

Home Bias and High Turnover: Dynamic Portfolio Choice with Incomplete Markets*

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Abstract

Why do investors trade a lot in foreign assets and hold so little of them in their portfolios? This paper shows that both observations can arise naturally in the presence of nondiversifiable nontraded consumption risk when each country specializes in production, preferences exhibit consumption home bias, and asset markets are incomplete. Using a general equilibrium two-country, two-sector (tradable and nontradable) model of the world economy with production I show that low diversification occurs because variations in relative prices (i) increase the riskiness of foreign assets and (ii) facilitate risk-sharing across countries. Large and volatile capital flows are necessary to take advantage of international risk premia differentials that occur in response to productivity changes in the nontradable sector. I characterize the optimal portfolio holdings, the evolution of the investment opportunity set, the risk premium, and the dynamics of capital flows using a new methodology for solving dynamic general equilibrium models with incomplete markets and portfolio choice.

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Introduction

Investors around the world allocate most of their portfolios to domestic assets despite the apparent diversification gains to be had from holding foreign assets. The potential welfare gains from international diversification and investors' unwillingness to diversify is a long-standing puzzle in international finance – termed home bias. Another aspect of the home bias puzzle concerns portfolio flows. A number of studies document that cross-border equity flows by domestic residents are large in magnitude and volatile, suggesting that investors do try to take advantage of diversification opportunities abroad.³ In fact, size and volatility of U.S. portfolio flows (equity and debt inflows and outflows) exceed the volatility of U.S. current account to GDP ratio. Furthermore, the size and volatility of portfolio flows across borders exceed the size and volatility of portfolio flows in and out of domestic assets.⁴ Overall, it seems as though investors trade too much *and* hold too little of their portfolios in foreign assets. This paper develops a model that can reconcile these two seemingly contradictory observations. It generates a bias in country portfolios towards domestic assets together with large and variable international asset flows.⁵

The key feature that distinguishes this paper from earlier research on home bias is its analysis of a model with incomplete asset markets. Specifically, my analysis is based on a set of primitive assumptions regarding investors' access to equity markets rather than on an assumption about the degree of risk-sharing achieved in equilibrium. The novelty of my approach is that the degree of risk-sharing is determined endogenously as part of the competitive equilibrium of the model I study. Consequently, I can address the fundamental question of why investors hold most of their portfolios in domestic assets when it appears, a priori, that international diversification could bring welfare gains by facilitating greater risk-sharing.⁶ Addressing this question is the central task of this paper.

³For example, based on U.S. international investment position data from the U.S. Bureau of Economic Analysis (BEA), U.S. holdings of foreign equities amounted to 37% of U.S. GDP and to 26% of U.S. stock market capitalization in 2007. At the same time, according to the International Financial Statistics (IFS) from the International Monetary Fund (IMF), U.S. annual gross portfolio flows amounted to almost 11% of GDP and 7.4% of capitalization in 2006. In terms of volatility, the standard deviation of U.S. net equity assets (or equity outflows) and liabilities (or equity inflows) to GDP during 1990:1-2007:1 was in the order of 0.17%, for U.S. net debt assets (or debt outflows) to GDP the number was 0.18%, while for U.S. net debt liabilities (or debt inflows) to GDP the number was 0.54%. To put these numbers in perspective, during the same period, the standard deviation of the ratio of the U.S. current account to GDP (Hodrick-Prescott filtered) was only 0.12%. Similar pattern also holds in annual data on capital flows and current account. Lewis (1999) provides a survey of the literature on home bias in equities and consumption, capital flows, as well as on welfare gains from diversification.

⁴Tesar and Werner (1994, 1995) provide detailed evidence on high volume and high turnover rate of foreign equity investments in a sample of OECD countries. They show that the turnover rate on foreign equity is significantly larger than the turnover rate on domestic equity. Warnock (2002) updates these statistics using the results of comprehensive benchmark surveys of foreign equity holdings for U.S. and Canada, but still finds that foreign equity turnover is above domestic equity turnover.

⁵Throughout the paper I use the definition of capital flows that is conventionally employed by the national statistical agencies and by the IMF's IFS database to record the transaction in the financial account of country's balance of payments. It records capital flows as changes in the holdings of assets measured at market prices. Any changes in market value while securities are still in the holders possession (valuation changes) are omitted under this definition. An alternative definition of capital flows will also include valuation effects that may arise from asset price and exchange rate variations. My focus in this paper is on the former definition.

⁶Of course, the presence of home bias does not in itself imply that risk-sharing is incomplete. For example, Heathcote and Perri (2004) present a model in which world equilibrium is Pareto efficient and portfolios exhibit home bias.

My analysis uses a two-country general equilibrium model with two sectors: a tradable sector and a nontradable sector. Both sectors are subject to stochastic productivity shocks. Each country specializes in the production of its tradable good. Household preferences are defined over the consumption of three goods: a domestic nontradable and a basket of domestic and foreign tradable goods. Households finance these consumption expenditures by trading in equities issued by tradable firms in both countries, equities indexed to domestic nontradable production, and bonds. In this setting, the home bias (in tradable portfolios) arises as households try to hedge the fluctuations in their nontradable consumption even though productivity changes are independent across sectors and countries. Variations in nontradable consumption also lead to changes in risk premia, which in turn drive portfolio flows across borders.

The intuition for these results is the following. When preferences are complementary in the consumption of a nontradable and a basket of tradable goods, any increase in nontradable consumption must be accompanied by an increase in tradable consumption, both local and imported. Thus, households allocate a larger share of their portfolios to tradable assets whose payoffs are high when their nontraded consumption is high. When, in addition, local and imported tradable goods are imperfect substitutes, variations in the relative price of the imported good perform two roles: First they correlate negatively with the relative supply of foreign tradable dividends; second, they increase the variability of foreign tradable returns in the eyes of domestic households. The first effect decreases the relative value of foreign equity payouts to domestic households. The second effect makes the return on foreign assets riskier. The combination of these two effects inclines households to increase their holdings of domestic equity and skew their portfolios against foreign assets. Variations in the terms of trade are also associated with significant changes in the risk premia on tradable assets. In equilibrium, the shifts in expected excess returns are accompanied by the changes in desired households' portfolios, which in turn trigger large and volatile international capital flows.

This paper builds on a large literature that studies the role of real exchange rate fluctuations for international risk-sharing. Cole and Obstfeld (1991) studied a two-country economy with complete markets and showed that when preferences are symmetric Cobb-Douglas or separable, any variation in relative endowments induces an exactly off-setting change in relative price. As a result, any portfolio ensures perfect risk-sharing across countries. Kollmann (2005) uses constant elasticity of substitution (CES) preferences with home bias in consumption to generate portfolio home bias in an endowment economy. Uppal (1993) reaches an opposite conclusion in a complete market general equilibrium setting with shipping costs.⁷ Heathcote and Perri (2004) extend this analysis by introducing production. As in Cole and Obstfeld (1991), changes in relative prices facilitate pooling of risks across countries.⁸

A big strand of literature extends this analysis to introduce non-traded goods, so that the relative prices of non-traded goods contribute to the variation in real exchange rate.⁹ Stockman and Dellas (1989) started

⁷Uppal (1993) shows that observed portfolios can not be justified by the consumption bias towards domestic goods. On the contrary, investors that are more risk-averse than a log investor will prefer foreign stocks. The reason for his result is that the real exchange rate is negatively correlated with foreign returns, making returns on foreign assets less risky than domestic returns in the eyes of home investors.

⁸Recent work has extended the analysis by introducing additional shocks, assets, etc. (Coeurdacier, Kollmann, and Martin (2007, 2008), Coeurdacier and Gourinchas (2008), Engel and Matsumoto (2008)).

⁹Another source of non-traded risk is labor income risk. Its role for equity home bias was studied in Baxter and Jermann

this literature by studying an endowment economy in which investor preferences are separable in tradable and nontradable consumption. They find that asset holdings are constant and the equilibrium portfolio shares are equally split between home and foreign tradable equity, while domestic nontradable equity is completely held by domestic investors. The implications of a richer preference specification are examined in Tesar (1993), Pesenti and van Wincoop (2002), Baxter, Jermann and King (1998), Serrat (2001), Matsumoto (2007), and Collard et al. (2007).

Tesar (1993) shows that when tradable and nontradable goods are complementary in consumption, the deviations from an equally-weighted tradable portfolio towards a home biased tradable portfolio will be welfare-improving if domestic nontradable productivity is more strongly correlated with domestic than foreign tradable productivity. Pesenti and van Wincoop (2002) derive an analogous result in a partial equilibrium framework and confirm it empirically using a sample of 14 OECD countries. Serrat (2001) solves for the optimal portfolios in an endowment general equilibrium model in which preferences are defined over a nontradable good and a basket of home and foreign tradable goods. In his framework domestic investors are the sole owners of local nontraded assets, while the home bias in traded portfolios arises under conditions analogous to those in Tesar (1993) and Pesenti and van Wincoop (2002). Kollmann (2006) revisits Serrat's model and shows that when the consumption aggregator is Cobb-Douglas and asset markets are complete, the optimal tradable portfolio remains equally-weighted, while the nontradable portfolio split becomes indeterminate. This result is reminiscent of Baxter, Jermann and King (1998). They argued that home bias in tradable equity cannot arise in a static economy with complete markets and international trade in claims to tradable and nontradable goods. Matsumoto (2007) studies asset allocations in the model with both nontraded goods and nontraded factors and finds that the optimal traded portfolios are sensitive to parameter values, especially the elasticity of substitution between different consumption goods and nontraded factors. Collard et al. (2007) and Coeurdacier (2008) confirm this result and emphasize the role of consumption home bias for optimal equity portfolios.

The key conclusion that emerges from this literature is that the degree of equity home bias that can be attributed to the presence of nontraded goods or factors is sensitive to the preference and technology parameters. In this paper I continue to focus on non-traded risk arising from the presence of nontraded good in households' consumption basket. However, relative to the existing literature, I add two new features to the model – capital accumulation and incomplete asset markets. The first feature allows me to eliminate portfolio sensitivity to some technology parameters, while the second feature turns out to be vital for understanding both equity home bias and portfolio flows.

This paper is also related to the literature studying capital flows. Much of this literature limits international trade in assets to bonds, i.e. risk-free bonds (Baxter and Crucini (1995), Ghironi (2006)) or nominal bonds (Persson and Svensson (1989), Bacchetta and van Wincoop (2000), Devereux and Saito (1997)), or complete markets (Stockman and Svensson (1987), Kollmann (2005), Obstfeld and Rogoff (1996)). In the complete markets case with isoelastic utility and symmetric economies, equity holdings are constant and thus equity flows are zero. Under the broader definition of capital flows, the latter can arise only due to

(1997), Engel and Matsumoto (2006), Matsumoto (2007).

changes in the value of assets.¹⁰ This is counterfactual, given that financial flows contribute almost 50% to the variations in the U.S. net foreign asset position.¹¹ More recently, Evans and Hnatkowska (2005a) and Tille and van Wincoop (2007) study capital flows in a general equilibrium setting with incomplete asset markets.¹² Evans and Hnatkowska (2005a) focus on the properties of capital flows and do not study equity home bias. Tille and van Wincoop (2007) present a very stylized model of endowment economy and do not provide tests of their framework. In contrast, this paper allows for a richer modelling environment that is frequently used in the study of international business cycles and conducts a test of the model by comparing the properties of predicted asset holdings and portfolio flows with their data counterparts.

This paper studies equity home bias and high asset turnover jointly in a unified framework.¹³ In doing so, it extends the existing literature along three dimensions. First, the optimal portfolios are studied in a general equilibrium framework with incomplete asset markets. As noted above, existing research focuses primarily on complete markets equilibria, which in combination with isoelastic utility and countries' symmetry implies that portfolio holdings are time-invariant. Relaxing the assumption of complete markets allows me to explore the dynamics of international portfolio holdings as well as the properties of capital flows and factors driving them. However, finding an equilibrium of a model with incomplete markets is challenging. In such an economy any shift in the distribution of wealth affects the dynamics of interest rates and asset returns, which in turn determine the variations in risk premia and investors' portfolios. This requires that wealth in each country is included in the state vector, which immediately leads to a number of technical difficulties.¹⁴ The equilibrium of this model is obtained by using a methodology developed in Evans and Hnatkowska (2005b).

Second, I assume that households' preferences are specified in terms of a nontradable good and a basket of tradable goods; one produced locally and one that must be imported from abroad. Preferences are assumed to be nonseparable in the consumption of all three goods. Such a generalization allows me to nest the existing results in the literature as special cases of my model.

The third dimension concerns production. I introduce production in the tradable sector. This allows for tradable dividends to be endogenously determined by firms in response to different shocks. This feature

¹⁰Stockman and Svensson (1987) show that in the economy with perfect risk pooling capital flows can arise only due to changes in equity prices, rather than portfolio holdings. Bacchetta and van Wincoop (2000) allow for endowment and preference asymmetry to generate nonzero capital flows. In an overlapping generations model with trades in an international risk-free bond, Ghironi (2006) shows that bond flows may be non-zero even with complete markets when elasticity of substitution between traded goods is different from 1. That paper, however, does not allow for trades in equity shares. When households face labor-leisure decisions, asset holdings may be changing over time, but current account remains zero (Coeurdacier, Kollmann, Martin (2008)).

¹¹This number is obtained from a variance decomposition of the U.S. net foreign asset position over 1989-2006 period using dataset on the "Components of Changes in the Net International Investment Position" compiled by BEA.

¹²Devereux and Sutherland (2007) develop a method for studying variation in asset holdings, however, their paper focuses on the methodological contribution and does not analyze portfolio flows explicitly.

¹³In a partial equilibrium mean-variance framework Amadi and Bergin (2006) show that a particular combination of fixed and proportional costs on equity holdings is required to explain home bias in holdings and high turnover rates on equities. In my paper both home bias and high equity turnover arise as a result of preference for local goods and incomplete risk sharing.

¹⁴The challenges relate to the dimensionality of the state vector, its conditional heteroskedasticity, and the nonstationarity of the wealth processes.

proves to be useful because when asset markets are incomplete, the stochastic properties of the shocks are of first-order importance for households' portfolio choices.¹⁵ However, the available estimates for sectoral productivity processes in the literature are very dissimilar, thus implying different equilibrium portfolios. I get around this problem by assuming independent productivity processes, but allowing them to transmit endogenously through the optimal investment decisions of the firms. Furthermore, I do not assume constant returns to scale in production. When constant returns to scale are present, the dynamics of wealth become monotonically related to the physical capital stock. It also implies that the expected returns on the risky assets and their variances become equivalent to the corresponding moments of the return on capital. In this setting expected excess equity returns are constant and changes in the risk-free rate are the only source of variability in the investment opportunity set. In my model, variations in equity returns govern the variability of households' wealth and risk premia, which, in turn, are the key determinants of households' portfolios.

My analysis produces the following main findings:

1. Market incompleteness in my model is quantitatively important. I find that the restrictions on asset ownership significantly impede international risk-sharing in equilibrium.
2. The portfolios of households in each country exhibit a significant degree of home bias. Moreover, the degree of bias is comparable with the bias in the international positions of U.S. investors.
3. The equilibrium bond and equity flows between countries are large and variable. There is nothing inherently contradictory between home bias in portfolio holdings and a large amount of international asset trade. In fact, equity flows generated by the model are larger than their empirical counterparts.

The paper is organized as follows. The next section sets up the model. Section 2 describes the equilibrium behavior of households and firms. Section 3 develops the intuition for the home bias. The solution procedure and calibration of the model are described in section 4. Section 5 analyses the equilibrium properties of the model. Section 6 concludes.

