d_t - \frac{d_{t-1}}{1 + \pi_t} \right) + z_t - \left(\frac{1 + i_t^g}{1 + \pi_t} \right) z_{t-1}. \quad (3.17)$$

²¹It is useful to note that consumption and labor supply both get affected by changes in i^g . Labor supply gets affected due to changes in employment induced by changes in the lending spread. Consumption too changes in response to changes in i^g due to changes in the transactions costs faced by households. Recall that changes in i^g change both i^d and i which alter the demand for cash and deposits. This changes the transactions costs s which, from equation (3.16), alter the consumable resources available to the economy. Indeed, changes in s induce changes in marginal utility itself due to a wealth effect.

This equation has one unknown i_t . For any given i_t^g equation (3.17) gives the equilibrium i_t . This, along with M_t , then gives the equilibrium E_t . Note that in this model the stock of money at any date t is predetermined but its growth for the next period is endogenous.

A casual reading of these relationships might suggest that the model implications for the exchange rate and the price level are identical since the law of one price holds and there is only one traded good in the model. Hence, the predictions for the exchange rate would also apply to the price level. While this is trivially true in this model, it would not be so under very small modifications of the basic structure of the model. As an example, introducing a non-traded good into the model while continuing to retain the flexible exchange rate assumption would immediately break the tight link between the exchange rate and the price level even though the predictions for the exchange rate in such an augmented model would remain exactly the same as here. Clearly, the model's predictions for the exchange rate are not robustly isomorphic to the price level to even small perturbations in the model environment. Hence, we do not pursue the predictions of the model for the price level in this paper and believe that it would be misguided to do so.

4 Exchange rates and interest rate policy: Analytical results

In order to build intuition about the workings of this model, in this section we derive some analytical results. In particular, we shall focus on stationary, perfect foresight environments in which both the policy-controlled interest rate, i^g , and government transfers, $\bar{\tau}$, are constant for all t . Throughout this section, we shall set the productivity parameter $A = 1$. Further we set the bank cost of making loans to firms to zero, i.e., $\phi^n = 0$.

4.1 Perfect foresight stationary equilibrium

We first derive the perfect foresight equilibrium path. Under perfect foresight, the first order conditions for optimal cash and demand deposits imply that $h_t = \tilde{h} \left(\frac{i_{t+1}}{1+i_{t+1}} \right)$ and $d_t = \tilde{d} \left(\frac{i_{t+1} - i_{t+1}^d}{1+i_{t+1}} \right)$. We also know that $i_t^d = (1 - \theta) i_t^g$ and $i_t^l = i_t^g$. Moreover, the first order condition for bank loans to the government implies an equilibrium relationship between deposits and bank loans to the government:

$$z_t = (1 - \theta) d_t - n_t + \left(\frac{i_{t+1} - r}{\gamma(1 + i_{t+1})} \right). \quad (4.18)$$

where we have used equation (3.5) to substitute out for $R(1 + \pi_{t+1})$.

Using the relationships above it is easy to see the government flow constraint (see equation

(3.17)) becomes a first-order difference equation in i . In the following we shall use $\eta_h \equiv -\frac{\tilde{h}'}{h} \frac{i}{1+i}$ to denote the absolute value of the interest elasticity of cash, $\eta_d \equiv -\frac{\tilde{d}'}{d} \frac{i-i^d}{1+i}$ for (the absolute value of) the opportunity-cost elasticity of demand deposits, and $\eta_n \equiv -\frac{\tilde{n}'}{n} \frac{i'-i}{1+i}$ to denote the corresponding elasticity of loans to firms. In order to economize on notation we shall also use $I^d = i - i^d$ and $I^g = i^g - i$ to denote the interest spreads on deposits and loans respectively.

It is easy to check that, in a local neighborhood of the steady state, $\left. \frac{di_{t+1}}{di_t} \right|_{i_t=i_{t+1}} > 1$, i.e., the government flow constraint is an unstable difference equation in i , if and only if

$$\begin{aligned} & Rh \left[1 - \left(\frac{i-r}{i} \right) \frac{\eta_h}{R} \right] + (1+i^d) Rd \left[1 - \left\{ 1 - \left(\frac{r(1+i^d)}{I^d} \right) \right\} \frac{\eta_d}{R} \right] \\ > & (1+i^g) Rn \left[1 - \left\{ 1 + \left(\frac{r(1+i^g)}{I^g} \right) \right\} \frac{\eta_n}{R} \right] + \frac{(r-i^g) I^g}{\gamma(1+i)} \left(1 + \frac{(1+2r)(1+i^g)}{I^g} \right). \end{aligned} \quad (4.19)$$

To ensure a unique convergent perfect foresight equilibrium path we shall restrict attention to parameter ranges for which the stability condition holds.²² As long as this condition is satisfied, along any perfect foresight equilibrium path with constant \bar{r} and i^g , i will also be constant over time.

A constant i and i^g imply that π , $i - i^d$ and $i^g - i$ must all be constant over time. In conjunction with the first order conditions for households, firms and banks, these results imply that consumption, c , employment, x , output, y , cash demand, h , deposit demand, d , and demand for bonds, z , must all remain constant as well. The constancy of both h and d implies that money demand is constant over time. Lastly, the stationary level of government transfers is given by

$$\bar{\tau} = \left(\frac{i-r}{1+i} \right) (h + \theta d) - \left(\frac{I^g + r(1+i^g)}{1+i} \right) z. \quad (4.20)$$

Before proceeding further it is useful to note that the left hand side of equation (4.19) reflects the well-known possibility of a Laffer curve relationship between revenues from money printing and the opportunity cost of holding money. As is standard, and to ensure that the economy is always operating on the “correct” side of the Laffer curve, we will assume throughout that $h \left[1 - \left(\frac{i-r}{i} \right) \frac{\eta_h}{R} \right] + (1+i^d) d \left[1 - \left\{ 1 - \left(\frac{r(1+i^d)}{I^d} \right) \right\} \frac{\eta_d}{R} \right] > 0$.

In the rest of this section we shall make one additional assumption which will simplify the analytics greatly. In particular, we shall assume that $\gamma = \infty$. This corresponds to assuming that

²²Note that if instead $\left. \frac{di_{t+1}}{di_t} \right|_{i_t=i_{t+1}} < 1$ then the economy would exhibit equilibrium indeterminacy. There would be an infinite number of equilibrium paths all converging to the unique steady state.

the cost of managing a non-zero net foreign asset position is prohibitively high for domestic banks. The implication of this assumption is that commercial banks will set $d_t^b = 0$ for all t . Hence, the commercial bank balance sheet will reduce to $z_t + n_t = (1 - \theta) d_t$ for all t .

4.2 Two special cases

We now turn to the central issue of the paper – the effects of interest rate policy on the nominal exchange rate. We want to ask the following questions: how does the *level* of the nominal exchange rate change when the policy-controlled interest rate, i^g , changes? What is the relationship between i^g and the *rate* of exchange rate depreciation? Our goal is to demonstrate that in the model just described, there is an inherent tendency for these relationships to be non-monotonic. Since the nominal interest rate is given by $1 + i_t = R(1 + \pi_t)$, for the *rate* of currency depreciation to be non-monotonic in i^g we need i to be a non-monotonic function of i^g . For the *level* of the nominal exchange rate to have a similar non-monotonicity, m ($= h + \theta d$) must be a non-monotonic function of i^g . With nominal money supply at time 0 given, a rise in m will imply that the price level must decrease on impact, i.e., the nominal exchange rate appreciates. Similarly, a fall in m will be associated with a nominal depreciation on impact.²³

We also want to determine the minimal elements that are needed to generate such non-monotonicities. To this effect, we will show that (i) in the presence of only one money (interest-bearing deposits), both the fiscal and the output effects are needed to generate a non-monotonic relationship between interest rates and the nominal exchange rate, and (ii) in the presence of two monies (cash and deposits), the fiscal effect is all that is needed to generate a non-monotonic relationship. As will be discussed below, these two cases illustrate the general principle that two sources of fiscal revenues are needed to generate a non-monotonic relationship between interest rates and exchange rates. In (i), the two sources of revenues are demand deposits and, indirectly, bank lending to firms (through its effect on banking lending to the government), whereas in (ii) the two sources of revenues are the two monies.

²³We should reiterate that the rate of depreciation of the currency and the initial level of the exchange rate are distinct variables in this model despite its lack of any transition dynamics. Indeed, this feature is typical in most standard monetary models where the initial price level gets determined separately from the rate of inflation. Our model is an example of an environment where shocks affect these two variables differently.

