Appendix to

Home Bias and High Turnover:

Dynamic Portfolio Choice with Incomplete Markets*

Viktoria Hnatkovska[†]

Department of Economics, University of British Columbia, Vancouver, BC V6T 1Z1, Canada.

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†Correspondence: Viktoria Hnatkovska, Department of Economics, University of British Columbia, 922 - 1873 East Mall, Vancouver, BC, Canada V6T 1Z1. Tel: +1 (604) 822-5941. Email: hnatkovs@interchange.ubc.ca.

Appendix

This appendix provides the derivations of some equations presented in the text.

A Model equations and the approximation point

The system of equations characterizing the equilibrium of the model consists of

1. Process for productivity

$$z_t = a z_{t-1} + e_t,$$

where e_t is a vector of i.i.d. normally distributed, mean zero shocks with covariance Ω_e.

2. H and F budget constraints

$$\begin{aligned} W_{t+1} &= R^{\text{W}}_{t+1}\left(W_t - C^{\text{H}}_t - Q^{\text{F}}_t C^{\text{F}}_t - Q^{\text{N}}_t C^{\text{N}}_t\right) \\ R^{\text{W}}_{t+1} &= R^1_t + \alpha^{\text{H}}_t (R^{\text{H}}_{t+1} - R^1_t) + \alpha^{\text{F}}_t (R^{\text{F}}_{t+1} - R^1_t) + \alpha^{\text{N}}_t (R^{\text{N}}_{t+1} - R^1_t) \end{aligned}$$

and

$$\begin{aligned} \hat{W}_{t+1} &= \hat{R}^{\text{W}}_{t+1}(\hat{W}_t - \hat{C}^{\text{H}}_t - Q^{\text{F}}_t \hat{C}^{\text{F}}_t - \hat{Q}^{\text{N}}_t \hat{C}^{\text{N}}_t) \\ \hat{R}^{\text{W}}_{t+1} &= R^1_t + \hat{\alpha}^{\text{H}}_t (R^{\text{H}}_{t+1} - R^1_t) + \hat{\alpha}^{\text{F}}_t (R^{\text{F}}_{t+1} - R^1_t) + \hat{\alpha}^{\text{N}}_t (\hat{R}^{\text{N}}_{t+1} - R^1_t). \end{aligned}$$

3. H and F bond and equity Euler equations

$$\begin{aligned} 1 = \mathbb{E}_t\left[M_{t+1} R^1_t\right], &\qquad 1 = \mathbb{E}_t\left[\hat{M}_{t+1} R^1_t\right], \\ 1 = \mathbb{E}_t\left[M_{t+1} R^{\text{H}}_{t+1}\right], &\qquad 1 = \mathbb{E}_t\left[\hat{M}_{t+1} R^{\text{H}}_{t+1}\right], \\ 1 = \mathbb{E}_t\left[M_{t+1} R^{\text{F}}_{t+1}\right], &\qquad 1 = \mathbb{E}_t\left[\hat{M}_{t+1} R^{\text{F}}_{t+1}\right], \\ 1 = \mathbb{E}_t\left[M_{t+1} R^{\text{N}}_{t+1}\right], &\qquad 1 = \mathbb{E}_t\left[\hat{M}_{t+1} \hat{R}^{\text{N}}_{t+1}\right], \end{aligned}$$

where $M_{t+1} = \beta \frac{\partial U_{t+1}/\partial C^{\text{H}}_{t+1}}{\partial U_t/\partial C^{\text{H}}_t}$. The US stochastic discount factor then becomes

$$M_{t+1} = \beta \left(\frac{C_{t+1}}{C_t}\right)^{-\phi} \left(\frac{C^{\text{T}}_{t+1}}{C^{\text{T}}_t}\right)^{\phi-\rho} \left(\frac{C^{\text{H}}_{t+1}}{C^{\text{H}}_t}\right)^{\rho-1},$$