1 The Model

The world consists of two symmetric countries: home (H) and foreign (F). In what follows, I will refer to the home country as the US and to the foreign country as the rest of the world (ROW). Each country is populated by a continuum of identical households who supply their labor inelastically to domestic firms. Two types of firms exist in each country: (i) firms that specialize in production of a tradable (T) good, and (ii) firms that receive an endowment of a nontradable (N) good. Both firm types issue equity on the domestic stock market.

¹⁵In the context of an endowment economy this result is shown in Tesar (1993) and Coeurdacier (2008).

1.1 Firms

There is a continuum of perfectly competitive tradable good firms in each country. A representative US firm owns all of its capital stock, K_t^H , and produces output, Y_t^H , according to

$$Y_t^H = Z_t^H (K_t^H)^\theta,$$

where Z_t^H denotes the exogenous state of productivity. The output of traded goods in the ROW, Y_t^F , is given by a symmetric production function using capital, K_t^F , and productivity Z_t^F . Hereafter, I use the term tradables to refer to traded goods produced by US and ROW firms.

At the beginning of each period, firms in the tradable sector observe the state of productivity and decide how to allocate their output between investment and consumption goods. Output allocated to consumption is supplied competitively to US and ROW households and the proceeds are used to finance dividend payments to the owners of the firm's equity. Output allocated to investment adds to the stock of physical capital available for production next period. I assume that firms allocate output to maximize the value of the firm to its shareholders. If the total number of shares issued by the US firm is normalized to one, then the optimization problem of the US firm producing tradables can be summarized as

$$\max_{I_t^H} \mathbb{E}_t \sum_{i=0}^{\infty} M_{t+i,t} D_{t+i}^H$$

subject to

$$I_t^H = K_{t+1}^H - (1 - \delta) K_t^H, \quad \text{and} \quad D_t^H = Y_t^H - I_t^H,$$

where $\delta > 0$ is the depreciation rate on physical capital, I_t^H is the real investment, and D_t^H is the dividend per share paid at t . $M_{t+i,t}$ is the US household's intertemporal marginal rate of substitution (IMRS) between the consumption of US tradables in period t and $t+i$, with $M_{t,t} = 1$. \mathbb{E}_t denotes expectation conditioned on information at the start of period t . The representative ROW firm producing tradables solves an analogous problem. It chooses I_t^F to maximize the expected present discounted value of dividends per share, D_{t+i}^F , using the ROW household's IMRS, $\hat{M}_{t+i,t}$. I assume that the US tradable good is numeraire. The ex-dividend price of a share in the representative US and ROW tradable firm (measured in terms of their respective tradables) is P_t^H and P_t^F .

There are two notable features in the formulation of the firm's problem above. First, firms in each country use the stochastic discount factor of domestic households to value the firm. This formulation embodies the idea that investment policy of the firm and its ownership are linked. In particular, the production decisions are made to represent interests of a majority of firm's shareholders. Such environment was formalized in DeMarzo (1993), Tvede and Crés (2005), Kelsey and Milne (1996). In the equilibrium of such model, the optimal investment and dividend policies of the firm are such that if shareholders choose their optimal portfolios based on the expectation of these policies, their expectations will be validated because these policies are optimal given the shareholdings. In other words, the optimal plans of the firm are consistent with the rational expectations of shareholders about the decision mechanism of the firm. In my model, this

corresponds to the objective function of the firm based on the domestic shareholders' stochastic discount factor.¹⁶

Second, the production technology exhibits decreasing returns to scale. This assumption allows me to break the link between the ex-dividend value of the firm and the market value of the end-of-period capital stock which arises when technology has constant returns to scale. In the latter case, if investment is perfectly reversible and there are no capital adjustment costs, then the unit price of capital in place is equal to the price of the numeraire consumption good. As a result, tradable equity returns and their moments are equal to return on capital and its corresponding moments. When the innovations to productivity are homoskedastic, the expected excess returns on tradable equities are constant and it is only changes in the risk-free rate that lead to the variability of household's investment opportunity set. One way to break the link between equity prices and capital stock is to allow the price of capital to deviate from 1. This can be accomplished by introducing frictions, such as investment irreversibility, capital adjustment costs (as in Tille and van Wincoop (2008)); or by adding shocks that are investment-specific (as in Coeurdacier, Kollmann, and Martin (2008)). In my paper, I continue to work with frictionless environment and standard productivity shocks, but instead assume decreasing returns to scale. This provides an endogenous way for breaking the link between the value of firm's equity and its capital and allows for endogenous variations in equity risk premia. The latter are key in determining equilibrium portfolios.

Production in the US and ROW nontradable sectors does not require capital. Outputs of nontraded goods in the US and ROW (hereafter nontradables), denoted by Y_t^N and \hat{Y}_t^N , are produced by

$$Y_t^N = \kappa Z_t^N, \quad \text{and} \quad \hat{Y}_t^N = \kappa \hat{Z}_t^N,$$

where $\kappa > 0$ is a constant, and Z_t^N and \hat{Z}_t^N denote the period- t state of nontradable productivity in the US and ROW respectively. Nontradables can only be consumed by domestic households. The proceeds are paid out as dividends to firm's shareholders. The number of shares issued by the representative nontradable firm is normalized to unity. As a result, $D_t^N = Y_t^N$, and $\hat{D}_t^N = \hat{Y}_t^N$. The ex-dividend price of a share in the representative US and ROW nontradable firm, measured in terms of nontradables, is P_t^N and \hat{P}_t^N , respectively.

The productivity processes in the tradable and nontradable sectors, summarized in $z_t \equiv [\ln Z_t^H, \ln Z_t^F, \ln Z_t^N, \ln \hat{Z}_t^N]'$, are governed by an AR(1) process:

$$z_t = az_{t-1} + e_t,$$

where e_t is a (4×1) vector of i.i.d. normally distributed, mean zero shocks with covariance Ω_e .

¹⁶Another way to define the objective of the firm when markets are incomplete is based on the considerations of efficiency: no alternative production plan can make some shareholders better off and none worse off. This approach is proposed in Dreze (1974), Grossman and Hart (1979). It implies that firms value dividends using a weighted average of domestic and foreign stochastic discount factors, where the weights are determined by the proportion of the firm that different households hold. Sabarwal (2007) shows that under some conditions firm's value maximization based on efficiency considerations is consistent with the decentralized process of shareholder control.

1.2 Households

Each country is populated by a continuum of households, whose preferences are defined over the consumption of several goods: a composite tradable and a domestic nontradable. The preferences of US households are represented by

$$\mathbb{E}_t \sum_{i=0}^{\infty} \beta^i U(C_{t+i}^T, C_{t+i}^N), \quad (1)$$

where $0 < \beta < 1$ is the discount factor, and $U(\cdot)$ is a concave sub-utility function defined over the consumption of a basket of tradable consumption, C_t^T , and US nontradable consumption, C_t^N :

$$U(C^T, C^N) = \ln \left[\lambda_T^{1-\phi} (C^T)^\phi + \lambda_N^{1-\phi} (C^N)^\phi \right]^{\frac{1}{\phi}},$$

with $\phi < 1$. λ_T and $\lambda_N = 1 - \lambda_T$ are the weights the household assigns to tradable and nontradable consumption, respectively. The elasticity of substitution between tradable and nontradable consumption is $(1 - \phi)^{-1} > 0$. Preferences for ROW households are similarly defined in terms of the consumption of a basket of tradables and ROW nontradables, \hat{C}_t^T and \hat{C}_t^N : $U(\hat{C}_t^T, \hat{C}_t^N)$.

The tradable consumption basket in the US, C_t^T , is given by a CES aggregator over tradables produced in the US and ROW:

$$C^T = \left[\lambda_H^{1-\rho} (C^H)^\rho + \lambda_F^{1-\rho} (C^F)^\rho \right]^{\frac{1}{\rho}},$$

where $\rho < 1$. The weights that households assign to the consumption of two tradable goods are λ_H and $\lambda_F = 1 - \lambda_H$, respectively. The elasticity of substitution between the two tradable goods is given by $(1 - \rho)^{-1} > 0$. The tradable consumption bundle in the ROW is defined symmetrically in terms of US tradables, \hat{C}^H , and ROW tradables, \hat{C}^F , as

$$\hat{C}^T = \left[\hat{\lambda}_H^{1-\rho} (\hat{C}^H)^\rho + \hat{\lambda}_F^{1-\rho} (\hat{C}^F)^\rho \right]^{\frac{1}{\rho}}.$$

Households finance their consumption expenditures by holding an array of financial assets. In each country households can allocate their wealth to equity providing claims to US and ROW tradable dividend streams, $\{D_t^H\}$ and $\{D_t^F\}$. I will use A^H and A^F to denote US holdings of tradable equity issued by US and ROW firms, respectively. Households also have access to a market for international borrowing/lending. B_t represents the bond holdings of US households in period t , while R_t^1 denotes the prevailing interest rate measured in terms of US tradables. Bonds are denominated in the units of a numeraire and thus pay one unit of US tradable good independent of the state of the world.¹⁷ I assume that households can not hold

¹⁷Note that this bond is truly risk-free only from the perspective of H households. As in Heathcote and Perri (2002), the denomination of the bond has no effect on the equilibrium real allocations. However, it introduces a small asymmetry in the equity holdings, which I discuss in section 5.1 below. An alternative approach would be to denominate bonds in units of a tradable consumption basket as in Ghironi (2006). However, this would not eliminate the asymmetry in the menu of assets across countries because there is home bias in consumption. The advantage of the first approach is that it allows me to access the robustness of home bias and portfolio flows dynamics to the presence of a risk-free asset.

foreign nontradable equity.¹⁸ The flow budget constraint for US households can be expressed as

$$\begin{aligned} & C_t^H + Q_t^F C_t^F + Q_t^N C_t^N + P_t^H A_t^H + Q_t^F P_t^F A_t^F + Q_t^N P_t^N A_t^N + \frac{1}{R_t^1} B_t \\ & \leq (P_t^H + D_t^H) A_{t-1}^H + Q_t^F (P_t^F + D_t^F) A_{t-1}^F + Q_t^N (P_t^N + D_t^N) A_{t-1}^N + B_{t-1}, \end{aligned} \quad (2)$$

where Q_t^F and Q_t^N denote the relative prices of ROW tradables and US nontradables, respectively. It proves convenient to rewrite the budget constraint in equation (2) in terms of financial wealth, W_t (measured in terms of US tradables), as

$$W_{t+1} = R_{t+1}^W (W_t - C_t^H - Q_t^F C_t^F - Q_t^N C_t^N), \quad (3)$$

where R_{t+1}^W is the (gross) return on wealth between period t and $t+1$, given by

$$R_{t+1}^W = R_t^1 + \alpha_t^H (R_{t+1}^H - R_t^1) + \alpha_t^F (R_{t+1}^F - R_t^1) + \alpha_t^N (R_{t+1}^N - R_t^1), \quad (4)$$

and

$$W_t = (P_t^H + D_t^H) A_{t-1}^H + Q_t^F (P_t^F + D_t^F) A_{t-1}^F + Q_t^N (P_t^N + D_t^N) A_{t-1}^N + B_{t-1}.$$

The returns on US and ROW tradable equity are R_{t+1}^H and R_{t+1}^F , while R_{t+1}^N is the return on US nontradable equity. All returns are measured in terms of US tradables. The shares α_t^H , α_t^F and α_t^N denote the fraction of wealth held by US households in US and ROW tradable equity and US nontradable equity. The budget constraint facing ROW households is similarly written as

$$\begin{aligned} \hat{W}_{t+1} &= \hat{R}_{t+1}^W (\hat{W}_t - \hat{C}_t^H - Q_t^F \hat{C}_t^F - \hat{Q}_t^N \hat{C}_t^N), \\ \hat{R}_{t+1}^W &= R_t^1 + \hat{\alpha}_t^H (R_{t+1}^H - R_t^1) + \hat{\alpha}_t^F (R_{t+1}^F - R_t^1) + \hat{\alpha}_t^N (\hat{R}_{t+1}^N - R_t^1), \end{aligned}$$

where $\hat{\alpha}_t^H$, $\hat{\alpha}_t^F$ and $\hat{\alpha}_t^N$ denote the shares of wealth allocated by ROW households into equity issued by US and ROW tradable firms, and ROW nontradable firms. \hat{R}_{t+1}^N denotes the return on ROW nontradable equity measured in terms of US tradables.

To complete the description of the economy, we need to relate the equity returns $\{R_{t+1}^H, R_{t+1}^F, R_{t+1}^N, \hat{R}_{t+1}^N\}$ to equity prices and dividends:

$$R_{t+1}^H = \frac{P_{t+1}^H + D_{t+1}^H}{P_t^H} \quad R_{t+1}^F = \frac{Q_{t+1}^F P_{t+1}^F + D_{t+1}^F}{Q_t^F P_t^F}, \quad (5)$$

$$R_{t+1}^N = \frac{Q_{t+1}^N P_{t+1}^N + D_{t+1}^N}{Q_t^N P_t^N} \quad \hat{R}_{t+1}^N = \frac{\hat{Q}_{t+1}^N \hat{P}_{t+1}^N + \hat{D}_{t+1}^N}{\hat{Q}_t^N \hat{P}_t^N}. \quad (6)$$

¹⁸This assumption is necessary to obtain less than perfect risk-sharing in equilibrium. When nontradable equities are not traded, households do not have enough assets to span the shocks hitting the economy. As a result, asset markets are not dynamically complete. There is support for limited tradability of claims to nontraded firms in the data. In particular, based on Table 2 in Denis and Huizinga (2004), it can be shown that foreign ownership in firms belonging to the traded sector in Europe is two times larger than in firms belonging to the nontraded sector. Kang and Stulz (1997) study foreign equity ownership in Japan and find that foreign holdings of Japanese shares are heavily biased towards firms in manufacturing (traded) industries; while foreign ownership is underweighted in Electric, Power and Gas industries and Services (nontraded). Similarly, using U.S. Treasury (TIC) data as of December 31, 2007, I calculated that U.S. holdings of foreign equities in traded sectors are twice as large as U.S. holdings of foreign equities in non-traded sectors. Furthermore, the Survey of Consumer Finances suggests that the share of privately held firms is larger in the nontraded sector than in the traded sector. I study the role of the assumption about nontradability of n equity in Section 5.3 of the paper.