4.2.1 Case 1: The one-money case

In this one-money case, higher interest rates have both an output effect and a fiscal effect. Formally, we set $v(h) \equiv 0$ for all h . Hence, the demand for cash is zero at all times (i.e., $\tilde{h} \left(\frac{i}{1+i} \right) \equiv 0$). This implies that the entire money base is held by the banking sector. In particular, $m = \theta d$. Hence, after setting $h = 0$ and $\gamma = \infty$, the stability condition (4.19) remains valid.²⁴

The relationship between the policy-controlled interest rate, i^g , and the nominal exchange rate, E , is summarized by the following proposition:

Proposition 4.1 *A permanent, unanticipated change in the policy-controlled interest rate, i^g , has a non-monotonic effect on the equilibrium nominal interest rate along three dimensions: (i) the initial level of the nominal exchange rate is falling or rising with i^g as $i^g \lesseqgtr \hat{i}^g$; (ii) the steady-state depreciation rate falls or rises with i^g as $i^g \lesseqgtr \bar{i}^g$ where $\bar{i}^g < \hat{i}^g$; and (iii) in the range $i^g \in (\bar{i}^g, \hat{i}^g)$, a rise in i^g appreciates the currency on impact but depreciates it at some point in the future.*

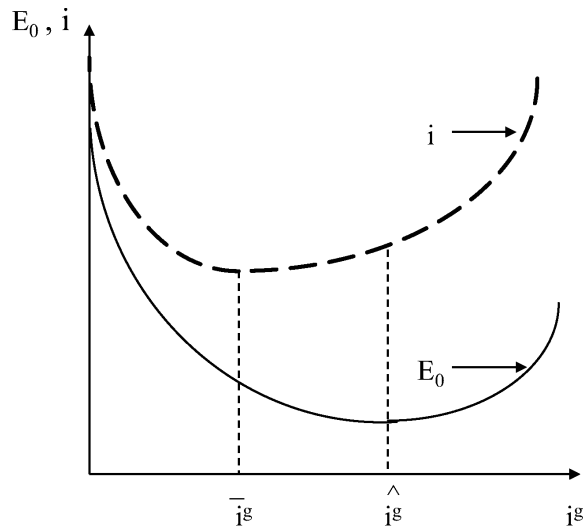
Proof. See appendix. ■

Figure 2 illustrates part (i) of this proposition. Specifically, the initial level of the exchange rate, E_0 , is a U-shaped function of the policy-controlled interest rate, i^g , with the minimum being reached at $i^g = \hat{i}^g$. The intuition is as follows. Recall that the opportunity cost of demand deposits is $\frac{I^d}{1+i} \equiv \frac{i-i^d}{1+i}$. A rise in i^g , in and of itself, increases the deposit rate, i^d – recall (3.13) – and therefore tends to reduce $\frac{I^d}{1+i}$ and appreciate the currency. A rising i^g , however, also raises I^g which induces a fall in bank credit to firms, n . This effect tends to reduce fiscal revenues because the counterpart of a falling n is an increase in z (i.e., an increase in liabilities of the central bank held by commercial banks), which increases the government’s debt service. In order to finance this fall in revenues, the inflation rate (i.e., the rate of depreciation) must increase. This effect tends to increase i and hence I^d . For all $i^g > \hat{i}^g$, the higher debt service overwhelms the increase in the deposit rate, and further increases in i^g actually raise I^d .

Figure 2 also illustrates part (ii) of this proposition, by showing that the market interest rate, i , is also a U-shaped function of i^g . For $i^g < \bar{i}^g$, the direct effect on revenues of an increase in i^g (due to a higher demand for real demand deposits) is so large that it facilitates a cut in the inflation tax. However, for $i^g > \bar{i}^g$ the indirect effect of a fall in n becomes large enough to require an increase in π (or equivalently, the rate of money growth μ) in order to finance fiscal spending.

²⁴Also notice that, in this particular case, the condition ensuring that the economy is operating on the correct side of the Laffer curve reduces to $1 - \left\{ 1 - \left(\frac{r(1+i^d)}{I^d} \right) \right\} \frac{\eta_d}{R} > 0$.

Figure 2: Exchange rate and the nominal interest rate



Aside from the non-monotonicity of both the initial level of the exchange rate and the steady-state depreciation rate, Proposition 4.1 also shows that an increase in the policy-controlled interest rate often induces an intertemporal trade-off in the path of the nominal exchange rate. In particular, the instantaneous appreciation in the level of the currency that is generated by a higher i^g is accompanied by a higher rate of currency depreciation (relative to the path with a lower stationary i^g). This occurs for values of $i^g \in (\bar{i}^g, \hat{i}^g)$ because in that range, as Figure 2 illustrates, a rise in i^g reduces E_0 but increases the rate of depreciation.

Finally, the results of Proposition 4.1 also imply that the relationship between the nominal exchange rate and the market interest rate is highly non-monotonic. This can be visualized using Figure 2. For “low” values of i^g (i.e., $i^g < \bar{i}^g$), both the initial exchange rate (E_0) and the market interest rate (i) fall (positive comovement). For “high” values of i^g (i.e., $i^g > \hat{i}^g$), both increase (also positive comovement). In contrast, for “intermediate” values of i^g , (i.e., $\bar{i}^g < i^g < \hat{i}^g$), E_0 falls while i increases, indicating a negative comovement.

4.2.2 Case 2: The two-money case

We now turn to our second special case of the general model. Here, we reintroduce a transactions role for cash so that $s_t = v(h_t) + \psi(d_t)$. Thus, this economy has two liquid assets – cash and demand deposits. However, we now assume that $\phi = 0$ so that loan demand by firms is zero (i.e., $\tilde{n}(I^g) \equiv 0$). Hence, there is no output effect of higher interest rates.

After setting $\tilde{n}(I^g) = 0$, the equilibrium transfer equation (4.20) continues to be valid. We

restrict parameter ranges so that the stability condition continues to hold in this case. Hence, the rate of devaluation remains constant along a convergent perfect foresight equilibrium path.²⁵

The interest elasticity of cash η_h is a function of the nominal interest rate i , which is an implicit function of the policy controlled interest rate i^g through the equilibrium transfer equation (4.20). Define \bar{i}^g by the relation $\eta_h \left(\frac{\bar{i}(\bar{i}^g, \bar{\tau})}{1 + \bar{i}(\bar{i}^g, \bar{\tau})} \right) = \frac{R\bar{i}(\bar{i}^g, \bar{\tau})}{\bar{i}(\bar{i}^g, \bar{\tau}) - r}$. We will now assume that the demands for cash and deposits satisfy the following conditions:

Condition 1: $\eta_h \left(\frac{\bar{i}(0, \bar{\tau})}{1 + \bar{i}(0, \bar{\tau})} \right) < \theta \eta_d \left(\frac{\bar{i}(0, \bar{\tau})}{1 + \bar{i}(0, \bar{\tau})} \right)$.

Condition 2: η_h and η_d are both increasing in their respective arguments.

Condition 1 requires that for “low” inflation rates (i.e., inflation rates corresponding to a non-activist interest rate policy), the interest elasticity of cash be lower than that of deposits (adjusted by the reserve requirement ratio). The idea is that cash is kept mainly for transactions and is therefore relatively interest inelastic for low inflation rates. This intuition is consistent with the evidence for the United States provided in Moore, Porter, and Small (1990).²⁶

We can now state the main proposition of this section:

Proposition 4.2 *Under Conditions 1 and 2, the initial nominal exchange rate is a non-monotonic (U-shaped) function of the policy-controlled interest rate, i^g . In particular there exists an $\hat{i}^g \in (0, \bar{i}^g)$ such that $\frac{\partial E_0}{\partial i^g} \leq 0$ as $i^g \leq \hat{i}^g$.*

Proof. See appendix. ■

This proposition shows that, as in the previous case, the initial level of the exchange rate is a U-shaped function of the policy-controlled interest rate. Intuitively, for low values of i^g , the positive money demand effect dominates the fiscal effect (i.e., the inflationary consequences of a higher i^g). Beyond a certain point, however, further increases in i^g have such a large impact on the rate of inflation that money demand begins to fall and hence the currency depreciates. The role of *Condition 1* is to ensure that, around $i^g = 0$, the demand for cash falls by less than the amount by which demand for bank deposits rises, so that overall real money demand increases.²⁷ Lastly, Proposition 4.2 also implies that, for $i^g < \bar{i}^g$, a rise in i^g appreciates the currency on impact but simultaneously increases the depreciation rate of the currency.