where $C \equiv \left(\lambda_{\text{T}}^{1-\phi}\left(C^{\text{T}}\right)^{\phi} + \lambda_{\text{N}}^{1-\phi}\left(C^{\text{N}}\right)^{\phi}\right)^{\frac{1}{\phi}}$ denotes the aggregate consumption basket in the US. Substituting in the definitions for C and C^{T}, and collecting the terms, allows me to express M_{t+1} as a growth rate of the US consumption expenditures:

$$M_{t+1} = \beta \frac{C^{\text{H}}_t + Q^{\text{F}}_t C^{\text{F}}_t + Q^{\text{N}}_t C^{\text{N}}_t}{C^{\text{H}}_{t+1} + Q^{\text{F}}_{t+1} C^{\text{F}}_{t+1} + Q^{\text{N}}_{t+1} C^{\text{N}}_{t+1}}.$$

When preferences are logarithmic, consumption expenditure is proportional to wealth, so that

$$C^{\text{H}}_t + Q^{\text{F}}_t C^{\text{F}}_t + Q^{\text{N}}_t C^{\text{N}}_t = (1-\beta)\, W_t, \tag{A1}$$

which allows to simplify the expression for M_{t+1} as $M_{t+1} = \beta W_t / W_{t+1}$. Similarly, in the ROW, $\hat{M}_{t+1} = \beta \hat{W}_t / \hat{W}_{t+1}$.

4. H and F optimality conditions determining relative goods prices

$$\begin{aligned} Q_t^{\text{N}} &= \left(\frac{\lambda_{\text{N}}}{\lambda_{\text{T}}}\right)^{1-\phi}\left(\frac{C_t^{\text{N}}}{C_t^{\text{T}}}\right)^{\phi-1}\lambda_{\text{H}}^{\rho-1}\left(\frac{C_t^{\text{H}}}{C_t^{\text{T}}}\right)^{1-\rho}, \\ \hat{Q}_t^{\text{N}} &= \left(\frac{\lambda_{\text{N}}}{\lambda_{\text{T}}}\right)^{1-\phi}\left(\frac{\hat{C}_t^{\text{N}}}{\hat{C}_t^{\text{T}}}\right)^{\phi-1}\hat{\lambda}_{\text{H}}^{\rho-1}\left(\frac{\hat{C}_t^{\text{H}}}{\hat{C}_t^{\text{T}}}\right)^{1-\rho}, \\ Q_t^{\text{F}} &= \left(\frac{\lambda_{\text{F}}}{\lambda_{\text{H}}}\right)^{1-\rho}\left(\frac{C_t^{\text{F}}}{C_t^{\text{H}}}\right)^{\rho-1}. \end{aligned}$$

5. Capital Euler equation at H and F

$$\begin{aligned} 1 &= \mathbb{E}_t\left[M_{t+1}R_{t+1}^{\text{K}}\right], \qquad \text{with } R_{t+1}^{\text{K}} \equiv \theta Z_{t+1}^{\text{H}}\left(K_{t+1}^{\text{H}}\right)^{\theta-1} + (1-\delta) \\ 1 &= \mathbb{E}_t\left[\hat{M}_{t+1}\hat{R}_{t+1}^{\text{K}}\right], \qquad \text{with } \hat{R}_{t+1}^{\text{K}} \equiv \left(Q_{t+1}^{\text{F}}/Q_t^{\text{F}}\right)\left[\theta Z_{t+1}^{\text{F}}(K_{t+1}^{\text{F}})^{\theta-1} + (1-\delta)\right] \end{aligned}$$

6. Market clearing conditions

   (a) traded goods

$$\begin{aligned} C_t^{\text{H}} + \hat{C}_t^{\text{H}} &= D_t^{\text{H}}, \\ C_t^{\text{F}} + \hat{C}_t^{\text{F}} &= D_t^{\text{F}}. \end{aligned}$$

   (b) nontraded goods

$$C_t^{\text{N}} = Y_t^{\text{N}} = D_t^{\text{N}} \qquad \text{and} \qquad \hat{C}_t^{\text{N}} = \hat{Y}_t^{\text{N}} = \hat{D}_t^{\text{N}}.$$