In this economy, the terms of trade are given by the relative price of ROW tradables, Q_t^F , so the terms of trade improves for the US when Q_t^F falls. The real exchange rate, \mathcal{E}_t , is defined as the relative price of foreign to domestic consumption, $\mathcal{E}_t = \hat{Q}_t/Q_t$, where Q_t and \hat{Q}_t are the US and ROW price indices given by

$$Q_t = \left(\lambda_T (Q_t^T)^{\frac{\phi}{\phi-1}} + \lambda_N (Q_t^N)^{\frac{\phi}{\phi-1}} \right)^{\frac{\phi-1}{\phi}} \quad \text{and} \quad \hat{Q}_t = \left(\hat{\lambda}_T (\hat{Q}_t^T)^{\frac{\phi}{\phi-1}} + \hat{\lambda}_N (\hat{Q}_t^N)^{\frac{\phi}{\phi-1}} \right)^{\frac{\phi-1}{\phi}}. \quad (7)$$

Q_t^T and \hat{Q}_t^T are the price indices of tradable consumption in the US and ROW:

$$Q_t^T = \left(\lambda_H + \lambda_F (Q_t^F)^{\frac{\rho}{\rho-1}} \right)^{\frac{\rho-1}{\rho}} \quad \text{and} \quad \hat{Q}_t^T = \left(\hat{\lambda}_H + \hat{\lambda}_F (Q_t^F)^{\frac{\rho}{\rho-1}} \right)^{\frac{\rho-1}{\rho}}. \quad (8)$$

2 Equilibrium

The first-order conditions for US households are given by

$$Q_t^F = (\partial U_t / \partial C_t^F) / (\partial U_t / \partial C_t^H), \quad (9a)$$

$$Q_t^N = (\partial U_t / \partial C_t^N) / (\partial U_t / \partial C_t^H), \quad (9b)$$

$$1 = \mathbb{E}_t [M_{t+1} R_t^1], \quad (9c)$$

$$1 = \mathbb{E}_t [M_{t+1} R_{t+1}^X], \quad \chi = \{H, F, N\} \quad (9d)$$

where $M_{t+1} \equiv M_{t+1,t}$. In the Appendix A, I show that $M_{t+1} = \beta W_t / W_{t+1}$. The first two equations define the relative prices of imported tradable and local nontradable consumption as ratios of their respective marginal utilities to the marginal utility of the numeraire, US tradables. The remaining equations are the standard pricing equations for the bond and equity. The first-order conditions for ROW households are symmetric.

The first-order condition for US tradable firms is

$$1 = \mathbb{E}_t [M_{t+1} R_{t+1}^K], \quad (10)$$

where $R_{t+1}^K = \theta Z_{t+1}^H (K_{t+1}^H)^{\theta-1} + (1 - \delta)$.

In equilibrium, households' and firms' decisions must also satisfy the market clearing conditions. By normalizing the household and firm populations in each country to unity, the output and consumption of tradables and nontradables can be obtained as the output and consumption of the representative firms and households. In the nontradable sector, consumption is equal to output, which is also equal to the dividends paid by nontradable firms to their shareholders:

$$C_t^N = D_t^N = Y_t^N,$$

$$\hat{C}_t^N = \hat{D}_t^N = \hat{Y}_t^N.$$

I assume that the tradables produced in each country are distinct, but can be costlessly transported internationally. Thus, in equilibrium, the world demand for each good must be equal to its corresponding production minus the amounts set aside for investment. For the tradable good produced in the US, market clearing requires that

$$C_t^H + \hat{C}_t^H = Y_t^H - I_t^H = D_t^H,$$

while for the tradables produced in the ROW, market clearing requires

$$C_t^F + \hat{C}_t^F = Y_t^F - I_t^F = D_t^F.$$

Market clearing in financial markets is equally straightforward. Since all nontradable equity is allocated domestically, $A_t^N = 1$ and $\hat{A}_t^N = 1$. The equity issued by US firms producing tradables (normalized to unity) must be held by US and ROW households, so $A_t^H + \hat{A}_t^H = 1$. Similarly, all tradable equity issued by ROW firms must be split by the households in the two countries, so that $A_t^F + \hat{A}_t^F = 1$. Finally, bonds are in zero net supply, so $B_t + \hat{B}_t = 0$, where \hat{B}_t denotes the bond holdings of ROW households.

An equilibrium in this economy consists of a set of goods' prices $\{Q^F, Q^N, \hat{Q}^N\}$, asset prices $\{P^H, P^F, P^N, \hat{P}^N\}$, and a risk-free rate R^1 , such that all markets clear when tradable firms optimally choose investment, and households optimally choose their consumption and portfolios, taking goods and asset prices as given.

3 Home Bias

In this section, I characterize the optimal consumption and portfolio decisions of households. I then derive the conditions under which there is home bias in equity holdings.

3.1 Decision Rules

The optimal consumption rules of US households are easy to obtain once we recognize that under logarithmic preferences the optimal consumption-wealth ratio is constant. Combining this result with the households' first-order conditions in (9a)-(9b) and taking logs gives equilibrium consumption as functions of wealth, relative prices and preference parameters:

$$c_t^H = w_t - \frac{\phi}{\phi-1} q_t + \left(\frac{\phi}{\phi-1} - \frac{\rho}{\rho-1} \right) q_t^T, \quad (11a)$$

$$c_t^F = w_t - \frac{\phi}{\phi-1} q_t + \left(\frac{\phi}{\phi-1} - \frac{\rho}{\rho-1} \right) q_t^T - \frac{1}{1-\rho} q_t^F, \quad (11b)$$

$$c_t^N = w_t - \frac{\phi}{\phi-1} q_t + \frac{1}{\phi-1} q_t^N. \quad (11c)$$

Hereafter, lowercase letters denote the natural logarithm of the uppercase counterpart in deviation from its steady state value or initial distribution (e.g., $c_t^H \equiv \ln C_t^H - \ln C_0^H$). Appendix A summarizes the approximation point of the economy and lists all equations used in the model's solution.

To understand how portfolio shares are determined, consider the first-order conditions for US households in (9d). These equations can be rewritten in log-approximate form as

$$\mathbb{E}_t r_{t+1}^\chi - r_t^1 + \frac{1}{2} \mathbb{V}_t (r_{t+1}^\chi) = -\mathbb{C}\mathbb{V}_t (m_{t+1}, r_{t+1}^\chi), \quad (12)$$

where r_{t+1}^χ is the log return for equity $\chi = \{H, F, N\}$, r_t^1 is the log risk free rate, and $m_{t+1} \equiv \ln M_{t+1} - \ln M_t$ is the log IMRS for US households. $\mathbb{V}_t(\cdot)$ and $\mathbb{C}\mathbb{V}_t(\cdot)$ denote the variance and covariance conditioned on period- t information. The left hand side of (12) is the equity risk premium on asset χ . Equation (12) shows that under an optimal consumption and portfolio plan, the risk premium is equal to the covariance of

that asset return with the log IMRS for US households.¹⁹ This covariance can be decomposed further using the fact that m_{t+1} is perfectly negatively correlated with the growth of household wealth, Δw_{t+1} . The risk premium in (12) can now be written as the covariance between wealth and the corresponding asset return:

$$\mathbb{E}_t r_{t+1}^\chi - r_t^1 + \frac{1}{2} \mathbb{V}_t (r_{t+1}^\chi) = \mathbb{C}\mathbb{V}_t (w_{t+1}, r_{t+1}^\chi). \quad (13)$$

This is a standard result. It says that households must choose their portfolios so that the expected excess returns match the covariance of returns with wealth.

The right-hand-side of (13) can also be approximated as

$$\gamma^H \mathbb{C}\mathbb{V}_t (c_{t+1}^H, r_{t+1}^\chi) + \gamma^F \mathbb{C}\mathbb{V}_t (q_{t+1}^F + c_{t+1}^F, r_{t+1}^\chi) + (1 - \gamma^H - \gamma^F) \mathbb{C}\mathbb{V}_t (q_{t+1}^N + c_{t+1}^N, r_{t+1}^\chi), \quad (14)$$

with

$$\gamma^F \equiv \left[\frac{1}{\lambda_H} (Q^T)^{\frac{\rho}{\rho-1}} - 1 \right] \gamma^H, \quad \text{and} \quad \gamma^H \equiv \left[\frac{1}{\lambda_T \lambda_H} (Q^T)^{\frac{\rho}{\rho-1} - \frac{\phi}{\phi-1}} (Q)^{\frac{\phi}{\phi-1}} \right]^{-1},$$

where Q^T and Q are the steady state values of Q_t^T and Q_t (see equations 7 and 8 above). Equation (14) implies that the equity risk premium on asset χ depends on a weighed average of three covariances, each between the log equity return and a corresponding log consumption: local and imported tradable and local nontradable. Detailed derivations of (12) - (14) are presented in the Appendix B.

To completely characterize the consumption and portfolio rules of US households, we need to specify the process for wealth and the distribution of returns. The process for US household wealth is obtained by log-linearizing the budget constraint in (3)²⁰:

$$\Delta w_{t+1} = r_{t+1}^W. \quad (15)$$

To characterize the log return on wealth, r_{t+1}^W , let $\mathbf{r}_t = [r_t^H \ r_t^F \ r_t^N]'$ denote the vector of log-returns on equity holdings in US and ROW tradables, and US nontradables, while let $\boldsymbol{\alpha}_t = [\alpha_t^H \ \alpha_t^F \ \alpha_t^N]'$ denote the vector of portfolio shares chosen by US households. Then, following Campbell et al. (2003), r_{t+1}^W can be approximated by

$$r_{t+1}^W = r_t^1 + \boldsymbol{\alpha}_t' (\mathbf{r}_{t+1} - r_t^1) + \frac{1}{2} \boldsymbol{\alpha}_t' (\text{diag}(\mathbb{V}_t(\mathbf{r}_{t+1})) - \mathbb{V}_t(\mathbf{r}_{t+1}) \boldsymbol{\alpha}_t). \quad (16)$$

The dynamics of ROW household wealth and its return are identified in a similar way.

The optimal portfolio policies of US households can now be easily obtained from (13) by substituting for wealth from the budget constraint (15) and combining the result with (16). Solving the resulting system of equations for $\boldsymbol{\alpha}_t$ gives

$$\boldsymbol{\alpha}_t = \mathbb{V}_t(\mathbf{r}_{t+1})^{-1} \left[\mathbb{E}_t \mathbf{r}_{t+1} - r_t^1 + \frac{1}{2} \text{diag}(\mathbb{V}_t(\mathbf{r}_{t+1})) \right]. \quad (17)$$

Equation (17) states that the share of wealth allocated into each asset is proportional to the vector of conditional log-Sharpe ratios. It also shows that if the conditional moments of asset returns are time-varying, so will be the portfolio shares. In this model households consume a constant fraction of wealth

¹⁹Excess returns are adjusted by the addition of one half times the return variance, a Jensen's inequality term, to account for the fact that I am working with log returns.

²⁰This approximation also uses the fact that the consumption-wealth ratio is constant in the model.

every period and invest the remaining wealth according to the risk premia on the available assets, scaled by the inverse of the variance-covariance matrix of returns.

The demand for equities and bonds by US and ROW households are obtained from the optimal portfolio shares in (17) as

	US households	ROW households
US tradable equity:	$A_t^H = \alpha_t^H W_t^C / P_t^H,$	$\hat{A}_t^H = \hat{\alpha}_t^H \hat{W}_t^C / P_t^H,$
ROW tradable equity:	$A_t^F = \alpha_t^F W_t^C / Q_t^F P_t^F,$	$\hat{A}_t^F = \hat{\alpha}_t^F \hat{W}_t^C / Q_t^F P_t^F,$
nontradable equity:	$A_t^N = \alpha_t^N W_t^C / Q_t^N P_t^N,$	$\hat{A}_t^N = \hat{\alpha}_t^N \hat{W}_t^C / \hat{Q}_t^N \hat{P}_t^N,$
bonds	$B_t = \alpha_t^B W_t^C R_t^1,$	$\hat{B}_t = \hat{\alpha}_t^B \hat{W}_t^C R_t^1,$

(18)

where $W_t^C \equiv W_t - C_t^H - Q_t^F C_t^F - Q_t^N C_t^N$ and $\hat{W}_t^C \equiv \hat{W}_t - \hat{C}_t^H - Q_t^F \hat{C}_t^F - \hat{Q}_t^N \hat{C}_t^N$ denote period- t wealth net of consumption expenditure, while $\alpha_t^B \equiv 1 - \alpha_t^H - \alpha_t^F - \alpha_t^N$ and $\hat{\alpha}_t^B \equiv 1 - \hat{\alpha}_t^H - \hat{\alpha}_t^F - \hat{\alpha}_t^N$ are the shares of wealth allocated into bonds.

3.2 When is Home Bias Optimal?

The equity portfolios of US households display home bias if the share of the wealth they allocate to equity issued by US firms producing tradables exceeds the share of their wealth allocated to the equity issued by ROW firms producing tradables: $\alpha_t^H > \alpha_t^F$. Similarly, the portfolios of ROW households display home bias if $\hat{\alpha}_t^H < \hat{\alpha}_t^F$. From equations in (18) one can see that in this model home bias in portfolio shares also implies a home bias in the asset holdings. For US households home bias implies $A_t^H > A_t^F$, while for ROW households it implies $\hat{A}_t^H < \hat{A}_t^F$. The discussion in this section focuses on home bias in portfolio shares. I will study home bias in equity holdings numerically in section 5.1 below.

To provide intuition for the existence of equity home bias, consider the portfolio rule in (17). In particular, consider the share of wealth allocated by US households to the equity issued by US tradable firms (the first row of α_t vector):

$$\alpha_t^H = \frac{\mathbb{V}_t(r_{t+1}^F) [\mathbb{C}\mathbb{V}_t(w_{t+1}, r_{t+1}^H) - \mathbb{C}\mathbb{V}_t(r_{t+1}^N, r_{t+1}^H) \alpha_t^N]}{\mathbb{V}_t(r_{t+1}^H) \mathbb{V}_t(r_{t+1}^F) - \mathbb{C}\mathbb{V}_t^2(r_{t+1}^H, r_{t+1}^F)} - \frac{\mathbb{C}\mathbb{V}_t(r_{t+1}^H, r_{t+1}^F) [\mathbb{C}\mathbb{V}_t(w_{t+1}, r_{t+1}^F) - \mathbb{C}\mathbb{V}_t(r_{t+1}^N, r_{t+1}^F) \alpha_t^N]}{\mathbb{V}_t(r_{t+1}^H) \mathbb{V}_t(r_{t+1}^F) - \mathbb{C}\mathbb{V}_t^2(r_{t+1}^H, r_{t+1}^F)}. \quad (19)$$

This equation determines the optimal share of US tradable equity held by US households for a given nontradable equity share, α_t^N . It is also a function of the covariance between wealth and returns in the US and ROW. The following proposition allows us to examine the determinants of α_t^H in detail.

Proposition 1 *When (i) all uncertainty is resolved after period $t + 1$; (ii) countries are symmetric in preferences, technology, and initial distribution of wealth; and (iii) the share of nontradable equity in household portfolios is equal to 1/2, then the share of domestic assets in US tradable portfolios is given by*

$$\alpha_t^H = \frac{1}{4} + bias$$

with

$$bias = \frac{1}{4\sigma_t^2} [\beta_t^{\text{HF}} \mathbb{C}\mathbb{V}_t(q_{t+1}^F + d_{t+1}^F, \Delta\eta^N) - \mathbb{C}\mathbb{V}_t(d_{t+1}^H, \Delta\eta^N)], \quad (20)$$

where $\Delta\eta^N \equiv q_{t+1}^N + d_{t+1}^N - (\hat{q}_{t+1}^N + \hat{d}_{t+1}^N)$ denotes the relative value of nontradable endowment in US versus ROW; β_t^{HF} is the conditional beta of the payoff on US tradable equity with respect to the payoff on ROW tradable equity, $\beta_t^{\text{HF}} \equiv \mathbb{C}\mathbb{V}_t(d_{t+1}^H, q_{t+1}^F + d_{t+1}^F) / \mathbb{V}_t(q_{t+1}^F + d_{t+1}^F)$, while $\sigma_t^2 \equiv \mathbb{V}_t(d_{t+1}^H)(1 - \rho_{\text{HF}}^2) > 0$ is the conditional variance in this projection; ρ_{HF} is the correlation coefficient between US and ROW tradable dividends.