²⁵To be consistent with the one-money case, we will also maintain the assumption that $\left(1 - \frac{1^d - r}{1^d} \eta_d\right) d > 0$.

²⁶Condition 2 is satisfied by, among others, Cagan money demands, which provide the best fit for developing countries (see Easterly, Mauro, and Schmidt-Hebbel (1995)).

²⁷If condition 1 is not satisfied, then an increase in i^g would always lead to a *depreciation* of the currency.

5 Numerical verification

Could the theoretical possibility of a non-monotonic relationship between interest rates and the exchange rate actually emerge in a plausibly calibrated economy? In this section we address this issue by conducting policy experiments to examine the quantitative relevance of the central result of the paper: the relationship between interest rates and exchange rates may be non-monotonic. Towards this end we calibrate the model developed above and assess its quantitative relevance for understanding the relationship between interest rates and the exchange rate.

5.1 Calibration

We start by describing the calibration of the key model parameters. For our benchmark calibration we use the data for Argentina during 1983-2002. The calibration is such that one period in the model corresponds to one quarter.

Following Rebelo and Vegh (1995), we assume that the transactions costs functions $v(\cdot)$ and $\psi(\cdot)$ have quadratic forms given by

$$s_{\varkappa} \left(\varkappa^2 - \lambda \varkappa + \left(\frac{\lambda}{2} \right)^2 \right),$$

where \varkappa represents cash or demand deposits, $\varkappa = \{h, d\}$, while s_{\varkappa} and λ are the level parameters. This formulation implies that the demand for money components are finite and that transaction costs are zero when the nominal interest rate is zero.²⁸ The level coefficients in the transactions costs technology, s_h , s_d , and λ are set to 10, 5, and 0.4 respectively, in order to match the ratio of liquid liabilities (M3) to GDP in Argentina. Our calibration of the transaction costs parameters also ensures that the deposit elasticity for low i^g is greater than the elasticity of cash demand.

The transaction technology for the banks is given by a quadratic function

$$q_t = \frac{\gamma}{2} \left(d_{t+1}^b - \bar{d}^b \right)^2,$$

where $d_{t+1}^b = \frac{N_t + Z_t - (1-\theta)D_t}{P_t}$ and \bar{d}^b is a constant. We pick parameters γ and \bar{d}^b to match the private sector external debt to GDP ratio equal to 15% of GDP.²⁹ The steady state bond holdings

²⁸Notice as well that condition 2 is met.

²⁹During the time period under consideration, the external debt to GNI ratio was 48%. Of it, the share of private sector long-term debt was 15%, while the share of short-term external debt was around 16%. The data is from the World Development Indicators (WDI), compiled by the World Bank.

of households and firms in the model are not determined. We set them such that in combination with banks' foreign debt the economy-wide external debt to GDP ratio in the model is 48% – the observed ratio is Argentina.

Most of the remaining parameter values are taken from Neumeyer and Perri (2005). In particular, we set the coefficient of relative risk aversion, σ , to 5, while the curvature of the labor, v , is set to 1.6, which is within the range of values used in the literature.³⁰ This implies the elasticity of labor demand with respect to real wage, $\frac{1}{v-1}$, equal to 1.67, consistent with the estimates for the U.S. Labor weight parameter ζ in the utility function is chosen to match the average working time of 1/5 of total time and is set to 2.48. Subjective discount factor, β , is set to 0.97, as in Uribe and Yue (2006).

We calibrate the share of wage bill paid in advance, ϕ , to be equal to 0.16, which is chosen to match the ratio of domestic private business sector credit to GDP in Argentina over our sample period. The number for private credit to GDP in Argentina is borrowed from the Financial Structure Dataset assembled by Beck, Demirgüç-Kunt, and Levine (2000) and is equal to 0.21. The share of business sector credit in the total private sector credit is calculated using activity based financing reports from the Central Bank of Argentina, and is equal to 78% over our sample period.

Parameter θ determines the reserve requirement ratio in the model and is set to match its value of 0.4 in Argentina. This value is reported in Pou (2000) and Montoro and Moreno (2011). We also obtain the same number for reserve requirement ratio based on our own calculations using the data for various measures of money in Argentina. In particular, we compute reserve requirement ratio in two ways: (i) as the ratio of reserve money excluding cash to M2 excluding cash; and (ii) as the ratio of reserve money excluding cash to M1 excluding cash. In the first case we obtain a reserve requirement ratio of 0.09, while in the second case we get 0.70. We proxy reserve requirement ratio by the average of the two numbers, giving us the value of 0.4 for the benchmark calibration.³¹

To calibrate the average policy-controlled interest rate i^g , we use data on the money market rate in Argentina during 1991-2002 period. During this period the average level of i^g was 18%. The proportional cost parameter ϕ^n in the banking sector' problem is chosen to match the average spread of nominal lending rate over the market rate equal 4% in Argentina over our sample period. The lump-sum transfers paid by the government to the private sector, $\bar{\tau}$, is measured as the average (seasonally-adjusted) ratio of government consumption to GDP over our sample period, which gives

³⁰For example, Mendoza (1991) uses v equal to 1.455 for Canada, while Correia, Neves, and Rebelo (1995) set v to 1.7 for Portugal.

³¹Data on various measures of money is from the IFS.

$\bar{\tau} = 13\%$.³²

Table 2 summarizes parameter values under our benchmark parametrization.³³

Table 2: Benchmark parameter values

PREFERENCES		
discount factor	β	0.971
risk-aversion	σ	5
labor curvature	ν	1.6
labor weight	ζ	2.48
TECHNOLOGY		
share of wage-in-advance	ϕ	0.16
MONEY		
reserve requirement	θ	0.4
transaction cost technology	$\sum_{\varkappa=h,d} s_{\varkappa}(\varkappa^2 - \lambda\varkappa + (\lambda/2)^2)$	$s_h = 10, s_d = 5, \lambda = 0.4$
banks cost technology	γ	8
OTHER		
policy interest rate	i^g	18%
lump-sum transfers	$\bar{\tau}$	13% of GDP

5.2 Results

We analyze the equilibrium properties of the model through the steady state comparative statics in which we compare the steady state implications of the model for different levels of policy-controlled interest rate. Three points regarding measurement are worth mentioning before we report the results. First, to maintain comparability with the data results we reported in Section 2, we normalize the level of the exchange rate at the initial date ($t = 0$) to one. Hence, exchange rates at all other dates are measured relative to the date 0 level. Second, nominal variables in the model are non-stationary since there is nominal money growth in steady state. We use the standard method of deflating the nominal exchange rate by the nominal money stock in order to introduce stationarity in levels. All results involving *levels* of the exchange rate use these normalized, stationary exchange rates.³⁴ Third, since both nominal money and the nominal exchange rate grow at the same rate along a steady state equilibrium path, it follows that $\frac{E_t}{M_t} = \frac{E_0}{M_0}$ as the model does not exhibit any transition dynamics. Hence, the average level of the normalized exchange rate is also the initial

³²The data on government consumption is from Neumeyer and Perri (2005).

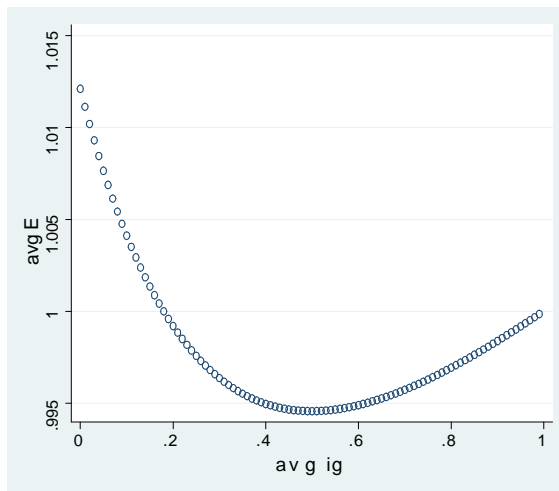
³³For all the experiments reported below we confirmed that the implied inflation tax revenues are on the upward sloping portion of the Laffer-curve, and the stability condition holds.

³⁴Without normalizing the nominal exchange rate one would not be able to compute averages of levels in this infinite horizon model. Fortunately, the normalization here is straightforward since the source of the non-stationarity in the model is well understood.

level of the normalized exchange rate.