   (c) bond

$$0 = B_t + \hat{B}_t.$$

   (d) tradable equity

$$1 = A_t^{\text{H}} + \hat{A}_t^{\text{H}} \qquad \text{and} \qquad 1 = A_t^{\text{F}} + \hat{A}_t^{\text{F}},$$

   which can equivalently be written as

$$\begin{aligned} P_t^{\text{H}} &= \alpha_t^{\text{H}}\beta W_t + \hat{\alpha}_t^{\text{H}}\beta\hat{W}_t \\ Q_t^{\text{F}}P_t^{\text{F}} &= \alpha_t^{\text{F}}\beta W_t + \hat{\alpha}_t^{\text{F}}\beta\hat{W}_t \end{aligned}$$

   (e) nontradable equity

$$1 = A_t^{\text{N}} \qquad \text{and} \qquad 1 = \hat{A}_t^{\text{N}},$$

   which is equivalent to

$$\alpha_t^{\text{N}} = Q_t^{\text{N}}P_t^{\text{N}}/\beta W_t \qquad \text{and} \qquad \hat{\alpha}_t^{\text{N}} = \hat{Q}_t^{\text{N}}\hat{P}_t^{\text{N}}/\beta\hat{W}_t.$$

The approximation point is given by $R = R^{\text{K}} = \hat{R}^{\text{K}} = R^{\text{H}} = R^{\text{F}} = R^{\text{N}} = \hat{R}^{\text{N}} = R^{\text{W}} = \hat{R}^{\text{W}} = \frac{1}{\beta}$. $K \equiv K^{\text{H}} = K^{\text{F}} = (\beta\theta)^{1/(1-\theta)}\left(1-\beta+\beta\delta\right)^{1/(\theta-1)}$, $D \equiv D^{\text{H}} = D^{\text{F}} = K^{\theta} - \delta K$, $P^{\text{H}} = P^{\text{F}} = \beta D/(1-\beta)$. $D^{\text{N}} = \hat{D}^{\text{N}} = \kappa$, so that $C^{\text{N}} = \hat{C}^{\text{N}} = \kappa$ and $P^{\text{N}} = \hat{P}^{\text{N}} = \beta\kappa/(1-\beta)$. Wealth at H and F is approximated around an initial level, W_0 and $\hat{W}_0$. When $W_0 = \hat{W}_0$, then $C_0^{\mathrm{H}} = \hat{C}_0^{\mathrm{H}} = D^{\mathrm{H}}$ and $C_0^{\mathrm{F}} = \hat{C}_0^{\mathrm{F}} = D^{\mathrm{F}}$. This implies that initial aggregate consumption is also equalized across countries: $C_0 = \hat{C}_0$. Then portfolios are approximated around $\alpha^{\mathrm{N}} = \hat{\alpha}^{\mathrm{N}} = \lambda_{\mathrm{N}}^{1-\phi}\left(C_0^{\mathrm{N}}/C_0\right)^{\phi}$ and $\alpha^{\mathrm{H}} = \alpha^{\mathrm{F}} = \lambda_{\mathrm{T}}^{1-\phi}\left(C_0^{\mathrm{T}}/C_0\right)^{\phi}$, where α^{H} and α^{F} denote the initial values of $(\alpha_t^{\mathrm{H}} + \hat{\alpha}_t^{\mathrm{H}})$ and $(\hat{\alpha}_t^{\mathrm{F}} + \alpha_t^{\mathrm{F}})$, respectively, as before.

B Derivation of equations (12)-(14)

Equation (12) is obtained by using second-order Taylor series expansion and the log-normality of asset returns. The first-order condition for bond in equation (9c) can be expressed as

$$1 = \mathbb{E}_t\left[\exp(m_{t+1} + r_t^1)\right] \simeq \exp\left[\mathbb{E}_t\left(m_{t+1} + r_t^1\right) + \tfrac{1}{2}\mathbb{V}_t\left(m_{t+1}\right)\right].$$

Taking logs on both sides yields a log-approximate version of the consumption Euler equation:

$$0 = r_t^1 + \mathbb{E}_t m_{t+1} + \tfrac{1}{2}\mathbb{V}_t\left(m_{t+1}\right). \tag{A2}$$