Proof. See Appendix C. ■

Condition (i) assumes that productivity in traded and nontraded sectors in both countries is set at its steady state level after $t + 1$. This assumption collapses the dynamics in the model to a two-period (t and $t + 1$) problem. At the start of period t , households and firms learn the period- t productivity shocks in the tradable and nontradable sectors. Households then decide on their consumption and asset allocation demands, while firms determine their optimal production and dividend policies. Under condition (i), from period $t + 2$ onwards, the dividend-price ratio and future returns for each asset are constant, so all the risk from holding equities between t and $t + 1$ comes from unanticipated variations in period $t + 1$ dividend payments, which in turn are determined by the variations in firms' optimal production plans in response to unanticipated changes in their productivity in $t + 1$. Conditions (i) and (ii) imply that bond holdings are zero. The share of wealth allocated by US households into nontradable equity, α_t^N , is endogenous. However, in equilibrium, market clearing ensures that holdings of nontradable equity are equal to unity, so all variations in α_t^N must be due to changes in wealth and/or the price of nontradable equity. To abstract from these variations, condition (iii) sets α_t^N equal to its steady state value of $1/2$.²¹

The *bias* term in (20) measures the degree to which US households skew their portfolios towards the equity issued by US firms producing tradables. The bias arises due to the hedging demand for tradable equity, which develops as households seek to insure against the fluctuations in their uninsurable nontradable consumption.

The *sign* of the bias depends on the sign of the terms in brackets, which are determined by the co-movements in the value of dividend streams on different assets. The *extent* of the bias depends on the relative ability of US versus ROW tradable equity in hedging unexpected fluctuations in the relative nontradable endowment in the US.²² In particular, when the amount of US nontradable risk co-moves more strongly with the payoffs of a particular tradable asset (say, ROW tradable equity), the diversification benefit from holding the alternative tradable asset (say, US tradable equity) will be larger. Notice that US households assign weight β_t^{HF} to the hedging ability of the ROW tradable asset. This β_t^{HF} term can be interpreted as

²¹It can be shown that in the steady state, the value for α^N is proportional to the share of nontraded consumption in the total consumption expenditure: $\alpha^N = \lambda_N^{1-\phi}(C^N/C_0)^\phi$. When tradable and nontradable sectors have equal size ($\lambda_N = 1/2$), as in my model, then $\alpha^N = 1/2$.

²²Recall that nontradable output is only consumed domestically and is produced without capital. Thus, in equilibrium, nontradable dividends and consumption are determined entirely by the exogenous state of productivity in the domestic nontraded sector.

a coefficient in the regression of US tradable equity payoffs on ROW tradable equity payoffs and thus, in the spirit of factor asset pricing models, measures the amount of risk that US asset carries relative to ROW asset.²³ The signs of the covariances entering the *bias* term will depend on the structural parameters of the model, in particular on ϕ and ρ which determine the intratemporal elasticities of substitution between consumption goods; and on the degree of consumption home bias.

In general, the covariances in the bias term will also be affected by the covariance structure of the underlying productivity shocks. Given there is considerable uncertainty about the properties of productivity across sectors and countries, I assume that they are independent in my model. Instead, I introduce capital accumulation in the traded sector. As a result, the properties of tradable dividends are endogenously determined by firms through the optimal investment decisions in response to productivity shocks. The bias term in (20) illustrates the importance of capital accumulation in the model. Consider what happens to the bias term in an endowment economy when productivity shocks across sectors and across countries are independent. In response to an unanticipated positive productivity innovation in the US nontraded sector, the dividends in the US and ROW traded sectors can not change, so that $\mathbb{C}\mathbb{V}_t(d_{t+1}^H, \Delta r^N) = 0$ and $\beta_t^{\text{HF}} = 0$, giving *bias* = 0. As a result, in an incomplete markets endowment economy with independent productivity processes across sectors and countries, traded equity portfolios are fully diversified. The studies that analyze equity home bias in endowment economies, therefore, must rely on productivity processes that are correlated across sectors or countries or both (see Tesar (1993), Coeurdacier (2008), Collard, Dellas, Diba and Stockman (2007), Matsumoto (2007)). By working with independent productivity processes but allowing for capital accumulation in the model, I can abstract from the portfolio results driven by the assumptions on productivity.

I next show that by varying ϕ and ρ my model encompasses a wide variety of models and results developed in the home bias literature. First consider the scenario when $\frac{1}{1-\phi} \rightarrow \infty$ and $\frac{1}{1-\rho} \rightarrow \infty$. In this case all goods in the economy (i.e., both types of tradables and nontradables) are perfect substitutes. If productivity shocks are independent across sectors and countries, the *bias* term in (20) disappears, giving $\alpha_t^H = 1/4$ and $\alpha_t^F = 1/4$. As a result, the optimal tradable portfolios of households in both countries are fully diversified. When $\frac{1}{1-\phi} = 1$, $\frac{1}{1-\rho} = 1$ no bias in tradable portfolios arises in equilibrium. A similar result was shown by Cole and Obstfeld (1991) and is due to the fact that any increase in nontradable or tradable dividends causes a proportional fall in the corresponding relative price, which makes all the covariances in the numerator of (20) equal to zero.

Next, we can analyze the role of nontradable sector in explaining portfolio home bias. Consider the case when $\frac{1}{1-\rho} \rightarrow \infty$, so that tradables are perfect substitutes, but $\frac{1}{1-\phi} \neq 1$ and $\neq \infty$. When $\frac{1}{1-\phi} > 1$ tradable and nontradable goods are considered substitutes and each covariance term, including β_t^{HF} , in the *bias* is

²³With tradable dividends being determined endogenously, β_t^{HF} also provides a convenient indicator for how the productivity shocks are transmitted across countries. In the terms of Cole and Obstfeld (1991), when countries are specialized in production of tradable goods, the tradable productivity shocks are “transmitted positively” between countries due to the relative price movements. As a result, $\mathbb{C}\mathbb{V}_t(d_{t+1}^H, q_{t+1}^F + d_{t+1}^F) > 0$, so β_t^{HF} is positive; when $\mathbb{C}\mathbb{V}_t(d_{t+1}^H, q_{t+1}^F + d_{t+1}^F) < 0$, shocks to tradable production are “transmitted negatively” across countries, and so β_t^{HF} becomes negative. This latter case occurs when both countries produce a homogenous tradable good.

positive. In this setup, an increase in nontradable endowment lowers the demand for tradable consumption. If US tradable firms are more likely to pay higher dividends than ROW firms when those dividends are less valued, US households would bias their portfolios in favor of ROW equity. This case corresponds to the result in Baxter and Jermann (1997) with application to nontraded labor income risk.²⁴ Now consider the scenario with $\frac{1}{1-\phi} < 1$. In this case any change in nontradable dividends is more than offset by the change in the relative price of nontradables, making the nontradable covariance terms in (20) negative. Furthermore, when $\frac{1}{1-\phi} < 1$, households like to consume a balanced basket of tradable and nontradable goods (i.e., the two goods are complements). Using the logic above, any increase in nontradable endowment will lead to a higher demand for tradable consumption. If US tradable firms are more likely to deliver higher dividends than ROW firms in those states of the world, then home bias will arise. Based on the empirical estimates in the literature, in the numerical analysis below I focus on the case with $\frac{1}{1-\phi} < 1$.²⁵

Finally, consider the case with $\frac{1}{1-\phi} < 1$ and $\frac{1}{1-\rho} \neq 1$ and $\rightarrow \infty$. Here each country specializes in the production of a distinct tradable good; and nontradables and a basket of tradables are complements in consumption. In this case variations in the terms of trade, q_t^F , add another dimension to the portfolio problem. As before, US portfolios will deviate in favor of US assets when the relative endowment of nontradables is more correlated with the value of US than ROW tradable dividends. Intuitively, when nontradables and tradables are complements, any increase in nontradable endowment is associated with an increase in the demand for tradable goods. How this higher tradable demand is distributed across the local and imported tradables is determined by two parameters: (i) the degree of consumption home bias, which determines the desirability of domestically produced good relative to foreign produced good and thus triggers relative price adjustments; and (ii) the elasticity of substitution between the two tradables, which controls the degree of adjustment in the relative price of the two tradables necessary to accommodate the changes in their relative demand. Below I numerically analyze this case in detail.

4 Solving the Model

While the two-period framework developed in the previous section provides a useful shortcut for developing the intuition for the home bias, it relies on the simplifying conditions in Proposition 1. Next, I study the competitive equilibrium in the full dynamic model numerically.

Parameter values for the benchmark calibration of the model are summarized in Table 1. The world economy is constructed as consisting of two symmetric countries, matching the properties of US economy in quarterly data. Most of the preference parameter values are borrowed from Corsetti et al. (2008) who use the same consumption aggregators as this paper. In particular, the value for ϕ is chosen to set the elasticity of substitution between tradable and nontradable consumption to 0.74. The share of tradables and

²⁴Baxter and Jermann (1997) show that when labor income correlates positively with capital income, investors take large short positions in domestic assets, thus generating a reverse home bias.

²⁵A number of studies estimate the intratemporal elasticity of substitution between tradable and nontradable consumption to be below one. For a sample of industrial countries, Mendoza (1991) estimates this elasticity to be 0.74. Corsetti, et al. (2008) use the same value in their calibration. Tesar (1993), Stockman and Tesar (1995) set $\frac{1}{1-\phi} = 0.44$.

nontradables in aggregate consumption expenditure, λ_T and $\lambda_N = 1 - \lambda_T$, are set to 0.5 in both countries. This number is calculated using OECD STructural ANalysis (STAN) database.²⁶

Table 1. Benchmark Model Parameters

PREFERENCES	β	λ_T	λ_N	$1/(1 - \phi)$
	0.99	0.5	0.5	0.74
		λ_H	$\hat{\lambda}_F$	$1/(1 - \rho)$
		0.72	0.72	1.10
PRODUCTION	θ	δ		
	0.36	0.02		
PRODUCTIVITY	a_{ii}^H	a_{ii}^N	Ω_e	
	0.78	0.99	0.0001	

Preferences over the consumption of local and imported tradables are calibrated using a range of possible values. Under the baseline calibration, the value of ρ is set to obtain the elasticity of substitution between US and ROW tradables equal to 1.1. Given considerable uncertainty about the value of this elasticity in the literature, the model is also solved for a sequence of values for $1/(1 - \rho)$ ranging from 1.05 to 115.²⁷ I also allow countries to exhibit consumption home bias in their preferences by setting $\lambda^H = \hat{\lambda}^F = 0.72$ as estimated by Corsetti et al. (2008).

On the production side, the capital share in tradable production, θ , is set to 0.36, and the depreciation rate, δ , is set to 0.02. These values are consistent with the estimates in Backus, Kehoe and Kydland (1995). I assume that each of the four productivity processes (i.e., $\ln Z_t^H, \ln Z_t^F, \ln Z_t^N$, and $\ln \hat{Z}_t^N$) follows an AR(1) process. However, in order to isolate the role of relative prices and endogenous dividend policies in explaining portfolio choices, I assume that productivity changes are independent across sectors and countries. The AR(1) coefficients in the processes for tradable productivity, $\ln Z_t^H$ and $\ln Z_t^F$, are 0.78, while the coefficients for nontradable productivity, $\ln Z_t^N$, and $\ln \hat{Z}_t^N$, are 0.99. These values are comparable to those used by

²⁶These numbers are similar to the estimates in the literature. For instance, Corsetti et al. (2008) and Dotsey and Duarte (2006) use $\lambda_N = 0.55$, Stockman and Tesar (1995) report λ_N close to 0.5; Pesenti and van Wincoop (2002) also argue that 0.5 of consumers budget is allocated to nontradables; Benigno and Thoenissen (2008) assume $\lambda_N = 0.45$.

²⁷In the literature, there is range of estimates available for the elasticity of substitution between tradable goods. In a model with tradable goods only Backus, Kehoe, and Kydland (1995) use a value of 1.5. Kollmann (2005) uses traded elasticity values as low as 0.6; Corsetti, et.al. (2008) assume the value of 0.85; Heathcote and Perri (2004) use elasticity equal to 0.9 in their benchmark parametrization; Chari, Kehoe, and McGrattan (2002), Engel and Matsumoto (2006), and Collard, et al. (2007) all use 1.5; Matsumoto (2007) uses a value of 2; Coeurdacier (2008) uses 5 in the benchmark calibration. Given the multitude of estimates, there does not seem to be a well-established reference value for this elasticity. Therefore, I provide the portfolio results for a range of the values for this elasticity. Under the benchmark parametrization I use the value of 1.1, which is about an average in the range of estimates used in the literature.

Corsetti et al. (2008). Shocks to all four productivity processes have a variance of 0.0001. This specification implies that all shocks have persistent but temporary effects on productivity.

To find the competitive equilibrium for the model, I apply the solution method developed in Evans and Hnatkovska (2005b). Their procedure extends the work of J. Campbell and co-authors (for instance, Campbell et al. (2003)) on dynamic portfolio choice to a general equilibrium setting. In short, the technique combines a perturbation technique commonly used in solving macro models with continuous-time approximations common in solving finance models of portfolio choice. The numerical procedure produces the equilibrium dynamics of the state variables as functions of their past values, their squares, and cross-products. The variance-covariance matrix of the state vector is similarly obtained. The solution also provides me with a set of decision rules, all expressed as linear functions of a vector of state variables as well as their second moments.²⁸

One of the well-known complications associated with incomplete asset markets is that temporary shocks that have no long run effect on real variables can have very persistent effects on the wealth of individual households, that is they can lead to non-stationary wealth distribution. The solution method developed in Evans and Hnatkovska (2005b) allows me to characterize the equilibrium behavior of the economy in a neighborhood around a particular initial wealth distribution. The advantage of this approach is that it does not require an assumption about how the international distribution of wealth is affected by such shocks in the long run. The disadvantage is that the characterization of the equilibrium dynamics is only accurate while wealth remains close to the initial distribution. Evans and Hnatkovska (2005b) show that this does not appear to be an important limitation in practice. In my model I find that model solution remains accurate for simulations that are 200 periods long. This is the horizon I use to study the characteristics of the equilibrium. The statistics reported in the next section are derived from 200 simulation samples and so are based on 40,000 observations of simulated data in the neighborhood of the equal initial wealth distribution.

5 Results

This section presents the main findings from the numerical solution to the model. First, I characterize the optimal portfolio holdings. Second, I analyze the capital flows and the factors driving them. I then evaluate the importance of market incompleteness for portfolio choice and risk-sharing between countries. Finally, I examine the benefits of having dynamic portfolio choice in the model as opposed to static portfolio choice by comparing the conditional and unconditional moments of the portfolio returns; as well as welfare associated with dynamic and static portfolio rules.

²⁸The most recent version of Evans and Hnatkovska (2005b) paper contains detailed evaluation of the method's accuracy as well as its comparison with the related methodology proposed in Devereux and Sutherland (2007) and Tille and van Wincoop (2007). In that paper, based on a battery of standard accuracy tests, we show that our method works well for models with incomplete markets, portfolio choice and for different degrees of risk aversion.

5.1 Portfolios

I now turn to the analysis of the equilibrium portfolio rules predicted by the model. Figure 1 shows the distribution of tradable equity holdings $\{A_t^H, A_t^F, \hat{A}_t^H, \hat{A}_t^F\}$. The left hand panel illustrates US households' holdings of equity issued by US and ROW firms producing tradables, A_t^H and A_t^F . ROW holdings of equity issued by ROW and US tradable firms, \hat{A}_t^F and \hat{A}_t^H , are shown in the right hand panel. Two main results stand out: first, portfolio holdings are volatile, second, they are biased towards the equity issued by domestic tradable firms.