Figure 3 presents the results of our experiments. It shows the response of the average level of the exchange rate (starting from an initial steady state) to changes in the steady state level of the policy-controlled interest rate, i^g . Each point on the graph shows the exchange rate that is associated in equilibrium with the corresponding i^g . In terms of mapping this graph back to the data pictures we showed in Section 2, it is helpful to associate each i^g with a different country, i.e., the graph depicts a cross-sectional feature of the model by comparing the effects of different steady state i^g 's on the exchange rate. The graph reveals a non-monotonic relationship between i^g and the exchange rate. Small increases in i^g appreciate the currency; more aggressive increases in i^g , on the other hand, depreciate the currency.

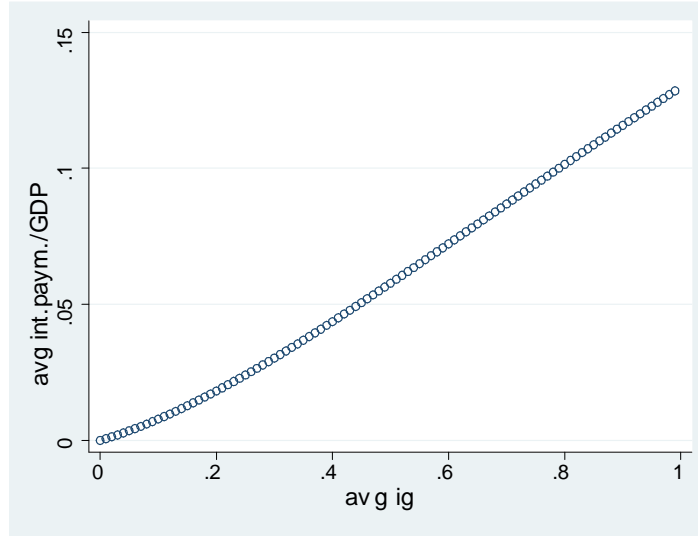
Figure 3: Steady state comparisons: Exchange rate level



The intuition for these results can be gained by examining the three effects that an increase in i^g has in our model. First, a higher i^g creates inflationary pressure on the government by increasing the interest burden on the outstanding government debt. Figure 4 illustrates this effect in the model by plotting average interest payments on government debt as a share of GDP against interest rate i^g . This “fiscal” effect raises the required seigniorage revenues and tends to increase the rate of currency depreciation and thus the nominal interest rate, i .

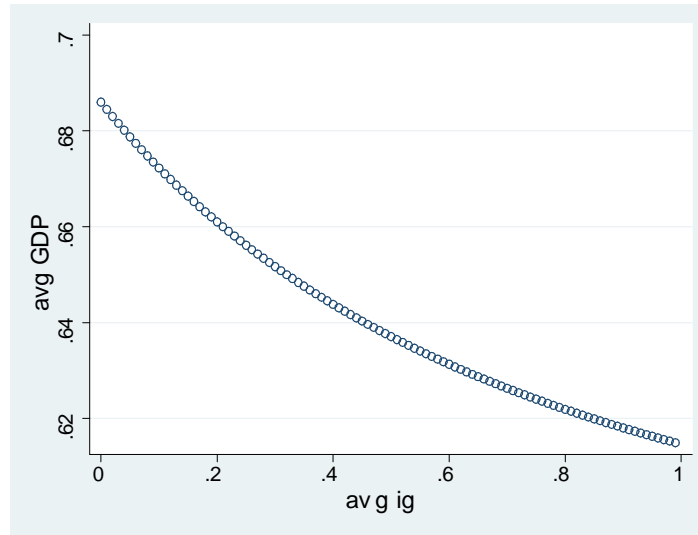
Second is the “output” effect, which arises due to the working capital constraint in this economy. Figure 5 illustrates the working of this effect in the model by showing the level of GDP as a function of different levels of i^g . When i^g rises, the cost of borrowing faced by the firm goes up, leading to lower employment and output. With lower outstanding loans, banks increase their holdings of

Figure 4: Steady state comparisons: Fiscal effect



government bonds to balance their balance sheets. This increases the government’s fiscal burden and raises the required seigniorage revenue to finance the budget. Thus, both the “fiscal” and “output” effects will push the market interest rate, i , up.

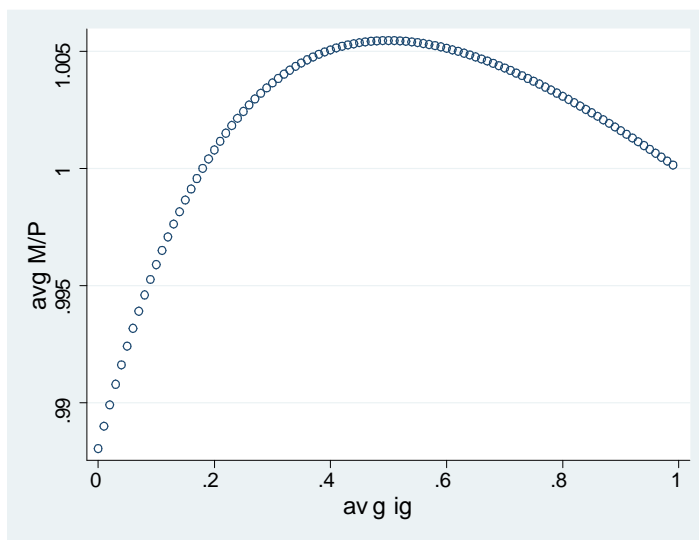
Figure 5: Steady state comparisons: Output effect



Our third effect is the “money demand” effect, which is summarized in Figure 6. It captures the response of the money base, $m(= h + \theta d)$, to changes in domestic interest rate i^g . With higher i , the opportunity cost of holding cash rises, thus lowering the demand for cash by households. At the same time, an increase in i^g is accompanied by an increase in the interest paid on demand

deposits, i^d . Lower opportunity costs of holding deposits lead to a higher demand for deposits, which is monotonically rising in i^g . The response of money demand, and thus the direction of the “money demand” effect, depends on which component dominates. For low steady state levels of i^g the increase in demand deposits is large enough to swamp the fall in cash demand. It is also strong enough to counterweight the “fiscal” and “output” effects, thus leading to an appreciation in the level of the currency. For high levels of i^g , deposits do not respond sufficiently to overtake the negative effect on cash demand coming through the fiscal and output effects. As a result, money demand declines leading to exchange rate depreciation.

Figure 6: Steady state comparisons: Money demand effect



Overall, we get a hump-shaped response of the money base in the economy and, therefore, a U-shaped response in the level of exchange rate.

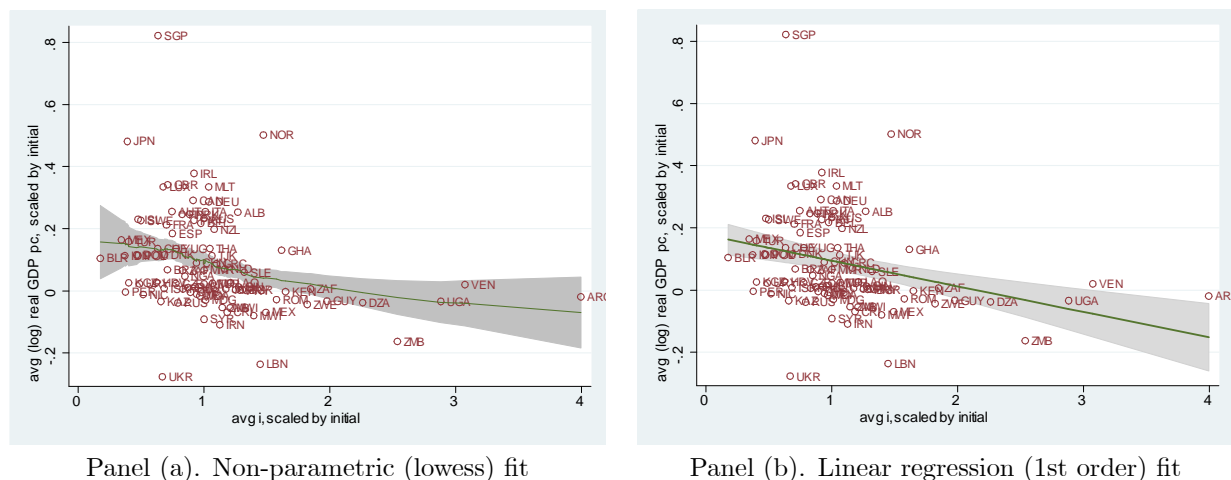
6 Empirical verification

In our model, the non-monotonic relationship between interest rates and the exchange rate results from an interaction of three effects: the output effect, the fiscal effect, and the money demand effect. Is there any evidence in support of the existence of these effects in the data? We now turn to this issue by examining the same cross-country data that we used to document the stylized facts regarding exchange rate behavior.