First-order condition for asset χ in equation (9) is log-approximated analogously:

$$1 = \mathbb{E}_t\left[M_{t+1}R_{t+1}^{\chi}\right] \simeq \exp\left[\mathbb{E}_t\left(m_{t+1} + r_{t+1}^{\chi}\right) + \tfrac{1}{2}\mathbb{V}_t\left(m_{t+1} + r_{t+1}^{\chi}\right)\right].$$

Again, taking logs and substituting in the bond Euler equation gives expression in (12):

$$\mathbb{E}_t r_{t+1}^{\chi} - r_t + \tfrac{1}{2}\mathbb{V}_t\left(r_{t+1}^{\chi}\right) = -\mathbb{CV}_t\left(m_{t+1}, r_{t+1}^{\chi}\right). \tag{A3}$$

Next, I characterize m_{t+1}. Recall its definition on page 10: $m_{t+1} \equiv \ln\left(M_{t+1}\right) - \ln M = \ln\left(\beta \frac{\partial U_{t+1}/\partial C_{t+1}^{\mathrm{H}}}{\partial U_t/\partial C_t^{\mathrm{H}}}\right) - \ln\beta$, which using the derivations in Appendix A can be written as $m_{t+1} = -\Delta w_{t+1}$. Substituting this result in (28) gives equation (13) in the text.

Finally, to obtain a representation for risk premium in terms of consumption, as given in equation (14), I log-linearized the consumption rule in equation (A1).

C Derivation of equation (19) and bias term in (20)

Start by revisiting the optimal decision rule for portfolio shares in equation (17). We can write it as

$$\begin{bmatrix} \alpha_t^{\mathrm{H}} \\ \alpha_t^{\mathrm{F}} \\ \alpha_t^{\mathrm{N}} \end{bmatrix} = \begin{bmatrix} \mathbb{V}_t(r_{t+1}^{\mathrm{H}}) & \mathbb{CV}_t(r_{t+1}^{\mathrm{H}}, r_{t+1}^{\mathrm{F}}) & \mathbb{CV}_t(r_{t+1}^{\mathrm{H}}, r_{t+1}^{\mathrm{N}}) \\ \mathbb{CV}_t(r_{t+1}^{\mathrm{H}}, r_{t+1}^{\mathrm{F}}) & \mathbb{V}_t(r_{t+1}^{\mathrm{F}}) & \mathbb{CV}_t(r_{t+1}^{\mathrm{F}}, r_{t+1}^{\mathrm{N}}) \\ \mathbb{CV}_t(r_{t+1}^{\mathrm{H}}, r_{t+1}^{\mathrm{N}}) & \mathbb{CV}_t(r_{t+1}^{\mathrm{F}}, r_{t+1}^{\mathrm{N}}) & \mathbb{V}_t(r_{t+1}^{\mathrm{N}}) \end{bmatrix}^{-1} \begin{bmatrix} \mathbb{CV}_t(w_{t+1}, r_{t+1}^{\mathrm{H}}) \\ \mathbb{CV}_t(w_{t+1}, r_{t+1}^{\mathrm{F}}) \\ \mathbb{CV}_t(w_{t+1}, r_{t+1}^{\mathrm{N}}) \end{bmatrix},$$

where I used equation (13) to substitute for the risk-premium. My objective is to find expressions for tradable portfolio shares, α_t^{H} and α_t^{F}, for a given α_t^{N}. For this purpose we can re-write the first two lines in the expression above as