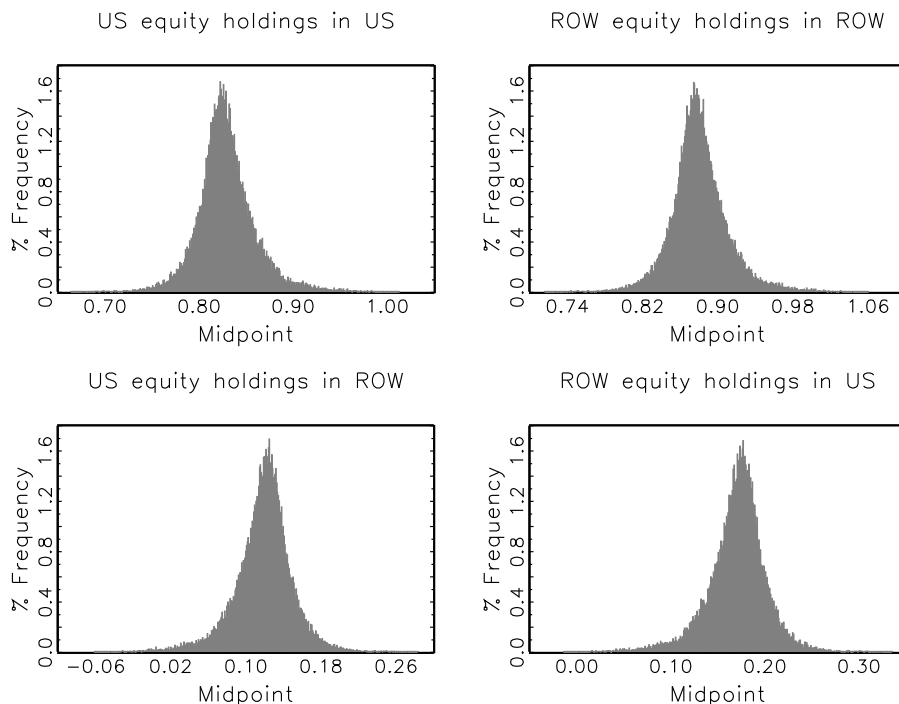


Figure 1. Distribution of portfolio holdings in the model

Table 2 reports descriptive statistics on the households equity holdings. Columns (i) and (iii) show that on average more than 80% of the equity issued by tradable firms is held by domestic households.²⁹ Consistent with the visual evidence in Figure 1, equity holdings are also very volatile.

The model correctly predicts both the direction and the magnitude of portfolio home bias. The *direction* of the bias is attributed to the consumption bias towards domestic tradable goods. When given a choice between two tradable equities (US and ROW), households increase their holdings of the asset whose payoffs are relatively high in states of the world in which the demand for those payoffs is also high. Due to the

²⁹A small asymmetry in the average degree of home bias across countries arises from the denomination of bonds in the model. In particular, because bonds are denominated in units of US tradables, US households find them to be safer means of borrowing and lending than ROW households do. As a result, US households tend to go long in the bond (i.e. $\alpha_t^B = 2.59$), while ROW households tend to go short in the bond (i.e. $\hat{\alpha}_t^B = -2.59$) on average. At the same time the net foreign asset positions in the model remain balanced and symmetric across countries. In other words, the asymmetry in the equity distribution is matched by the distribution of bond holdings across countries in my model.

Table 2. International Equity Positions

	US T equity holdings in		ROW T equity holdings in	
	US, A_t^H	ROW, A_t^F	ROW, \hat{A}_t^F	US, \hat{A}_t^H
	(i)	(ii)	(iii)	(iv)
mean	0.829	0.119	0.881	0.171
std. dev.	0.032	0.031	0.031	0.032
min	0.663	-0.062	0.715	-0.014
max	1.014	0.285	1.062	0.337
5th percentile	0.783	0.066	0.835	0.117
95th percentile	0.883	0.166	0.934	0.217

Notes: Statistics reported in the table are based on 200 simulations of quarterly series, each 200 periods long.

complementarity in preferences over tradable and nontradable goods, such states of the world occur when the relative domestic nontradable endowment is high. Therefore, when preferences in each country are biased in favor of local tradables, this condition requires that the relative nontradable endowment is correlated with the value of domestic tradable dividends in excess of the corresponding covariance with the value of foreign tradable dividends. As explained in section 3.2, in this case households bias their portfolios towards domestic tradable assets.

The *strength* of the portfolio bias is determined by the differential in these covariances, which in turn is critically related to the elasticity of substitution between US and ROW tradable consumption, $1/(1 - \rho)$. Figure 2 graphs the average equilibrium holdings of tradable equity by US and ROW households for a range of elasticities.³⁰ When tradables are perfect substitutes, and the productivity processes are independent across sectors and across countries, the optimal portfolios of US and ROW households are equally split between the equity issued by US and ROW tradable firms even in the presence of consumption home bias. As the elasticity of substitution between tradables falls, the share of local equity in portfolios initially declines until $1/(1 - \rho) = 2.5$, implying foreign bias, but then rises as the elasticity falls further.³¹ When $1/(1 - \rho) = 1.1$ (the baseline parametrization), household holdings of domestic tradable equity are well above their holdings of foreign tradable equity.

³⁰These graphs are computed by solving the model for different values of ρ and then simulating each solution 200 times over 200 quarters. Figure 2 plots the average values of A_t^H , A_t^F , \hat{A}_t^H and \hat{A}_t^F from these simulations.

³¹The result that equity portfolios are sensitive to the elasticity of substitution between traded goods is not new to this paper. For instance, Kollmann (2005) and Heathcote and Perri (2004) show that a one-sector model with complete markets exhibits significant sensitivity of portfolio positions to the elasticity of substitution between tradable goods. Matsumoto (2007) shows a similar result in a two-sector (tradable and nontradable) model with complete markets. By adopting a very traditional international business cycle model to study home bias and capital flows together in a unified framework, this paper inherited the sensitivity of portfolio holdings to the elasticity parameter.

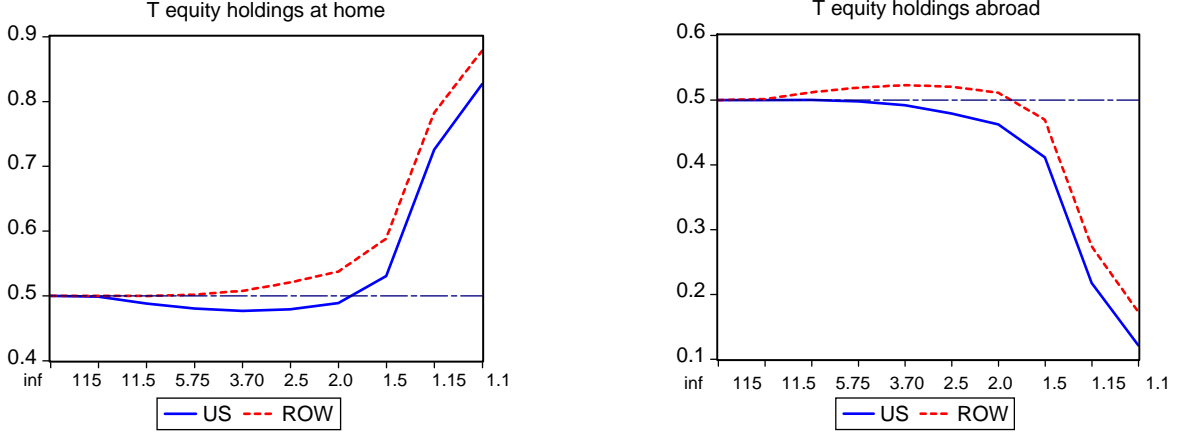


Figure 2. Varying $1/(1 - \rho)$, the elasticity of substitution between T's in consumption

To understand this pattern, note that changes in the elasticity mostly affect the behavior of the terms of trade, which in turn influence the hedging capacity of foreign equity. Consider first the case of infinite elasticity of substitution between tradables, so that US and ROW tradables are perfect substitutes. Under this parametrization changes in relative consumption demand for tradables, arising from nontradable productivity shocks, do not induce any shifts in the relative price, q^F . US and ROW tradable firms increase their dividend payout in response to positive nontradable productivity shocks by the same amount. This implies $\beta_t^{\text{HF}} = 1$. With no movements in q^F , the relative value of US to ROW dividend payments remains unaffected, and the rationale for home bias disappears. As a result, T equity holdings are diversified, $A_t^{\text{H}} = \hat{A}_t^{\text{F}} = 1/2$. As the elasticity of substitution $1/(1 - \rho)$ declines, any changes in ROW dividends are counterbalanced by the changes in the relative price. As a result, $\text{CV}_t(q_{t+1}^{\text{F}} + d_{t+1}^{\text{F}}, d_{t+1}^{\text{N}} - \hat{d}_{t+1}^{\text{N}})$ falls relative to $\text{CV}_t(d_{t+1}^{\text{H}}, d_{t+1}^{\text{N}} - \hat{d}_{t+1}^{\text{N}})$, thus strengthening the degree of portfolio home bias.³²

Optimal portfolio choice also depends on the beta, β_t^{HF} , defined by $\text{CV}_t(d_{t+1}^{\text{H}}, q_{t+1}^{\text{F}} + d_{t+1}^{\text{F}}) / \mathbb{V}_t(q_{t+1}^{\text{F}} + d_{t+1}^{\text{F}})$. Because nontradable shocks lead to higher dividend payments by both US and ROW tradable firms, $\beta_t^{\text{HF}} > 0$ for all values of $1/(1 - \rho) > 1$. The response of β_t^{HF} to the fall in $1/(1 - \rho)$ is however non-monotonic. The covariance between US and ROW tradable dividends which appears in the numerator of β_t^{HF} declines monotonically as $1/(1 - \rho)$ falls. The variance of ROW equity payoffs which appears in the denominator of β_t^{HF} is the source of non-monotonicity in β_t^{HF} . To see this, we can write the variance term as $\mathbb{V}_t(q_{t+1}^{\text{F}}) + \mathbb{V}_t(d_{t+1}^{\text{F}}) + 2\text{CV}_t(q_{t+1}^{\text{F}}, d_{t+1}^{\text{F}})$. A decline in the elasticity is associated with an increase in $\mathbb{V}_t(q_{t+1}^{\text{F}})$ as larger shifts in the terms of trade are necessary to induce changes in the relative consumption of tradables. At the same time, the covariance, $\text{CV}_t(q_{t+1}^{\text{F}}, d_{t+1}^{\text{F}})$, while negative, falls in absolute value. The latter effect dominates when the elasticity is high, so $\mathbb{V}_t(q_{t+1}^{\text{F}} + d_{t+1}^{\text{F}})$ initially declines. This can lead to β_t^{HF} exceeding 1, so that US equity carries more risk relative to ROW equity. As a result, the *bias* becomes smaller. When the

³²Recall that when $1/(1 - \phi) < 1$, the nontradable covariance terms in (20) are negative. In this case, the sign of the bias is determined by the sign of $-\beta_t^{\text{HF}} \text{CV}_t(q_{t+1}^{\text{F}} + d_{t+1}^{\text{F}}, d_{t+1}^{\text{N}} - \hat{d}_{t+1}^{\text{N}}) + \text{CV}_t(d_{t+1}^{\text{H}}, d_{t+1}^{\text{N}} - \hat{d}_{t+1}^{\text{N}})$.

elasticity falls below 2.5, the effect of higher $\mathbb{V}_t(q_{t+1}^F)$ dominates and the variance of ROW equity payouts becomes larger. This can reduce β_t^{HF} below 1, making US equity less risky relative to ROW equity. The *bias* term thus increases. The threshold elasticity of 2.5 produces an equilibrium in which investors in both countries choose portfolios that are equally-weighted in the equities issued by US and ROW tradable firms.

The source of home bias arising from the presence of nontraded goods has been studied in the empirical literature. Tesar (1993) documents empirically that for a sample of OECD economies during 1961-1985 the correlation of domestic output of non-traded goods with domestic output of traded goods exceeded the corresponding correlation with foreign output of traded goods. She then shows that in combination with positive cross-derivative of utility (with respect to traded and nontraded consumption), this finding is supportive of the hypothesis that nontraded goods help explain equity home bias in industrial countries. Pesenti and van Wincoop (2002) compute more direct measures of returns to tradables and nontradables and find that their correlation pattern is similar to that in Tesar (1993). Overall, the existing empirical evidence on correlations between returns to nontraded equity and domestic versus foreign traded equities is qualitatively consistent with the pattern required to generate home bias in the model.

In summary, two results stand out from the analysis of households' average asset holdings. First, the degree of home bias in traded portfolios is sensitive to the elasticity of substitution between domestic and foreign traded goods. As shown in Figure 2, the model predicts equity home bias for the values of elasticity below 2, which seems to be the most commonly used range of values for this elasticity in the literature. Second, at the baseline value for the elasticity equal to 1.1, the lower relative riskiness of domestic tradable equity combined with the reduced incentives for pooling risks generate home bias, the magnitude of which is comparable with its empirical counterpart (see footnote 1).

5.2 Asset Turnover

In symmetric isoelastic economies with complete risk-sharing, portfolio holdings are time-invariant and capital flows are zero. The model developed in this paper generates variable asset holdings and thus allows me to study the properties of capital flows. Table 3 describes the behavior of the bond and equity flows between countries measured relative to stock market capitalization. Panel I summarizes the findings for the size and volatility of the flows in the model, while panel II contrasts those numbers with the data. The bond flows are computed as $\frac{1}{R_t}(B_t - B_{t-1}) \left[= -\frac{1}{R_t}(\hat{B}_t - \hat{B}_{t-1}) \right]$, net US purchases of foreign assets (or US equity outflow) as $Q_t^F P_t^F (A_t^F - A_{t-1}^F)$, and net foreign purchases of US assets (or US equity inflow) as $P_t^H (\hat{A}_t^H - \hat{A}_{t-1}^H)$ using the equilibrium portfolio shares and wealth as shown in (18).³³

Columns (i) and (ii) show that both equity flows are large and volatile. To share risks, households have to frequently adjust their positions and these adjustments tend to be large. The size and volatility of bond flows, shown in column (iii), are much below those for equity. Overall, the model seems to overpredict the size and volatility of equity flows, and underpredict those characteristics for the bond flows.

A perspective on the magnitude of the capital flows can be gained by considering their contribution to

³³These definitions of capital flows do not take into account capital gains or losses on the existing assets. Instead they allow me to focus on the portfolio flows resulting from changes in holdings of different assets.