To analyze the three effects in the data we employ two empirical strategies. First, we use the lowest technique to fit the interest rate variable to a measure capturing each of the three effects

non-parametrically. Second, we use a traditional regression approach in which we fit a high-order polynomial in interest rate to a measure capturing each of the three effects – that is we estimate the regression specification in equation (2.1).

Figure 7: Output effect, 1974-2009



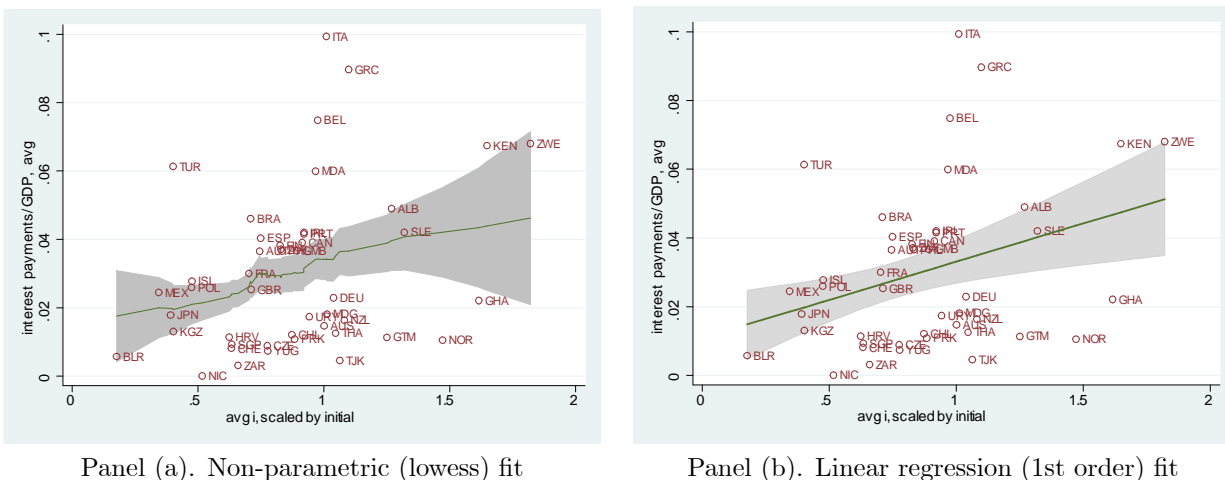
Notes: The figures plot average interest rate against average real GDP per capita for a sample of 88 country-episodes during 1974-2009 period, with each point obtained over a flexible exchange rate period. For each episode, both interest rate and GDP are scaled by their respective initial levels in the episode. Panel (a) presents the non-parametric fit of the data using lowess technique; Panel (b) presents the results from the linear fit of interest rate to the GDP level.

We start with the output effect. The output effect in the model arises due to working capital needs of the firms and predicts that, by raising the lending rate to firms, a higher domestic interest rate leads to lower employment and output. Therefore, the output effect predicts a negative correlation between interest rates and aggregate output. To test the output effect in the data we use real per capita GDP data for our sampled countries during flexible exchange rate episodes. The data is from the WDI. As before, to facilitate cross-country comparisons, we re-scale the average GDP series for each country and each episode using the initial level of GDP in that episode. The details on data sources and calculations are provided in the Appendix A.

The results are presented on Figure 7. Panel (a) of that Figure presents the results from a non-parametric (lowess) fit (shaded area represents the 90% pointwise confidence interval), while Panel (b) shows the fit obtained from a linear regression. As equation (2.1) prescribed, we regressed the re-scaled average (log) real GDP per capita on a 5th-order polynomial in interest rate, but found that a simple linear regression was preferred as all higher-order interest rate terms were insignificant. Fitted values from this regression together with the 90% confidence interval are plotted in Panel

(b) of Figure 7. Consistent with the predictions of the theory, the relationship between output and interest rate is negative in the data. The results are also statistically significant.³⁵

Figure 8: Fiscal Effect, 1974-2009



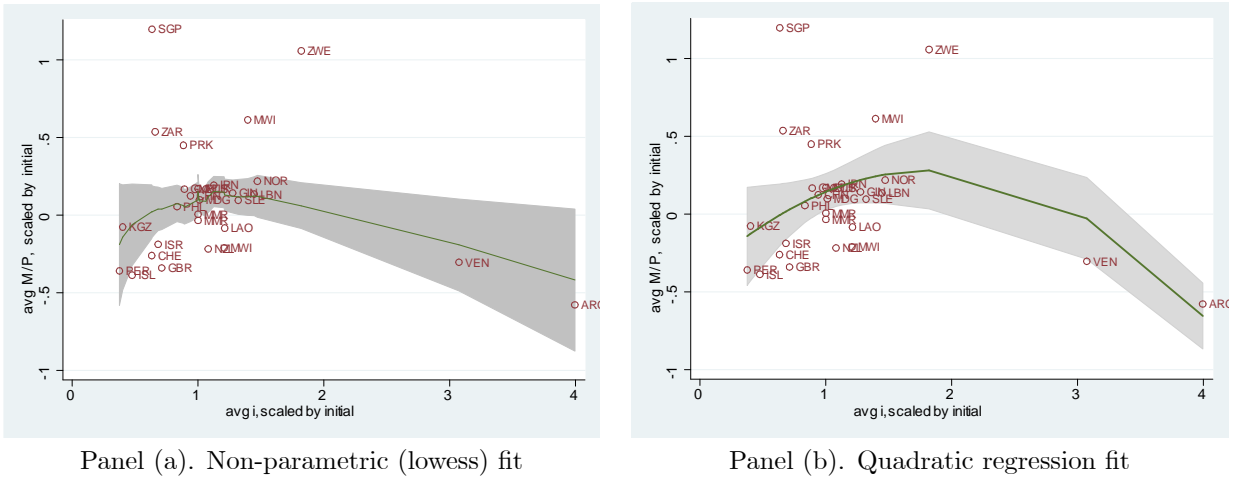
Notes: The figures plot average interest rate against average interest payments to GDP ratio for a sample of 88 country-episodes during 1974-2009 period, with each point obtained over a flexible exchange rate period. For each episode, interest rate is scaled by its initial level in the episode. We excluded episodes during which a country went through debt default. Panel (a) presents the non-parametric fit of the data using lowess technique; Panel (b) presents the results from the linear fit of interest rate to the interest payment data.

Next, we test for the presence of the fiscal effect in the data. The fiscal effect says that a higher interest rate raises the interest servicing burden directly and reduces fiscal revenues indirectly due to its negative effect on firm borrowing. The two effects jointly increase the government’s debt servicing burden. The fiscal effect, therefore, predicts a positive relationship between the interest rate and government’s debt payments. To evaluate the fiscal effect, we collected the data on interest payments by the central government from the WDI for our sample of countries. We computed the average interest payment to GDP ratio for each country during each flexible regime episode. The results are presented in Figure 8. Again, Panel (a) shows the results under non-parametric (lowess) fit, while Panel (b) presents the results using linear fit to the interest payment-to-GDP data.³⁶ Clearly, using either approach, the relationship between interest rates and debt payments is positive, in line with the theory and conventional wisdom. Note that this evidence is more a test of the assumption that fiscal spending is exogenous. Clearly, if we had not assumed $\bar{\tau}$ to be exogenous, the fiscal authority could reduce τ to prevent debt from rising in response to an increase

³⁵ We should note that the results are robust to using the growth rate of GDP per capita instead of the level.

³⁶ As in the case with the output effect, a linear model was preferred to a model with higher-order terms in interest rate.

Figure 9: Money Demand Effect, 1974-2009



Notes: The figures plot average interest rate against average real reserve money for a sample of 88 country-episodes during 1974-2009 period, with each point obtained over a flexible exchange rate period. For each episode, both interest rate and reserve money are scaled by their respective initial levels in the episode. Panel (a) presents the non-parametric fit of the data using lowess technique; Panel (b) presents the results from the quadratic fit of interest rate to the reserve money.