$$\begin{bmatrix} \mathbb{V}_t(r_{t+1}^{\mathrm{H}}) & \mathbb{CV}_t(r_{t+1}^{\mathrm{H}}, r_{t+1}^{\mathrm{F}}) \\ \mathbb{CV}_t(r_{t+1}^{\mathrm{H}}, r_{t+1}^{\mathrm{F}}) & \mathbb{V}_t(r_{t+1}^{\mathrm{F}}) \end{bmatrix} \begin{bmatrix} \alpha_t^{\mathrm{H}} \\ \alpha_t^{\mathrm{F}} \end{bmatrix} + \begin{bmatrix} \mathbb{CV}_t(r_{t+1}^{\mathrm{H}}, r_{t+1}^{\mathrm{N}}) \\ \mathbb{CV}_t(r_{t+1}^{\mathrm{F}}, r_{t+1}^{\mathrm{N}}) \end{bmatrix} \alpha_t^{\mathrm{N}} = \begin{bmatrix} \mathbb{CV}_t(w_{t+1}, r_{t+1}^{\mathrm{H}}) \\ \mathbb{CV}_t(w_{t+1}, r_{t+1}^{\mathrm{F}}) \end{bmatrix}.$$

Solving for $\begin{bmatrix} \alpha_t^{\text{H}} \\ \alpha_t^{\text{F}} \end{bmatrix}$ gives

$$\begin{bmatrix} \alpha_t^{\text{H}} \\ \alpha_t^{\text{F}} \end{bmatrix} = \begin{bmatrix} \mathbb{V}_t(r_{t+1}^{\text{H}}) & \mathbb{CV}_t(r_{t+1}^{\text{H}}, r_{t+1}^{\text{F}}) \\ \mathbb{CV}_t(r_{t+1}^{\text{H}}, r_{t+1}^{\text{F}}) & \mathbb{V}_t(r_{t+1}^{\text{F}}) \end{bmatrix}^{-1} \left(\begin{bmatrix} \mathbb{CV}_t(w_{t+1}, r_{t+1}^{\text{H}}) \\ \mathbb{CV}_t(w_{t+1}, r_{t+1}^{\text{F}}) \end{bmatrix} - \alpha_t^{\text{N}} \begin{bmatrix} \mathbb{CV}_t(r_{t+1}^{\text{H}}, r_{t+1}^{\text{N}}) \\ \mathbb{CV}_t(r_{t+1}^{\text{F}}, r_{t+1}^{\text{N}}) \end{bmatrix} \right).$$

Isolating the solution for α_t^{H} gives equation (19) in the text:

$$\begin{aligned} \alpha_t^{\text{H}} &= \frac{\mathbb{V}_t(r_{t+1}^{\text{F}}) \left[\mathbb{CV}_t(w_{t+1}, r_{t+1}^{\text{H}}) - \mathbb{CV}_t(r_{t+1}^{\text{N}}, r_{t+1}^{\text{H}}) \alpha_t^{\text{N}} \right]}{\mathbb{V}_t(r_{t+1}^{\text{H}}) \mathbb{V}_t(r_{t+1}^{\text{F}}) - \mathbb{CV}_t^2(r_{t+1}^{\text{H}}, r_{t+1}^{\text{F}})} \\ &\quad - \frac{\mathbb{CV}_t\left(r_{t+1}^{\text{H}}, r_{t+1}^{\text{F}}\right) \left[\mathbb{CV}_t(w_{t+1}, r_{t+1}^{\text{F}}) - \mathbb{CV}_t(r_{t+1}^{\text{N}}, r_{t+1}^{\text{F}}) \alpha_t^{\text{N}} \right]}{\mathbb{V}_t(r_{t+1}^{\text{H}}) \mathbb{V}_t(r_{t+1}^{\text{F}}) - \mathbb{CV}_t^2(r_{t+1}^{\text{H}}, r_{t+1}^{\text{F}})}. \end{aligned}$$

To derive the expression for bias in (20) I used the model equilibrium conditions as follows. First, using the definition of wealth from households' problem, the aggregate world wealth in the model is given by

$$\begin{aligned} W_t + \hat{W}_t &= \left(P_t^{\text{H}} + D_t^{\text{H}}\right) \left(A_{t-1}^{\text{H}} + \hat{A}_{t-1}^{\text{H}}\right) + Q_t^{\text{F}} \left(P_t^{\text{F}} + D_t^{\text{F}}\right) \left(A_{t-1}^{\text{F}} + \hat{A}_{t-1}^{\text{F}}\right) \\ &\quad + Q_t^{\text{N}} \left(P_t^{\text{N}} + D_t^{\text{N}}\right) A_{t-1}^{\text{N}} + \hat{Q}_t^{\text{N}} \left(\hat{P}_t^{\text{N}} + \hat{D}_t^{\text{N}}\right) \hat{A}_{t-1}^{\text{N}} + \left(B_{t-1} + \hat{B}_{t-1}\right). \end{aligned}$$