Table 3. International Portfolio Flows, % Market Capitalization

	I: Model			II: Data		
	equity outflow	equity inflow	net bond flow	equity outflow	equity inflow	net bond flow
	(i)	(ii)	(iii)	(iv)	(v)	(vi)
std. dev.	0.38	0.38	0.01	0.13	0.13	0.37
min	-2.65	-2.77	-0.05	-0.61	-0.20	-0.30
max	2.59	2.60	0.06	0.12	0.44	1.37

Notes: Statistics reported in panel I (Model) are based on 200 simulations of quarterly series, each 200 periods long. The properties of capital flows are studied relative to stock market capitalization, measured as $P_t^H A_t^H + Q_t^F P_t^F A_t^F + Q_t^N P_t^N A_t^N + \frac{1}{R_t} B_t$ in US and a symmetric expression in ROW. Statistics in panel II (Data) are calculated for the US portfolio debt and equity flows vis-a-vis the rest of the world using IFS Balance of Payments data. Net bond flows are obtained as a sum of US debt inflows (+) and outflows (-). Stock market capitalization for US is obtained from World Bank's World Development Indicators (WDI). The sample period is 1990:1-2007:1.

the current account. For this purpose, I decompose the current account from the model into the components attributable to bond and equity flows. The results of such a decomposition are presented in Table 4. As the first row indicates, the current account balance has volatility of 0.92% in the US. Measured relative to GDP the volatility of current account is 0.13% in the US. This number matches well the corresponding statistic in the US data equal to 0.12% during 1990:1-2007:1. Variation in equity holdings contribute the most to the current account variance. Thus, a 1% improvement in the US current account in the model is accompanied by: (i) a 10.78% decrease in US foreign equity assets, which is matched by an inflow of US tradables, (ii) a 10.01% fall in US foreign equity liabilities, accompanied by a corresponding outflow of US tradables, and (iii) a 1.77% outflow of resources due to an increase in US bond holdings.³⁴

The directions of the flows are determined by how households adjust their asset holdings in response to changes in the economy's productivity. In particular, the change in US foreign equity assets can be decomposed using the definition of A_t^F in (18) as:

$$Q_t^F P_t^F \Delta A_t^F = \Delta \alpha_t^F W_t^c + \alpha_{t-1}^F \Delta W_t^c - [Q_t^F P_t^F / Q_{t-1}^F P_{t-1}^F - 1] \alpha_{t-1}^F W_{t-1}^c. \quad (21)$$

Equation (21) shows that US households may change their asset position in order to accommodate: (i) shifts in absolute risk premia across countries, as captured by $\Delta \alpha_t^F W_t^c$; (ii) changes in wealth, $\alpha_{t-1}^F \Delta W_t^c$; and (iii) capital gains or losses on the existing portfolios, $[Q_t^F P_t^F / Q_{t-1}^F P_{t-1}^F - 1] \alpha_{t-1}^F W_{t-1}^c$.³⁵ The contribution of each

³⁴It is worth noting that the correlation among different classes of portfolio flows predicted by the model is consistent with U.S. data. For instance, during 1990:1-2007:1 net U.S. purchases of foreign equities and net foreign purchases of U.S. equities were positively correlated, while both equity flows were negatively correlated with net bond flows. This is also the pattern predicted by the model.

³⁵In the terminology of Kraay and Ventura (2003), (i) is a portfolio rebalancing factor, while (ii) and (iii) are portfolio growth factors. Bohn and Tesar (1996) refer to (i) as return chasing component of capital flows, and to (ii) and (iii) as portfolio rebalancing components of capital flows.

Table 4. Variance Decomposition of Current Account (CA)

	US
	(i)
std.dev. of CA	0.92%
Δ bonds	1.77
Δ equity assets	-10.78
Δ equity liab.	-10.01

Notes: Statistics reported in the table are based on 200 simulations of quarterly series, each 200 periods long. The decomposition is obtained based on the following identity for US: $CA_t = Q_t^F P_t^F \Delta A_t^F - P_t^H \Delta \hat{A}_t^H + \frac{1}{R_t} \Delta B_t$. It expresses US current account as the sum of its net equity and net bond flows. The corresponding identity for ROW is $\widehat{CA}_t = P_t^H \Delta \hat{A}_t^H - Q_t^F P_t^F \Delta A_t^F + \frac{1}{R_t} \Delta \hat{B}_t$.

component can be assessed by taking a variance decomposition of $Q_t^F P_t^F \Delta A_t^F$. I find that 99% of variation in the capital flows can be explained by the changes in the risk premia. Changes in wealth and capital gains/losses play only an insignificant role and split the remaining 1% of variation in the capital flows.

What are the factors driving the risk-premium? Recall from equation (17) that each portfolio share is a weighted average of the risk premia on *all* assets. Therefore, $\Delta \alpha_t^\chi$, $\chi = \{H, F, N\}$ provide a convenient way of summarizing the effects of changes in the risk-premia on holdings of asset χ in the US. In equation (14) I also showed that the risk premium on asset χ could be approximated by

$$\gamma^H \mathbb{C}\mathbb{V}_t(c_{t+1}^H, r_{t+1}^X) + \gamma^F \mathbb{C}\mathbb{V}_t(q_{t+1}^F + c_{t+1}^F, r_{t+1}^X) + (1 - \gamma^H - \gamma^F) \mathbb{C}\mathbb{V}_t(q_{t+1}^N + c_{t+1}^N, r_{t+1}^X).$$

From this expression, it is easy to see that variations in relative prices are important determinants of risk premium. Thus, changes in q_t^F affect the risk premium directly via the covariance terms, and indirectly via its impact on the components of consumption: c_t^H , c_t^F and c_t^N (see equations in (11)). To quantify this dependence, Table 5 summarizes the correlations between the changes in absolute risk premia on different assets in each country, measured by the first-difference of the respective portfolio share, and the change in q_t^F .

As columns (i) and (ii) of Table 5 show, an increase in q_t^F is associated with an increase in risk premia on domestic assets and a fall in risk premia on foreign assets in both countries. Recall that higher risk premium on an asset is associated with an increase in the holdings of that asset, while the opposite is true when the risk premium falls. The relation between the risk premia and the change in q_t^F is particularly strong if I consider variations in $\Delta \alpha_t^\chi$ and Δq_t^F driven solely by nontradable productivity shocks. In this case, as shown in columns (iii) and (iv), the sign of the correlations remains the same, but their magnitude increases dramatically.

To gain some intuition about the mechanism generating capital flows, consider the responses of US net purchases of US and ROW tradable equity, international bonds, and q^F , the relative price of ROW tradable

Table 5. Correlation between asset χ 's risk premium, $\Delta\alpha_t^X$, and Δq_t^F

	US household	ROW household	US household	ROW household
			N shock only	N shock only
	(i)	(ii)	(iii)	(iv)
US T equity	0.070	-0.075	0.790	-0.831
ROW T equity	-0.089	0.093	-0.859	0.876
bonds	0.172	-0.150	0.921	-0.961

Notes: Statistics reported in the table are based on 200 simulations of quarterly series, each 200 periods long. Columns (i) and (ii) report results obtained from simulations of the model with both T and N productivity shocks. The results in columns (iii) and (iv) are driven by N productivity shocks alone.

goods, to productivity shocks in the US nontradable and tradable sectors. All capital flows are expressed as percentages of US stock market capitalization, while relative prices are in percent deviation from their steady state values. Figures 3a and 3b summarize these responses. First, consider a positive one standard deviation innovation to productivity in the US nontradable sector, as presented in Figure 3a.

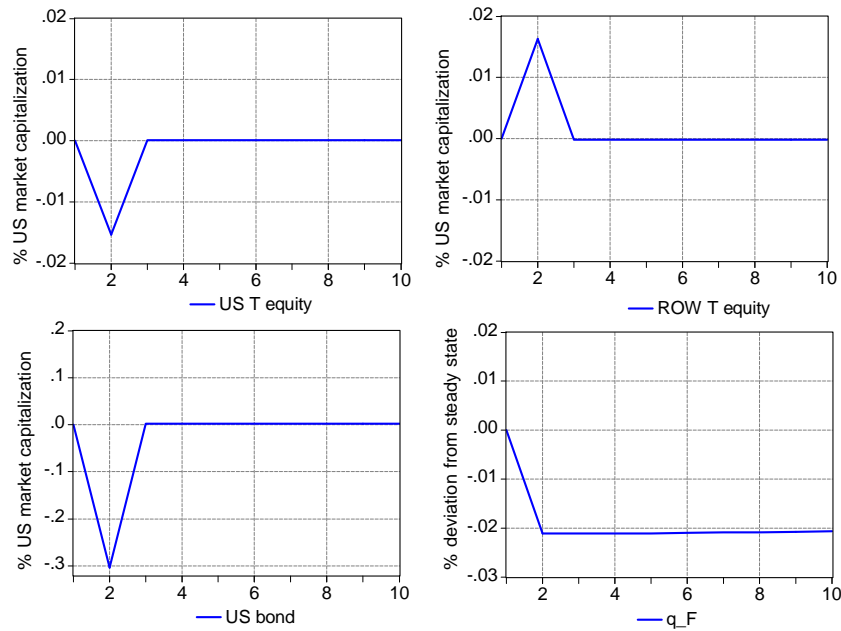


Figure 3a. US portfolio flows and price impulse responses: Nontradable shock

Here we see that a higher supply of nontradable goods in the US depresses their relative price thus stimulating domestic demand for these goods in the US. The higher consumption of nontradables that results is accompanied by an increase in the consumption demand for tradables and a fall in the relative

price of ROW tradables due to consumption home bias. US households then substitute ROW tradables for US tradables, which leads to a trade deficit in the US.

A nontradable productivity shock also induces households to adjust their portfolio positions. Several factors trigger the adjustment. First, the risk premia on the available assets change. Table 5 predicts that in response to a nontradable shock risk premia on domestic assets in each country must fall, while the risk premia on foreign assets must rise. The latter effect increases US holdings of foreign equity, while the former implies a fall in the US holdings of its domestic equity. As a result, US foreign equity assets as well as US foreign equity liabilities increase. Intuitively, a fall in q^F after a nontradable shock reduces the payoffs from ROW tradable equity exactly when consumption demand for ROW tradable goods is high. To induce US households to continue holding ROW tradable equity, the risk-premium on this asset must rise. In equilibrium, higher risk premium on an asset is associated with a larger portfolio share allocated into that asset. US tradable equity, on the other hand, pays in terms of US tradable goods. As demand for US tradable good rises, US equity becomes more attractive, and thus commands lower risk-premium. In equilibrium this leads to lower US holdings of US tradable equity.

Second, households want to rebalance their portfolios due to a wealth effect. In response to a nontradable productivity shock, wealth in both US and ROW declines. In the US this decline in wealth is driven by a fall in q^N , while in the ROW - it is due to a fall in q^F .³⁶ This implies that households in both economies would want to reduce their holdings of financial assets. In the model, however, this effect is weak relative to the risk-premium effect.

Finally, households adjust their equity positions due to capital gains or losses on their portfolio. An initial drop in q^F generates a capital loss on the existing US holdings of ROW equity. Driven by the desire to maintain a balanced portfolio, US households purchase more of ROW tradable equity following the shock.

On aggregate, US economy experiences a net *equity* outflow following a nontradable shock. This implies that US households purchased ROW equity in excess of the US equity purchased by ROW households, so that $(Q_t^F P_t^F \Delta A_t^F - P_t^H \Delta \hat{A}_t^H) > 0$. To finance the net purchases of ROW assets and imports of ROW goods, US households increase their borrowing, $\frac{1}{R_t}(B_t - B_{t-1}) < 0$. The associated inflow of resources covers the net *equity* outflow and pays for the current account deficit in the US.

Because productivity shocks are temporary, an initial fall in q^F will be reversed over time as supply of nontradable and demand for tradable goods subside. In the absence of new shocks, this will result in future capital gains on ROW tradable equity and its sale by US households. As households wealth recovers, their purchases of ROW equity also pick up. Both effects however remain very small.

The outcomes of the tradable shocks can be analyzed in a similar manner. However, in this economy changes in tradable productivity can be almost completely insured with the menu of available assets. Figure 3b presents the responses of equity and bond flows, as well as q^F , to a positive unanticipated shock to productivity in US tradable sector. The equity and bond flows, as well as changes in the current account, are very small and are dominated by the capital flows induced by shocks to nontradable productivity.

³⁶This does not imply that households are worse off following a positive N productivity shock. Firms in the tradable sectors in both countries pay higher dividends after the shock. As a result, H household's consumption of both traded and nontraded goods goes up following the shock in H country.

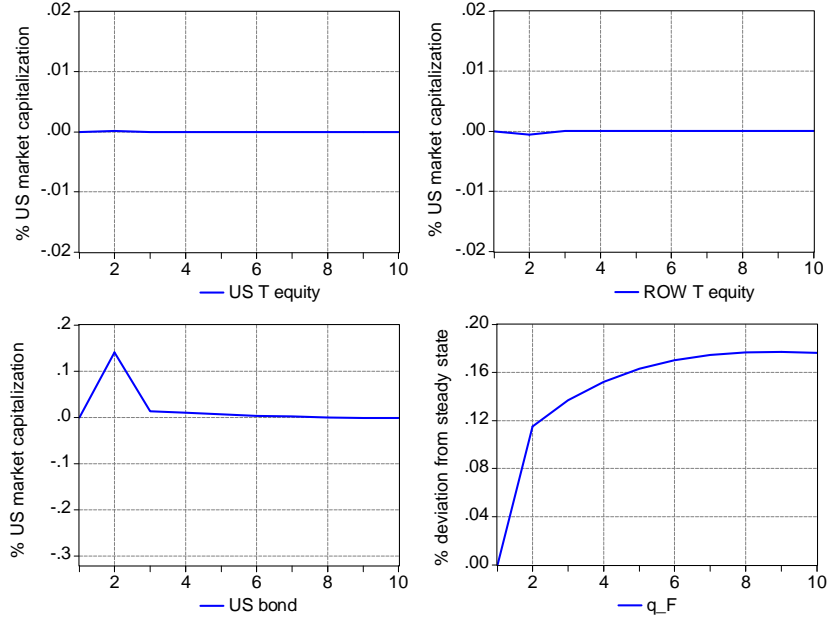


Figure 3b. US portfolio flows and price impulse responses: Tradable shock

5.3 How Important is Market Incompleteness?

As noted in the introduction, risk-sharing plays a conceptually important role in the determination of portfolio allocations and capital flows. In this subsection, I quantify the degree of risk-sharing that arises in the competitive equilibrium of the model. Recall that households cannot hold equity issued by foreign firms producing nontradables. The questions I now ask are: to what extent does this restriction on households' portfolios impede international risk-sharing? what are the implications of this restriction on households' portfolios for equity home bias and capital flows? In other words, is market incompleteness in the model quantitatively important?

To address these questions, I return to the first-order conditions governing households' holdings of non-tradable equity:

$$1 = \mathbb{E}_t [M_{t+1} R_{t+1}^N], \quad (22a)$$

$$1 = \mathbb{E}_t [\hat{M}_{t+1} \hat{R}_{t+1}^N], \quad (22b)$$

where, as before, M_{t+1} and \hat{M}_{t+1} respectively denote the IMRS for US and ROW households. Substituting for M_{t+1} with the identity $M_{t+1} \equiv \hat{M}_{t+1} + M_{t+1} - \hat{M}_{t+1}$ in (22a), we can rewrite the first-order condition for US households as

$$1 = \mathbb{E}_t [\hat{M}_{t+1} R_{t+1}^N] + \mathbb{E}_t [(M_{t+1} - \hat{M}_{t+1}) R_{t+1}^N]. \quad (23)$$

Similarly, the first-order condition for ROW households in (22b) can be rewritten as

$$1 = \mathbb{E}_t [M_{t+1} \hat{R}_{t+1}^N] - \mathbb{E}_t [(M_{t+1} - \hat{M}_{t+1}) \hat{R}_{t+1}^N]. \quad (24)$$

Equations (23) and (24) allow us to quantify the degree of risk-sharing in the model and examine its consequences for households' portfolio choices. To see why, suppose (counter-factually) that the equilibrium in my model permitted complete risk-sharing. In this case $M_t = \hat{M}_t$ for all t , so that the second terms on the right-hand-side of (23) and (24) disappear, leaving $1 = \mathbb{E}_t[\hat{M}_{t+1}R_{t+1}^N]$ and $1 = \mathbb{E}_t[M_{t+1}\hat{R}_{t+1}^N]$. Notice that these expressions would be the first-order conditions for ROW and US households had they had access to foreign nontradable equity. Thus, had the equilibrium in the model permitted complete risk-sharing, households would find it optimal not to hold any foreign nontradable equity. Intuitively, if households are able to achieve complete risk-sharing in a competitive equilibrium in which they are prohibited from holding foreign nontradable equity, then households will not benefit from gaining access to the foreign nontradable equity market.