Finally, we turn to the money demand effect. This effect captures the response of the monetary base to interest rate changes. We showed that relationship between the interest rate and money demand in the model is non-monotonic and takes the form of an inverted U-shape. We test this implication of the model empirically.

To proxy the real money balances in the model, we used the cross-country data on reserve money from the IFS converted into real terms using GDP deflator (from WDI database).³⁸ To facilitate cross-country comparisons, as before, we re-normalize the average real money balances for each country and each flexible regime episode using the initial level of the variable for that country and episode. Figure 9 plots this re-scaled real balances against re-scaled interest rate for all available country-episode pairs. Panel (a) shows the results under non-parametric (lowess) fit, while Panel (b) presents the results using a quadratic fit to the real money data.³⁹ The figure reveals an inverted U-pattern, as the theory predicted. Note, however, that due to data limitations, the

³⁷We should note that a number of countries for which the WDI reports debt servicing data actually went through debt default episodes during their flexible exchange rate episodes. Based on the debt default chronologies contained in Reinhart and Rogoff (2011), we dropped all countries whose flexible exchange rate episodes overlapped with default episodes since their debt servicing data is clearly contaminated.

³⁸The stock of reserve money comprises currency in circulation, deposits of the deposit money banks, and deposits of other residents, apart from the central government, with the monetary authorities.

³⁹In this case, 5th-, 4th-, and 3rd-order terms in interest rate turned out to be insignificant, and thus were dropped.

sample of countries for which reserve money is available is smaller than our benchmark exchange rate sample, so the results on Figure 9 must be taken with caution.

Table 3 shows the regression results underlying Panels (b) of Figures 7, 8, and 9. These correspond, respectively, to columns (i), (ii), and (iii) in Table 3. Column (i) uses the average (log) real GDP per capita normalized by its initial level as the dependent variable. Column (ii) uses interest payments by the central government, as a share of GDP as the dependent variable, while column (iii) uses average real money balances, normalized by their initial level as the dependent variable. In all cases we start by estimating equation (2.1), but sequentially drop higher-order terms in interest rate if those turn out to be insignificant. This approach leaves us with a linear regression for the output and fiscal effect and a quadratic regression for the monetary effect. We then computed the predicted values from those regression and plot them together with the 90% confidence intervals in Panel (b) of Figures 7, 8, and 9.

Table 3: Regressions underlying empirical validation figures

	$\ln(GDP/GDP_0)$	$int.paym./GDP$	$\frac{M/P}{M_0/P_0}$
	(i)	(ii)	(iii)
i/i_0	-0.0822*** (0.0233)	0.0222** (0.0089)	0.7320* (0.3906)
$(i/i_0)^2$			-0.1999** (0.0801)
<i>constant</i>	0.1765*** (0.0336)	0.0109 (0.0075)	-0.3891 (0.3208)
N	85	46	29
R^2	0.093	0.108	0.209

Notes: This Table presents the results from the cross-sectional regressions underlying Panel (b) in Figures 7, 8 and 9. We started with flexible regression specification that allowed for the 5th order polynomial in the interest rate given in equation 2.1 where y_k is the dependent variable representing output ($\ln(GDP/GDP_0)$), fiscal ($int.paym./GDP$) and monetary ($\frac{M/P}{M_0/P_0}$) effects. Interest rate variable is normalized by its initial level. Robust standard errors are in parenthesis. * p-value \leq 0.10, ** p-value \leq 0.05, *** p-value \leq 0.01.

7 Conclusions

The relationship between interest rates and the exchange rate has been the focus of a spirited academic and policy debate for a long time. In this paper we have presented some new evidence on this relationship. In particular, we have examined a cross-country data set of flexible exchange rate

episodes in 80 countries to show empirically that there is a non-monotonic relationship between the average levels of interest rates and the exchange rate. We then developed a simple monetary model to rationalize these empirical relationship. Our model incorporated three key effects of interest rate increases – a negative effect on output through a credit channel, a fiscal effect due to a higher debt servicing burden, and a positive money demand effect due to higher deposit interest rates. The first two effects tend to depreciate the currency while the third tends to appreciate the currency. We have shown analytically and numerically that the interactions between these three effects render the relationship between changes in interest rates and the exchange rate inherently non-monotonic. For small to moderate increases in interest rates the exchange rate tends to appreciate but the effect reverses for larger increases in the domestic interest rate. Lastly, we have documented empirical support for the output, fiscal and money demand effects, which were the three key mechanisms embedded in the model.

In separate work in Hnatkovska, Lahiri, and Vegh (2011), we have explored the time series implications of the channels formalized in the model. In that work we have shown that a dynamic version of this model including physical capital can generate both appreciations and depreciations on impact of temporary increases in interest rates. Hence, the basic model structure can also account for some of the conflicting time-series evidence regarding the exchange rate and interest rate relationship that researchers have found in the data. In sum, we believe that these non-monotonic results are quite general and not specific to the particular formulation that we may have chosen. Hence, there is no reason to expect to find a linear, monotonic relationship between the two variables in the data.

The analysis in our model has treated the policy-controlled interest rate as an exogenous driving process. This simplified treatment of the interest rate allowed us to isolate the channels through which it can affect the nominal exchange rate. While being convenient for our purposes, this is an over-simplification. Understanding equilibrium exchange rate behavior in an environment with endogenous interest rate policy is an area that we intend to pursue in future work.

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Appendix

A Data sources and calculations

In this Appendix we describe our data sources and transformations used in the empirical sections of the paper. Our primary data sources are the International Financial Statistics (IFS) compiled by the International Monetary Fund (IMF) and the World Development Indicators (WDI) compiled by the World Bank. In our analysis we considered all countries in the IFS and WDI datasets for which annual data on exchange rates and interest rates was available for any fraction of the 1974-2009 period.

Data description and sources are summarized in Table A1.⁴⁰

Table A1: Data description and sources

Variable	Definition	Source
Exchange rate	official exchange rate, period average	IFS by IMF
	market exchange rate, period average	IFS by IMF
	principal exchange rate, period average	IFS by IMF
	commercial exchange rate, period average	IFS by IMF
Interest rate	T-bill rate, period average	IFS by IMF
	discount rate, period average	IFS by IMF
	money market rate, period average	IFS by IMF
Output	Gross Domestic Product (per capita), in constant local currency unit	WDI by World Bank
Interest payments to GDP ratio	Interest payments/GDP, where Interest payments by the central government, in current local currency units	WDI by World Bank
	GDP, in current local currency units	WDI by World Bank
Real money balances	Nominal reserve money/GDP deflator, where Nominal reserve money, in national currency units	IFS by IMF
	GDP deflator	WDI by World Bank

As we mentioned in the main text, in our empirical analysis we restrict the sample to only those countries and time periods that are characterized by a flexible exchange rate regime. To perform the selection, we rely on the Reinhart and Rogoff (2004) classification of historical exchange rate regimes. In particular, we classify a country as having a flexible exchange rate regime if, in a given year, its exchange rate was either (i) within a moving band that is narrower than or equal to $\pm 2\%$

⁴⁰We choose to work with output measured in local currency units since all the GDP numbers in each episode are normalized by the initial level of GDP in that episode. This converts the included GDP series into an index and hence comparable across countries. Measuring output in local currency units has the advantage that sudden changes in the exchange rate do not change GDP simply through revaluation effects – an issue that becomes problematic when output is measured in dollars.

(i.e., allows for both appreciation and depreciation over time); or (ii) was classified as managed floating; or (iii) was classified as freely floating; or (iv) was classified as freely falling according to Reinhart and Rogoff (2004). These correspond to their fine classification indices of 11, 12, 13, and 14, respectively. We only focus on the post-Bretton Woods period for all countries. High income OECD countries are included in our sample, irrespective of their exchange rate classification. The selection leaves us with a sample of 80 countries, and 88 country-episode pairs. These country-episode pairs are listed in Table A2. The average duration across episodes in our sample is 11 years, although this number differs significantly across developed countries, for whom the average duration is 23 years, and developing countries, for which the average duration across episodes is 6 years.