Using the asset market clearing conditions, I get

$$W_t + \hat{W}_t = \left(P_t^{\text{H}} + D_t^{\text{H}}\right) + Q_t^{\text{F}} \left(P_t^{\text{F}} + D_t^{\text{F}}\right) + Q_t^{\text{N}} \left(P_t^{\text{N}} + D_t^{\text{N}}\right) + \hat{Q}_t^{\text{N}} \left(\hat{P}_t^{\text{N}} + \hat{D}_t^{\text{N}}\right).$$

The expression above can also be written in terms of returns as

$$W_t + \hat{W}_t = R_t^{\text{H}} P_{t-1}^{\text{H}} + R_t^{\text{F}} Q_{t-1}^{\text{F}} P_{t-1}^{\text{F}} + R_t^{\text{N}} Q_{t-1}^{\text{N}} P_{t-1}^{\text{N}} + \hat{R}_t^{\text{N}} \hat{Q}_{t-1}^{\text{N}} \hat{P}_{t-1}^{\text{N}}$$

Log-approximating this condition around a symmetric steady state and equal initial wealth distribution yields

$$\tfrac{1}{2} w_t + \tfrac{1}{2} \hat{w}_t = \tfrac{1}{4}\left(r_t^{\text{H}} + p_{t-1}^{\text{H}}\right) + \tfrac{1}{4}\left(r_t^{\text{F}} + q_{t-1}^{\text{F}} + p_{t-1}^{\text{F}}\right) + \tfrac{1}{4}\left(r_t^{\text{N}} + q_{t-1}^{\text{N}} + p_{t-1}^{\text{N}}\right) + \tfrac{1}{4}\left(\hat{r}_t^{\text{N}} + \hat{q}_{t-1}^{\text{N}} + \hat{p}_{t-1}^{\text{N}}\right). \quad \text{(A4)}$$

Bond market clearing condition in conjunction with logarithmic utility also imply

$$\beta \left(W_t + \hat{W}_t\right) = P_t^{\text{H}} + Q_t^{\text{F}} P_t^{\text{F}} + Q_t^{\text{N}} P_t^{\text{N}} + \hat{Q}_t^{\text{N}} \hat{P}_t^{\text{N}},$$

which can be log-approximated as follows

$$\tfrac{1}{2} w_t + \tfrac{1}{2} \hat{w}_t = \tfrac{1}{4} p_t^{\text{H}} + \tfrac{1}{4}\left(q_t^{\text{F}} + p_t^{\text{F}}\right) + \tfrac{1}{4}\left(q_t^{\text{N}} + p_t^{\text{N}}\right) + \tfrac{1}{4}\left(\hat{q}_t^{\text{N}} + \hat{p}_t^{\text{N}}\right).$$

Substituting this result into equation (A4) and forwarding it by one period gives

$$\tfrac{1}{2} \Delta w_{t+1} + \tfrac{1}{2} \Delta \hat{w}_{t+1} = \tfrac{1}{4} r_{t+1}^{\text{H}} + \tfrac{1}{4} r_{t+1}^{\text{F}} + \tfrac{1}{4} r_{t+1}^{\text{N}} + \tfrac{1}{4} \hat{r}_{t+1}^{\text{N}}.$$