Now consider the case where the competitive equilibrium only permits incomplete risk-sharing. Here M_t and \hat{M}_t will be less than perfectly correlated but it is still *possible* that households would not benefit from gaining access to the foreign market for nontradable equity. So long as the difference between M_t and \hat{M}_t is conditionally uncorrelated with the returns on nontradables, R_{t+1}^N and \hat{R}_{t+1}^N , the second terms on the right-hand-side of (23) and (24) disappear, again leaving $1 = \mathbb{E}_t[\hat{M}_{t+1}R_{t+1}^N]$ and $1 = \mathbb{E}_t[M_{t+1}\hat{R}_{t+1}^N]$.

This observation suggests a simple way to quantify the impact of incomplete-risk sharing on households' portfolio choices. Namely, consider the projections of R_t^N and \hat{R}_t^N on $M_t - \hat{M}_t$:

$$R_t^N = \mathcal{P}(R_t^N | M_t - \hat{M}_t) + \epsilon_t, \quad (25a)$$

$$\hat{R}_t^N = \mathcal{P}(\hat{R}_t^N | M_t - \hat{M}_t) + \hat{\epsilon}_t. \quad (25b)$$

Clearly, these projections only make sense when M_t and \hat{M}_t are imperfectly correlated. In the equilibrium of my model the correlation between M_t and \hat{M}_t is 0.76, so the projections in (25) can be estimated without difficulty.³⁷ Now, if the estimated projection coefficients are significantly different from zero, we can reject the hypothesis that the returns on nontradables are orthogonal to $M_t - \hat{M}_t$. And, as a consequence, we can also reject the hypothesis that households would never benefit from gaining access to the market for foreign nontradable equity.³⁸ This turns out to be the case. Using simulated data from the model, the estimated projection coefficients in (25a) and (25b) are -0.95 and 1.09 respectively. Both of these estimates are highly statistically significant.³⁹

We can also use the projections in (25) to think about how households would adjust their portfolio holdings if they were given the opportunity to hold foreign nontradable equity. Although households vary their

³⁷A more direct measure of risk-sharing achieved in the equilibrium of the model can be obtained as the correlation between relative consumption and real exchange rate (RER). I find this correlation to be equal to 0.63. Furthermore, with incomplete markets, the risk-sharing condition implies that the *growth rates* of RER and relative consumption are equalized in expected values. So, I also compute the correlation between the realized growth rates of the two variables, and find that this correlation is equal to 0.66. Therefore, both correlations in the model are significantly below 1, suggesting that the equilibrium of the model admits far from perfect risk sharing, consistent with the empirical evidence.

³⁸Recently, the same approach based on households' optimality conditions was used by Chue (2007) to analyze empirically whether the existing US equity portfolios are optimal.

³⁹Although (25) can only be used to test the unconditional moment condition $E[(M_{t+1} - \hat{M}_{t+1})\chi_{t+1}]$ for $\chi_{t+1} = \{R_{t+1}^N, \hat{R}_{t+1}^N\}$, rejecting the null that $E[(M_{t+1} - \hat{M}_{t+1})\chi_{t+1}] = 0$ implies that $E_t[(M_{t+1} - \hat{M}_{t+1})\chi_{t+1}] \neq 0$ for at least some t .

portfolio shares through time, we can characterize their “average shares” by considering the unconditional version of (22): $1 = \mathbb{E}[M_{t+1}R_{t+1}^N]$ and $1 = \mathbb{E}[\hat{M}_{t+1}\hat{R}_{t+1}^N]$, where $\mathbb{E}[\cdot]$ denotes unconditional expectations.⁴⁰ My estimates of (25) imply that $\mathbb{E}[R_{t+1}^N(M_{t+1} - \hat{M}_{t+1})] < 0$ and $\mathbb{E}[\hat{R}_{t+1}^N(M_{t+1} - \hat{M}_{t+1})] > 0$. Combining these inequalities with the unconditional version of (22), gives

$$\mathbb{E}[\hat{M}_{t+1}R_{t+1}^N] > 1 \quad \text{and} \quad \mathbb{E}[M_{t+1}\hat{R}_{t+1}^N] > 1.$$

Thus, on average, both US and ROW households would like to acquire a long position in foreign nontradable equity given the prevailing behavior of returns. Furthermore, had they had access to foreign nontradable equity, the home bias in the tradable portfolios would disappear. This can be easily seen by extending the optimal portfolio rules in (17) to include foreign nontradable equity. In this case US and ROW households would face the same investment opportunity set and therefore would choose the same vectors of portfolio shares, $\alpha_t = \hat{\alpha}_t$. This would imply perfectly diversified portfolios in each country.⁴¹ As a result, if households’ start with the same initial wealth, their wealth would also be equalized across countries in all periods. With log-utility, consumption would also be equalized, as implied by complete risk-sharing. Under these circumstances, portfolio flows would arise only due to changes in asset prices. Based on the definitions adopted in this paper, portfolio flows will be zero in the model. In sum, asset market incompleteness is vital for home bias and portfolio flows in the model.

5.4 How Important is Dynamic Portfolio Choice?

I now evaluate the importance of dynamic portfolio choice in the model by examining the conditional and unconditional moments of portfolio returns. Figure 4 plots the US mean-variance frontier for the risky assets using the unconditional sample moments of the simulated returns from the model. Only returns on assets available to US households are used to calculate the frontier. Thus, it consists of the set of minimum-variance portfolios, composed of US and ROW tradable equity and US nontradable equity, in the return mean-standard deviation space. Every point on the frontier delivers portfolios with the lowest standard deviation of return, given mean return. Point **A** on the frontier corresponds to the average portfolio implied by the model; the asset weights are computed as the sample averages of simulated portfolio shares.

⁴⁰Note that these “average shares” are not the same as the true averages of the portfolio shares determined by the households’ first-order conditions because the latter are nonlinear functions of conditioning information.

⁴¹This conclusion is consistent with a number of results in the literature. For example, in a one-sector model with complete markets and logarithmic utility, Kollmann (2005) shows that portfolios are fully-diversified across countries. Coeurdacier (2008) shows a similar result in a two-sector version of the model (see equations (46) and (47) in that paper with $\gamma = 1$).

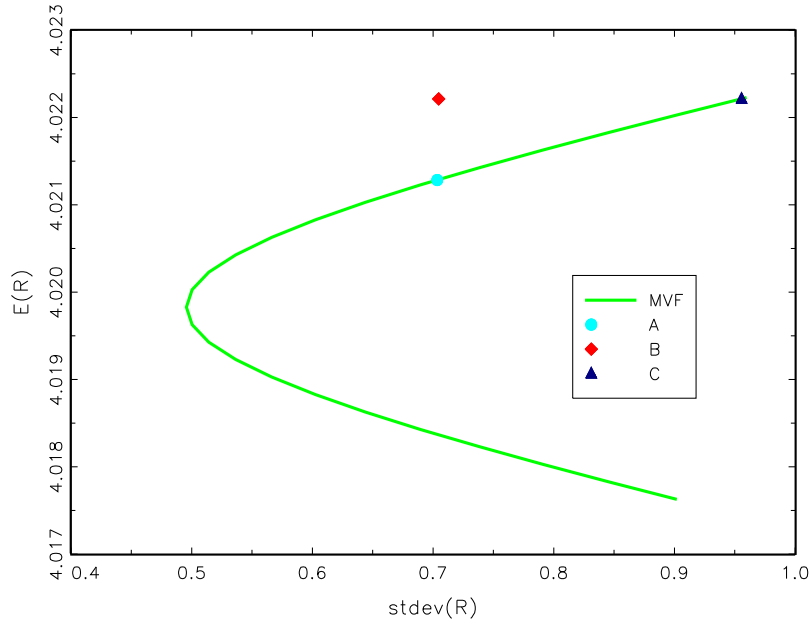


Figure 4. US Mean-Variance Frontier: Conditioning Information

Figure 4 characterizes portfolios using unconditional moments of returns. However, throughout this paper I have used conditional moments to describe the decisions of firms and households. In particular, households incorporate the additional information embodied in the conditional moments of returns by constantly adjusting their portfolios. Points **B** and **C** in Figure 4 illustrate the importance of this conditioning information. According to the model, the conditional first and second moments of optimally invested wealth for US households are

$$\mathbb{E}_t r_{t+1}^W = r_t^1 + \frac{1}{2} \boldsymbol{\alpha}_t' \mathbb{V}_t(\mathbf{r}_{t+1}) \boldsymbol{\alpha}_t \quad \text{and} \quad \mathbb{V}_t(r_{t+1}^W) = \boldsymbol{\alpha}_t' \mathbb{V}_t(\mathbf{r}_{t+1}) \boldsymbol{\alpha}_t.$$

Notice that both moments vary through time with changes in the variance-covariance matrix of returns, $\mathbb{V}_t(\mathbf{r}_{t+1})$, and the vector of optimally chosen portfolio shares, $\boldsymbol{\alpha}_t$. To assess the importance of these variations, Figure 4 plots the unconditional mean and variance of the return on wealth as point **B**. The unconditional mean is calculated as a sample average of $\mathbb{E}_t r_{t+1}^W$, while the unconditional variance of the return on wealth is obtained as $\mathbb{V}(r_{t+1}^W) \equiv \mathbb{E}[\mathbb{V}_t(r_{t+1}^W)] + \mathbb{V}(\mathbb{E}_t r_{t+1}^W)$. Point **B** lies well above the frontier. To place this result in perspective, I also calculate the minimum-variance portfolio that delivers the expected return on optimally invested wealth (i.e., without the use of conditioning information and thus not allowing for period-by-period portfolio adjustments). This portfolio corresponds to point **C**, which lies well to the right of **B**. In fact, to achieve the same expected return with fixed portfolio shares, households would have to live with 39% more risk.⁴² Therefore, conditioning information and dynamic portfolio choice allow households to move significantly beyond the unconditional mean-variance frontier.

⁴²Note that while the model implies time-varying risk premium, its magnitude, as in much of the international portfolio choice literature, is small.

To further illustrate the importance of the dynamic portfolio choice in the model, I compare households' welfare achieved under a dynamic portfolio rule and a portfolio rule in which households (mistakenly) believe that returns are i.i.d., and as a result set their portfolio shares to a constant level equal to the average of portfolio shares in the dynamic model.⁴³ To perform the comparison I use the certainty equivalent return (CER), which is obtained as the rate of return on wealth which, if earned with certainty, will provide the household with the same level of welfare as a particular portfolio rule.⁴⁴ The welfare of US household is obtained as a period t expected discounted lifetime utility, equal to $\mathbb{E}_t \sum_{i=0}^{\infty} \beta^i U(C_{t+i})$. It can conveniently be written as

$$\frac{1}{1-\beta} c_t + \frac{1}{1-\beta} \sum_{i=1}^{\infty} \beta^i \mathbb{E}_t \Delta c_{t+i}.$$

With log utility, $\Delta c_{t+i} = r_{t+i}^W$, which allows to obtain the US household's CER, r_{CER}^W , from

$$\frac{1}{1-\beta} c_t + \frac{\beta}{(1-\beta)^2} r_{\text{CER}}^W = \frac{1}{1-\beta} c_t + \frac{1}{1-\beta} \sum_{i=1}^{\infty} \beta^i \mathbb{E}_t r_{t+i}^W.$$

I compute r_{CER}^W under the dynamic and static portfolio rule using average unconditional welfare in US and ROW. I find that by constantly adjusting their portfolios in responses to changes in the conditional moments of returns, households gain extra certainty equivalent return, which on average is equal to three times the risk-premium in the model. In sum, both results suggest that dynamic portfolio choice plays an economically significant role in the model.

6 Conclusion

This paper reconciles two puzzles in international finance: home bias in equity holdings and high turnover and volatility of international capital flows. These results are developed in a model of international portfolio choice with several distinct features: the production of tradable and nontradable goods in each country is specialized, preferences are nonseparable in the consumption of all goods, and asset markets are incomplete.

Both home bias and international capital flows are driven by variations in international relative prices which arise from productivity changes. Low diversification occurs because the variations in the terms of trade affect the hedging ability of foreign assets. Large and volatile capital flows occur in response to the international risk-premia differentials which are also driven by the terms of trade movements. Therefore, it is not inconsistent to have home bias in portfolios and significant capital flows. My results also emphasize the importance of market incompleteness and dynamic portfolio choice.

⁴³This rule does not characterize an equilibrium of the model, however, it allows me to evaluate the equilibrium contribution of the time-varying portfolio shares.

⁴⁴This approach has been used extensively in the literature studying the economic significance of dynamic portfolio choice (see McCulloch and Rossi (1990), Kandel and Stambaugh (1996), DeMiguel, Garlappi, and Uppal (2007), Lan (2008)).

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Appendix

This appendix provides the derivations of some equations presented in the text.

A Model equations and the approximation point

The system of equations characterizing the equilibrium of the model consists of

1. Process for productivity

$$z_t = az_{t-1} + e_t,$$

where e_t is a vector of i.i.d. normally distributed, mean zero shocks with covariance Ω_e .

2. H and F budget constraints

$$\begin{aligned} W_{t+1} &= R_{t+1}^W (W_t - C_t^H - Q_t^F C_t^F - Q_t^N C_t^N) \\ R_{t+1}^W &= R_t^1 + \alpha_t^H (R_{t+1}^H - R_t^1) + \alpha_t^F (R_{t+1}^F - R_t^1) + \alpha_t^N (R_{t+1}^N - R_t^1) \end{aligned}$$

and

$$\begin{aligned} \hat{W}_{t+1} &= \hat{R}_{t+1}^W (\hat{W}_t - \hat{C}_t^H - \hat{Q}_t^F \hat{C}_t^F - \hat{Q}_t^N \hat{C}_t^N) \\ \hat{R}_{t+1}^W &= R_t^1 + \hat{\alpha}_t^H (R_{t+1}^H - R_t^1) + \hat{\alpha}_t^F (R_{t+1}^F - R_t^1) + \hat{\alpha}_t^N (\hat{R}_{t+1}^N - R_t^1). \end{aligned}$$

3. H and F bond and equity Euler equations

$$\begin{aligned} 1 &= \mathbb{E}_t [M_{t+1} R_t^1], & 1 &= \mathbb{E}_t [\hat{M}_{t+1} R_t^1], \\ 1 &= \mathbb{E}_t [M_{t+1} R_{t+1}^H], & 1 &= \mathbb{E}_t [\hat{M}_{t+1} R_{t+1}^H], \\ 1 &= \mathbb{E}_t [M_{t+1} R_{t+1}^F], & 1 &= \mathbb{E}_t [\hat{M}_{t+1} R_{t+1}^F], \\ 1 &= \mathbb{E}_t [M_{t+1} R_{t+1}^N], & 1 &= \mathbb{E}_t [\hat{M}_{t+1} \hat{R}_{t+1}^N], \end{aligned}$$

where $M_{t+1} = \beta \frac{\partial U_{t+1} / \partial C_{t+1}^H}{\partial U_t / \partial C_t^H}$. The US stochastic discount factor then becomes

$$M_{t+1} = \beta \left(\frac{C_{t+1}}{C_t} \right)^{-\phi} \left(\frac{C_{t+1}^T}{C_t^T} \right)^{\phi - \rho} \left(\frac{C_{t+1}^H}{C_t^H} \right)^{\rho - 1},$$

where $C \equiv \left(\lambda_T^{1-\phi} (C^T)^\phi + \lambda_N^{1-\phi} (C^N)^\phi \right)^{\frac{1}{\phi}}$ denotes the aggregate consumption basket in the US. Substituting in the definitions for C and C^T , and collecting the terms, allows me to express M_{t+1} as a growth rate of the US consumption expenditures:

$$M_{t+1} = \beta \frac{C_t^H + Q_t^F C_t^F + Q_t^N C_t^N}{C_{t+1}^H + Q_{t+1}^F C_{t+1}^F + Q_{t+1}^N C_{t+1}^N}.$$

When preferences are logarithmic, consumption expenditure is proportional to wealth, so that

$$C_t^H + Q_t^F C_t^F + Q_t^N C_t^N = (1 - \beta) W_t, \tag{A1}$$

which allows to simplify the expression for M_{t+1} as $M_{t+1} = \beta W_t / W_{t+1}$. Similarly, in the ROW, $\hat{M}_{t+1} = \beta \hat{W}_t / \hat{W}_{t+1}$.