Table A2: Countries and episodes

code	country	time period	code	country	time period
ALB	Albania	1994-2001	LAO	Lao People's Dem. Rep.	1997-1999
DZA	Algeria	1988-1994	LBN	Lebanon	1984-1991
ARG	Argentina	1989-1990	LBY	Libya	1974-1985
AUS	Australia	1974-2009	LUX	Luxembourg	1980-1998
AUT	Austria	1974-1998	MDG	Madagascar	2000-2007
BLR	Belarus	1994-2002	MWI	Malawi	1983-1994; 1998-2003
BEL	Belgium	1974-1998	MLT	Malta	1987-2000
BRA	Brazil	1999-2007	MEX	Mexico	1982-1988; 1995-2007
BGR	Bulgaria	1992-1996	MDA	Moldova	1998-1999
CAN	Canada	1974-2009	MMR	Myanmar	1991-1993; 1997-1998
CHL	Chile	2000-2001	NLD	Netherlands	1978-1990
CHN	China, P.R. (Mainland)	1990-1992	NZL	New Zealand	1978-2009
ZAR	Congo, Dem. Rep. of	1994-2003	NIC	Nicaragua	1989-1990
CRI	Costa Rica	1981-1983	NGA	Nigeria	1991-2007
HRV	Croatia	1993-1994	NOR	Norway	1974-2008
CZE	Czech Republic	1997-2001	PRY	Paraguay	1989-1990
DNK	Denmark	1981-1988	PER	Peru	1990-1993
DOM	Dominican Republic	2004-2007	PHL	Philippines	1998-1999
EU	EURO Area	1999-2008	POL	Poland	2000-2007
FIN	Finland	1974-1998	PRT	Portugal	1980-1998
FRA	France	1974-1998	ROM	Romania	1995-2000
GMB	Gambia, The	1986-1991	RUS	Russian Federation	1994-1996
DEU	Germany	1975-1998	YUG	Serbia, Republic of	2003-2007
GHA	Ghana	1985-2000	SLE	Sierra Leone	2002-2005
GRC	Greece	1983-2000	SGP	Singapore	1974-2007
GTM	Guatemala	1985-1990	ZAF	South Africa	1974-1985; 1995-2007
GIN	Guinea	2000-2002	ESP	Spain	1979-1998
GUY	Guyana	1988-1991	SWE	Sweden	1993-2009
ISL	Iceland	1984-2009	CHE	Switzerland	1980-2009
IDN	Indonesia	1998-2007	SYR	Syrian Arab Republic	1982-1987
IRN	Iran, I. R. of	1977-1979	TJK	Tajikistan	1999-2002
IRL	Ireland	1974-1998	THA	Thailand	2001-2007
ISR	Israel	1984-1985	TUR	Turkey	1992-1995; 2001-2007
ITA	Italy	1977-1998	UGA	Uganda	1980-1986; 1990-1992
JAM	Jamaica	1991-1992	UKR	Ukraine	1993-1996
JPN	Japan	1974-2009	GBR	United Kingdom	1974-2009
KAZ	Kazakhstan	1994-1995	URY	Uruguay	1994-1995; 2002-2004
KEN	Kenya	1987-1995	VEN	Venezuela, Rep. Bol.	1984-1996
PRK	Korea, Republic of	1998-2007	ZMB	Zambia	1978-1982; 1985-2007
KGZ	Kyrgyz Republic	1994-1999	ZWE	Zimbabwe	1991-1998

B Proof of Proposition 4.1

(i) Since the nominal money supply is given at time 0, any change in m_0 due to a change in I_0^d has to be accommodated by a change in the exchange rate E_0 in the opposite direction. Hence, to uncover the effect of i^g on E_0 we need to determine the effect of i^g on I_0^d . Since $I^d = i - i^d$ we need to first determine the sign of $\frac{\partial i}{\partial i^g}$. After setting $\tilde{h}(i) = 0$, we can totally differentiate (4.20) to implicitly solve for i as a function of i^g and $\bar{\tau}$ with

$$\frac{\partial i}{\partial i^g} = \frac{(1 - \theta) d \left[1 - \left\{ 1 - \left(\frac{r(1+i^d)}{I^d} \right) \right\} \frac{\eta_d}{R} \right] - \left[1 - \left\{ 1 + \left(\frac{r(1+i^g)}{I^g} \right) \right\} \frac{\eta_n}{R} \right] \theta n}{\chi_1 / (1 + i)} \quad (\text{A1})$$

where $\chi_1 \equiv (1 + i^d) d \left[1 - \left\{ 1 - \left(\frac{r(1+i^d)}{I^d} \right) \right\} \frac{\eta_d}{R} \right] - (1 + i^g) n \left[1 - \left\{ 1 + \left(\frac{r(1+i^g)}{I^g} \right) \right\} \frac{\eta_n}{R} \right]$. Define this implicit solution for i as $\tilde{i}(i^g; \bar{\tau})$.

We can substitute $\tilde{i}(i^g; \bar{\tau})$ into I^g to implicitly solve for $\frac{I^g}{1+i} = \tilde{I}^g(i^g; \bar{\tau})$ where

$$\frac{\partial \tilde{I}^g}{\partial i^g} = \frac{\left[1 - \left\{ 1 - \left(\frac{r(1+i^d)}{I^d} \right) \right\} \frac{\eta_d}{R} \right] \theta d}{\chi_1 (1 + i)} > 0. \quad (\text{A2})$$

The sign of this expression follows directly from our assumption $1 > \left\{ 1 - \left(\frac{r(1+i^d)}{I^d} \right) \right\} \frac{\eta_d}{R}$ and the stability condition. Define \hat{i}^g such that $\hat{I}^g = \tilde{I}^g(\hat{i}^g; \bar{\tau})$.

Using equation (A1) along with the definition of I^d and the equilibrium relation $i^d = (1 - \theta) i^g$ we can derive the implicit function $\frac{I^d}{1+i} = \tilde{I}^d(i^g; \bar{\tau})$, where

$$\frac{\partial \tilde{I}^d}{\partial i^g} = - \frac{\left[1 - \left\{ 1 + \left(\frac{r(1+i^g)}{I^g} \right) \right\} \frac{\eta_n}{R} \right] \theta n}{\chi_1 (1 + i)}. \quad (\text{A3})$$

Under our assumed production function for firms (equation (3.6) the steady state loan demand is given by $n = \phi (1 - \alpha)^{\frac{\nu}{\nu-1+\alpha}} (\nu \zeta)^{\frac{\alpha-1}{\nu-1+\alpha}} \left[\frac{1}{1+\phi \left\{ \frac{i^g-i}{1+i} \right\}} \right]$. Using this to solve for $\eta_n (= -\frac{\tilde{n}'}{n} \frac{i^g-i}{1+i})$ in equation (A3) above, it is easy to check that $1 \gtrless \left[1 + \left(\frac{r(1+i^g)}{I^g} \right) \right] \frac{\eta_n}{R}$ as $1 + \phi \left(\frac{I^g}{1+i} \right) \lesseqgtr \nu \left(1 - \frac{\phi r}{R} \right)$. Hence, if $1 < \nu \left(1 - \frac{\phi r}{R} \right)$, then $\frac{\partial \tilde{I}^d}{\partial i^g} < 0$ for low values of $\frac{I^g}{1+i}$ but $\frac{\partial \tilde{I}^d}{\partial i^g} > 0$ for all $\frac{I^g}{1+i} > \left[\nu \left(1 - \frac{\phi r}{R} \right) - 1 \right] / \phi \equiv \hat{I}^g$. The proof then follows from the fact that $\partial \tilde{I}^g / \partial i^g > 0$.

(ii) From (i) above we know that $\frac{\partial i}{\partial i^g} \Big|_{i^g=\hat{i}^g} = (1 - \theta) \left(\frac{1+\tilde{i}(\hat{i}^g; \bar{\tau})}{1+(1-\theta)\hat{i}^g} \right) > 0$. Moreover, $1 < \left\{ 1 + \left(\frac{r(1+i^g)}{I^g} \right) \right\} \frac{\eta_n}{R}$ for all $i^g > \hat{i}^g$. Since $1 > \left\{ 1 - \left(\frac{r(1+i^d)}{I^d} \right) \right\} \frac{\eta_d}{R}$ by assumption and $\chi_1 > 0$ from the stability condition, it directly follows from equation (A1) that $\frac{\partial i}{\partial i^g} > 0$ for all $i^g \geq \hat{i}^g$.