I can now use the expression above to derive $\mathbb{CV}_t(w_{t+1}, r_{t+1}^{\text{H}})$ and $\mathbb{CV}_t(w_{t+1}, r_{t+1}^{\text{F}})$ terms in equation (19). For instance, the covariance between US wealth and US tradable return, r_{t+1}^{H}, can be obtained from

$$\begin{aligned} \tfrac{1}{2} \mathbb{CV}_t(w_{t+1}, r_{t+1}^{\text{H}}) + \tfrac{1}{2} \mathbb{CV}_t(\hat{w}_{t+1}, r_{t+1}^{\text{H}}) &= \tfrac{1}{4} \mathbb{V}_t(r_{t+1}^{\text{H}}) + \tfrac{1}{4} \mathbb{CV}_t(r_{t+1}^{\text{F}}, r_{t+1}^{\text{H}}) \\ &\quad + \tfrac{1}{4} \mathbb{CV}_t(r_{t+1}^{\text{N}}, r_{t+1}^{\text{H}}) + \tfrac{1}{4} \mathbb{CV}_t(\hat{r}_{t+1}^{\text{N}}, r_{t+1}^{\text{H}}). \end{aligned} \quad \text{(A5)}$$

Second, since both US and ROW households have access to tradable equity issued in both countries, Euler equations in (13) imply that $\mathbb{CV}_t(w_{t+1}, r^{\text{H}}_{t+1}) = \mathbb{CV}_t(\hat{w}_{t+1}, r^{\text{H}}_{t+1})$ and $\mathbb{CV}_t(w_{t+1}, r^{\text{F}}_{t+1}) = \mathbb{CV}_t(\hat{w}_{t+1}, r^{\text{F}}_{t+1})$. Similar steps are applied to derive an expression for $\mathbb{CV}_t(w_{t+1}, r^{\text{F}}_{t+1})$. Equipped with these equalities we can substitute the expressions for $\mathbb{CV}_t(w_{t+1}, r^{\text{H}}_{t+1})$ and $\mathbb{CV}_t(w_{t+1}, r^{\text{F}}_{t+1})$ into equation (19) in the text to get

$$\begin{aligned} \alpha^{\text{H}}_t \quad = \quad & \tfrac{1}{4} + \frac{1}{\mathbb{V}_t(r^{\text{H}}_{t+1})\left(1 - \frac{\mathbb{CV}^2_t(r^{\text{H}}_{t+1}, r^{\text{F}}_{t+1})}{\mathbb{V}_t(r^{\text{H}}_{t+1})\mathbb{V}_t(r^{\text{F}}_{t+1})}\right)} [\tfrac{1}{4}\mathbb{CV}_t(r^{\text{H}}_{t+1}, \hat{r}^{\text{N}}_{t+1}) - \left(\alpha^{\text{N}}_t - \tfrac{1}{4}\right)\mathbb{CV}_t(r^{\text{H}}_{t+1}, r^{\text{N}}_{t+1}) \\ & - \frac{\mathbb{CV}_t\left(r^{\text{H}}_{t+1}, r^{\text{F}}_{t+1}\right)}{\mathbb{V}_t(r^{\text{F}}_{t+1})}\left(\tfrac{1}{4}\mathbb{CV}_t(r^{\text{F}}_{t+1}, \hat{r}^{\text{N}}_{t+1}) - \left(\alpha^{\text{N}}_t - \tfrac{1}{4}\right)\mathbb{CV}_t(r^{\text{F}}_{t+1}, r^{\text{N}}_{t+1})\right)], \end{aligned}$$

or, using the notation in Proposition 1 in the text:

$$\begin{aligned} \alpha^{\text{H}}_t \quad = \quad & \tfrac{1}{4} + \frac{1}{\sigma^2_t}[\tfrac{1}{4}\mathbb{CV}_t(r^{\text{H}}_{t+1}, \hat{r}^{\text{N}}_{t+1}) - \left(\alpha^{\text{N}}_t - \tfrac{1}{4}\right)\mathbb{CV}_t(r^{\text{H}}_{t+1}, r^{\text{N}}_{t+1}) \\ & -\beta^{\text{HF}}_t\left(\tfrac{1}{4}\mathbb{CV}_t(r^{\text{F}}_{t+1}, \hat{r}^{\text{N}}_{t+1}) - \left(\alpha^{\text{N}}_t - \tfrac{1}{4}\right)\mathbb{CV}_t(r^{\text{F}}_{t+1}, r^{\text{N}}_{t+1})\right)]. \end{aligned}$$

Under conditions (i)-(iii) in the Proposition, the expression above simplifies to the bias term in the text.