Is it possible for $\frac{\partial i}{\partial i^g} < 0$ when $i^g < \hat{i}^g$? From part (i) of this Proposition we know that $\left\{1 + \left(\frac{r(1+i^g)}{I^g}\right)\right\} \frac{\eta_n}{R}$ is monotonically increasing in i^g . Since n is monotonically decreasing in \tilde{I}^g and \tilde{I}^g is increasing in i^g , we know that n is monotonically decreasing in i^g . Hence $\left[1 - \left\{1 + \left(\frac{r(1+i^g)}{I^g}\right)\right\} \frac{\eta_n}{R}\right] \theta n$ is monotonically falling in i^g for all i^g . We also know from part (i) of this proposition that d is increasing with i^g for $i^g < \hat{i}^g$ since $\frac{\partial \tilde{I}^d}{\partial i^g} > 0$ in this range. Define

$$A \equiv \left\{1 - \left(\frac{r(1+i^d)}{I^d}\right)\right\} \frac{\eta_d}{R}$$

Note that this can be rewritten as

$$A \equiv \left(1 - \frac{r}{R\tilde{I}^d}\right) \eta_d$$

It is easy to check that $\frac{\partial \eta_d}{\partial \tilde{I}^d} > 0$ (a maintained assumption) is a sufficient condition for A to be decreasing in i^g for $i^g < \hat{i}^g$. This implies that the first term in the numerator is $(1 - \theta) d [1 - A]$ is monotonically increasing in i^g for all $i^g < \hat{i}^g$ (since d is also rising with i^g in this range). Combining this with the fact that $\left[1 - \left\{1 + \left(\frac{r(1+i^g)}{I^g}\right)\right\} \frac{\eta_n}{R}\right] \theta n$ is monotonically decreasing in i^g for all i^g implies that the numerator of equation (A1) is monotonically increasing in i^g for all $i^g < \hat{i}^g$. Hence, as we lower i^g below \hat{i}^g , the numerator of equation (A1) falls monotonically. By arguments of continuity, there exists an $\bar{i}^g < \hat{i}^g$ such that $\frac{\partial i}{\partial i^g} \geq 0$ for all $i^g \geq \bar{i}^g$.⁴¹

(iii) From (i) and (ii) we know that for $i^g \in (\bar{i}^g, \hat{i}^g)$ an increase in i^g appreciates the currency on impact but also increases the steady-state depreciation rate.

C Proof of Proposition 4.2

Totally differentiating the government transfer equation (4.20), evaluating it around the steady state and rearranging gives

$$\frac{\partial i}{\partial i^g} = \frac{(1 - \theta) d \left[1 - \left(1 - \frac{Ri^d}{i-r}\right) \left(\frac{i-r}{i-i^d}\right) \frac{\eta_d}{R}\right]}{\chi_2/(1+i)} > 0 \quad (\text{A4})$$

where

$$\chi_2 = h \left[1 - \left(\frac{i-r}{Ri}\right) \eta_h\right] + (1+i^d) R d \left[1 - \left\{1 - \left(\frac{r(1+i^d)}{i-i^d}\right)\right\} \frac{\eta_d}{R}\right]. \quad (\text{A5})$$

⁴¹It is fairly easy to restrict parameters such that $\bar{I}^g > 0$ where $\bar{I}^g = \tilde{I}^g(\bar{i}^g, \bar{\tau})$. This restriction is necessary to guarantee a non-monotonicity of i within the permissible range of I^g .

The inequality follows from our restriction that $1 > \left(1 - \frac{Ri^d}{i-r}\right) \left(\frac{i-r}{i-i^d}\right) \frac{\eta_d}{R}$ (which we imposed to maintain comparability with the one money case). It is easy to check that

$$\frac{\partial \tilde{I}^d}{\partial i^g} = \frac{-(1-\theta)h}{(1+i)\chi_2} \left[1 - \left(\frac{i-r}{Ri}\right) \eta_h\right]. \quad (\text{A6})$$

where $\tilde{I}^d = \frac{i-i^d}{1+i}$ and where χ_2 is given by equation (A5).

To determine the relationship between the level of the nominal exchange rate and i^g we need to determine the effect of i^g on total real money demand $m(= h + \theta d)$. Using equation (A4) it follows that

$$\frac{\partial m}{\partial i^g} = \frac{(1-\theta)hd\eta_d}{i\chi_2} \left[\frac{\eta_d - \eta_h}{\eta_d} - \frac{r(1-\theta)i^g \eta_h}{i\tilde{I}^d R} + \frac{(1-\theta)I^g}{I^d} \left(1 - \frac{i-r}{i} \frac{\eta_h}{R}\right) \right]. \quad (\text{A7})$$

Since i is always rising in i^g , demand for cash (h) is always falling in i^g . Hence, m must necessarily fall with a higher i^g if demand deposits are non-increasing in i^g . Noting that the interest elasticity of cash η_h is a function of the nominal interest rate i , define \bar{i}^g by the relation $\eta_h \left(\frac{\tilde{i}(\bar{i}^g, \bar{\tau})}{1+\tilde{i}(\bar{i}^g, \bar{\tau})}\right) = \frac{R\tilde{i}(\bar{i}^g, \bar{\tau})}{\tilde{i}(\bar{i}^g, \bar{\tau})-r}$. Hence, from equation (A6) we have $\left.\frac{\partial \tilde{I}^d}{\partial i^g}\right|_{i^g=\bar{i}^g} = 0$. But this implies that $\left.\frac{\partial d}{\partial i^g}\right|_{i^g=\bar{i}^g} = 0$ and $\left.\frac{\partial m}{\partial i^g}\right|_{i^g=\bar{i}^g} < 0$.

Since $\left.\frac{\partial m}{\partial i^g}\right|_{i^g=\bar{i}^g} < 0$, the proof of the non-monotonicity of m in i^g hinges on showing that $\left.\frac{\partial m}{\partial i^g}\right|_{i^g=0} > 0$. First, note that since i is increasing in i^g from equation (A4) and $\frac{\partial \eta_h}{\partial i} > 0$ (from Condition 2) we must have $\eta_h \left(\frac{\tilde{i}(0, \bar{\tau})}{1+\tilde{i}(0, \bar{\tau})}\right) < \frac{R\tilde{i}(0, \bar{\tau})}{\tilde{i}(0, \bar{\tau})-r}$. Hence, $\left.\frac{\partial \tilde{I}^d}{\partial i^g}\right|_{i^g=0} < 0$. But this implies that $\left.\frac{\partial d}{\partial i^g}\right|_{i^g=0} > 0$. Noting that $I^d = i = -I^g$ around $i^g = 0$, it is easy to check that equation (A7) gives

$$\left.\frac{\partial m}{\partial i^g}\right|_{i^g=0} = \frac{(1-\theta)hd\eta_d}{i\chi_2} \left[\frac{\theta\eta_d \left(\frac{\tilde{i}(0, \bar{\tau})}{1+\tilde{i}(0, \bar{\tau})}\right) - \eta_h \left(\frac{\tilde{i}(0, \bar{\tau})}{1+\tilde{i}(0, \bar{\tau})}\right)}{\eta_d \left(\frac{\tilde{i}(0, \bar{\tau})}{1+\tilde{i}(0, \bar{\tau})}\right)} + \left(\frac{1-\theta}{R}\right) \left(\frac{\tilde{i}(0, \bar{\tau})-r}{\tilde{i}(0, \bar{\tau})}\right) \eta_h \left(\frac{\tilde{i}(0, \bar{\tau})}{1+\tilde{i}(0, \bar{\tau})}\right) \right].$$

Hence, Condition 1 is sufficient for $\left.\frac{\partial m}{\partial i^g}\right|_{i^g=0} > 0$. Moreover, $\frac{\partial i}{\partial i^g} > 0$ and $\frac{\partial \eta_h}{\partial i} > 0$ jointly imply that $\left(\frac{\tilde{i}(i^g, \bar{\tau})-r}{\tilde{i}(i^g, \bar{\tau})}\right) \eta_h \left(\frac{\tilde{i}(i^g, \bar{\tau})}{1+\tilde{i}(i^g, \bar{\tau})}\right)$ is rising in i^g . Moreover, for all $i^g < \bar{i}^g$ we know from above that $\left.\frac{\partial \tilde{I}^d}{\partial i^g}\right|_{i^g} < 0$. From Condition 2 $\frac{\partial \eta_d}{\partial I^d} > 0$. Hence, η_d must be falling with i^g for all $i^g < \bar{i}^g$. By arguments of continuity, there must therefore exist an \hat{i}^g such that $\left.\frac{\partial m}{\partial i^g}\right|_{i^g=\hat{i}^g} = 0$. The proof is completed by noting that $E_0 = M_0/m$ and that nominal money supply at time 0 is given (see equation (3.15)). Hence, E_0 moves inversely with m .