4. H and F optimality conditions determining relative goods prices

$$\begin{aligned} Q_t^N &= \left(\frac{\lambda_N}{\lambda_T}\right)^{1-\phi} \left(\frac{C_t^N}{C_t^T}\right)^{\phi-1} \lambda_H^{\rho-1} \left(\frac{C_t^H}{C_t^T}\right)^{1-\rho}, \\ \hat{Q}_t^N &= \left(\frac{\lambda_N}{\lambda_T}\right)^{1-\phi} \left(\frac{\hat{C}_t^N}{\hat{C}_t^T}\right)^{\phi-1} \hat{\lambda}_H^{\rho-1} \left(\frac{\hat{C}_t^H}{\hat{C}_t^T}\right)^{1-\rho}, \\ Q_t^F &= \left(\frac{\lambda_F}{\lambda_H}\right)^{1-\rho} \left(\frac{C_t^F}{C_t^H}\right)^{\rho-1}. \end{aligned}$$

5. Capital Euler equation at H and F

$$\begin{aligned} 1 &= \mathbb{E}_t [M_{t+1} R_{t+1}^K], & \text{with } R_{t+1}^K &\equiv \theta Z_{t+1}^H (K_{t+1}^H)^{\theta-1} + (1-\delta) \\ 1 &= \mathbb{E}_t [\hat{M}_{t+1} \hat{R}_{t+1}^K], & \text{with } \hat{R}_{t+1}^K &\equiv (Q_{t+1}^F/Q_t^F) [\theta Z_{t+1}^F (K_{t+1}^F)^{\theta-1} + (1-\delta)] \end{aligned}$$

6. Market clearing conditions

(a) traded goods

$$\begin{aligned} C_t^H + \hat{C}_t^H &= D_t^H, \\ C_t^F + \hat{C}_t^F &= D_t^F. \end{aligned}$$

(b) nontraded goods

$$C_t^N = Y_t^N = D_t^N \quad \text{and} \quad \hat{C}_t^N = \hat{Y}_t^N = \hat{D}_t^N.$$

(c) bond

$$0 = B_t + \hat{B}_t.$$

(d) tradable equity

$$1 = A_t^H + \hat{A}_t^H \quad \text{and} \quad 1 = A_t^F + \hat{A}_t^F,$$

which can equivalently be written as

$$\begin{aligned} P_t^H &= \alpha_t^H \beta W_t + \hat{\alpha}_t^H \beta \hat{W}_t \\ Q_t^F P_t^F &= \alpha_t^F \beta W_t + \hat{\alpha}_t^F \beta \hat{W}_t \end{aligned}$$

(e) nontradable equity

$$1 = A_t^N \quad \text{and} \quad 1 = \hat{A}_t^N,$$

which is equivalent to

$$\alpha_t^N = Q_t^N P_t^N / \beta W_t \quad \text{and} \quad \hat{\alpha}_t^N = \hat{Q}_t^N \hat{P}_t^N / \beta \hat{W}_t.$$

The approximation point is given by $R = R^K = \hat{R}^K = R^H = R^F = R^N = \hat{R}^N = R^W = \hat{R}^W = \frac{1}{\beta}$. $K \equiv K^H = K^F = (\beta\theta)^{1/(1-\theta)} (1-\beta + \beta\delta)^{1/(\theta-1)}$, $D \equiv D^H = D^F = K^\theta - \delta K$, $P^H = P^F = \beta D / (1-\beta)$. $D^N = \hat{D}^N = \kappa$, so that $C^N = \hat{C}^N = \kappa$ and $P^N = \hat{P}^N = \beta\kappa / (1-\beta)$. Wealth at H and F is approximated around

an initial level, W_0 and \hat{W}_0 . When $W_0 = \hat{W}_0$, then $C_0^H = \hat{C}_0^H = D^H$ and $C_0^F = \hat{C}_0^F = D^F$. This implies that initial aggregate consumption is also equalized across countries: $C_0 = \hat{C}_0$. Then portfolios are approximated around $\alpha^N = \hat{\alpha}^N = \lambda_N^{1-\phi} (C_0^N/C_0)^\phi$ and $\alpha^H = \alpha^F = \lambda_T^{1-\phi} (C_0^T/C_0)^\phi$, where α^H and α^F denote the initial values of $(\alpha_t^H + \hat{\alpha}_t^H)$ and $(\hat{\alpha}_t^F + \alpha_t^F)$, respectively, as before.

B Derivation of equations (12)-(14)

Equation (12) is obtained by using second-order Taylor series expansion and the log-normality of asset returns. The first-order condition for bond in equation (9c) can be expressed as

$$1 = \mathbb{E}_t [\exp(m_{t+1} + r_t^1)] \simeq \exp [\mathbb{E}_t (m_{t+1} + r_t^1) + \frac{1}{2} \mathbb{V}_t (m_{t+1})].$$

Taking logs on both sides yields a log-approximate version of the consumption Euler equation:

$$0 = r_t^1 + \mathbb{E}_t m_{t+1} + \frac{1}{2} \mathbb{V}_t (m_{t+1}). \quad (\text{A2})$$

First-order condition for asset χ in equation (9) is log-approximated analogously:

$$1 = \mathbb{E}_t [M_{t+1} R_{t+1}^\chi] \simeq \exp [\mathbb{E}_t (m_{t+1} + r_{t+1}^\chi) + \frac{1}{2} \mathbb{V}_t (m_{t+1} + r_{t+1}^\chi)].$$

Again, taking logs and substituting in the bond Euler equation gives expression in (12):

$$\mathbb{E}_t r_{t+1}^\chi - r_t + \frac{1}{2} \mathbb{V}_t (r_{t+1}^\chi) = -\mathbb{C}\mathbb{V}_t (m_{t+1}, r_{t+1}^\chi). \quad (\text{A3})$$

Next, I characterize m_{t+1} . Recall its definition on page 10: $m_{t+1} \equiv \ln(M_{t+1}) - \ln M = \ln\left(\beta \frac{\partial U_{t+1}/\partial C_{t+1}^H}{\partial U_t/\partial C_t^H}\right) - \ln\beta$, which using the derivations in Appendix A can be written as $m_{t+1} = -\Delta w_{t+1}$. Substituting this result in (A3) gives equation (13) in the text.

Finally, to obtain a representation for risk premium in terms of consumption, as given in equation (14), I log-linearized the consumption rule in equation (A1).

C Derivation of equation (19) and bias term in (20)

Start by revisiting the optimal decision rule for portfolio shares in equation (17). We can write it as

$$\begin{bmatrix} \alpha_t^H \\ \alpha_t^F \\ \alpha_t^N \end{bmatrix} = \begin{bmatrix} \mathbb{V}_t(r_{t+1}^H) & \mathbb{C}\mathbb{V}_t(r_{t+1}^H, r_{t+1}^F) & \mathbb{C}\mathbb{V}_t(r_{t+1}^H, r_{t+1}^N) \\ \mathbb{C}\mathbb{V}_t(r_{t+1}^H, r_{t+1}^F) & \mathbb{V}_t(r_{t+1}^F) & \mathbb{C}\mathbb{V}_t(r_{t+1}^F, r_{t+1}^N) \\ \mathbb{C}\mathbb{V}_t(r_{t+1}^H, r_{t+1}^N) & \mathbb{C}\mathbb{V}_t(r_{t+1}^F, r_{t+1}^N) & \mathbb{V}_t(r_{t+1}^N) \end{bmatrix}^{-1} \begin{bmatrix} \mathbb{C}\mathbb{V}_t(w_{t+1}, r_{t+1}^H) \\ \mathbb{C}\mathbb{V}_t(w_{t+1}, r_{t+1}^F) \\ \mathbb{C}\mathbb{V}_t(w_{t+1}, r_{t+1}^N) \end{bmatrix},$$

where I used equation (13) to substitute for the risk-premium. My objective is to find expressions for tradable portfolio shares, α_t^H and α_t^F , for a given α_t^N . For this purpose we can re-write the first two lines in the expression above as

$$\begin{bmatrix} \mathbb{V}_t(r_{t+1}^H) & \mathbb{C}\mathbb{V}_t(r_{t+1}^H, r_{t+1}^F) \\ \mathbb{C}\mathbb{V}_t(r_{t+1}^H, r_{t+1}^F) & \mathbb{V}_t(r_{t+1}^F) \end{bmatrix} \begin{bmatrix} \alpha_t^H \\ \alpha_t^F \end{bmatrix} + \begin{bmatrix} \mathbb{C}\mathbb{V}_t(r_{t+1}^H, r_{t+1}^N) \\ \mathbb{C}\mathbb{V}_t(r_{t+1}^F, r_{t+1}^N) \end{bmatrix} \alpha_t^N = \begin{bmatrix} \mathbb{C}\mathbb{V}_t(w_{t+1}, r_{t+1}^H) \\ \mathbb{C}\mathbb{V}_t(w_{t+1}, r_{t+1}^F) \end{bmatrix}.$$

Solving for $\begin{bmatrix} \alpha_t^H \\ \alpha_t^F \end{bmatrix}$ gives

$$\begin{bmatrix} \alpha_t^H \\ \alpha_t^F \end{bmatrix} = \begin{bmatrix} \mathbb{V}_t(r_{t+1}^H) & \mathbb{C}\mathbb{V}_t(r_{t+1}^H, r_{t+1}^F) \\ \mathbb{C}\mathbb{V}_t(r_{t+1}^H, r_{t+1}^F) & \mathbb{V}_t(r_{t+1}^F) \end{bmatrix}^{-1} \left(\begin{bmatrix} \mathbb{C}\mathbb{V}_t(w_{t+1}, r_{t+1}^H) \\ \mathbb{C}\mathbb{V}_t(w_{t+1}, r_{t+1}^F) \end{bmatrix} - \alpha_t^N \begin{bmatrix} \mathbb{C}\mathbb{V}_t(r_{t+1}^H, r_{t+1}^N) \\ \mathbb{C}\mathbb{V}_t(r_{t+1}^F, r_{t+1}^N) \end{bmatrix} \right).$$

Isolating the solution for α_t^H gives equation (19) in the text:

$$\alpha_t^H = \frac{\mathbb{V}_t(r_{t+1}^F) [\mathbb{C}\mathbb{V}_t(w_{t+1}, r_{t+1}^H) - \mathbb{C}\mathbb{V}_t(r_{t+1}^N, r_{t+1}^H)\alpha_t^N]}{\mathbb{V}_t(r_{t+1}^H)\mathbb{V}_t(r_{t+1}^F) - \mathbb{C}\mathbb{V}_t^2(r_{t+1}^H, r_{t+1}^F)} - \frac{\mathbb{C}\mathbb{V}_t(r_{t+1}^H, r_{t+1}^F) [\mathbb{C}\mathbb{V}_t(w_{t+1}, r_{t+1}^F) - \mathbb{C}\mathbb{V}_t(r_{t+1}^N, r_{t+1}^F)\alpha_t^N]}{\mathbb{V}_t(r_{t+1}^H)\mathbb{V}_t(r_{t+1}^F) - \mathbb{C}\mathbb{V}_t^2(r_{t+1}^H, r_{t+1}^F)}.$$

To derive the expression for bias in (20) I used the model equilibrium conditions as follows. First, using the definition of wealth from households' problem, the aggregate world wealth in the model is given by

$$\begin{aligned} W_t + \hat{W}_t &= (P_t^H + D_t^H) (A_{t-1}^H + \hat{A}_{t-1}^H) + Q_t^F (P_t^F + D_t^F) (A_{t-1}^F + \hat{A}_{t-1}^F) \\ &\quad + Q_t^N (P_t^N + D_t^N) A_{t-1}^N + \hat{Q}_t^N (\hat{P}_t^N + \hat{D}_t^N) \hat{A}_{t-1}^N + (B_{t-1} + \hat{B}_{t-1}). \end{aligned}$$

Using the asset market clearing conditions, I get

$$W_t + \hat{W}_t = (P_t^H + D_t^H) + Q_t^F (P_t^F + D_t^F) + Q_t^N (P_t^N + D_t^N) + \hat{Q}_t^N (\hat{P}_t^N + \hat{D}_t^N).$$

The expression above can also be written in terms of returns as

$$W_t + \hat{W}_t = R_t^H P_{t-1}^H + R_t^F Q_{t-1}^F P_{t-1}^F + R_t^N Q_{t-1}^N P_{t-1}^N + \hat{R}_t^N \hat{Q}_{t-1}^N \hat{P}_{t-1}^N.$$

Log-approximating this condition around a symmetric steady state and equal initial wealth distribution yields

$$\frac{1}{2}w_t + \frac{1}{2}\hat{w}_t = \frac{1}{4}(r_t^H + p_{t-1}^H) + \frac{1}{4}(q_t^F + q_{t-1}^F + p_{t-1}^F) + \frac{1}{4}(r_t^N + q_{t-1}^N + p_{t-1}^N) + \frac{1}{4}(\hat{r}_t^N + \hat{q}_{t-1}^N + \hat{p}_{t-1}^N). \quad (\text{A4})$$

Bond market clearing condition in conjunction with logarithmic utility also imply

$$\beta (W_t + \hat{W}_t) = P_t^H + Q_t^F P_t^F + Q_t^N P_t^N + \hat{Q}_t^N \hat{P}_t^N,$$

which can be log-approximated as follows

$$\frac{1}{2}w_t + \frac{1}{2}\hat{w}_t = \frac{1}{4}p_t^H + \frac{1}{4}(q_t^F + p_t^F) + \frac{1}{4}(q_t^N + p_t^N) + \frac{1}{4}(\hat{q}_t^N + \hat{p}_t^N).$$

Substituting this result into equation (A4) and forwarding it by one period gives

$$\frac{1}{2}\Delta w_{t+1} + \frac{1}{2}\Delta \hat{w}_{t+1} = \frac{1}{4}r_{t+1}^H + \frac{1}{4}r_{t+1}^F + \frac{1}{4}r_{t+1}^N + \frac{1}{4}\hat{r}_{t+1}^N$$

I can now use the expression above to derive $\mathbb{C}\mathbb{V}_t(w_{t+1}, r_{t+1}^H)$ and $\mathbb{C}\mathbb{V}_t(w_{t+1}, r_{t+1}^F)$ terms in equation (19). For instance, the covariance between US wealth and US tradable return, r_{t+1}^H , can be obtained from

$$\begin{aligned} \frac{1}{2}\mathbb{C}\mathbb{V}_t(w_{t+1}, r_{t+1}^H) + \frac{1}{2}\mathbb{C}\mathbb{V}_t(\hat{w}_{t+1}, r_{t+1}^H) &= \frac{1}{4}\mathbb{V}_t(r_{t+1}^H) + \frac{1}{4}\mathbb{C}\mathbb{V}_t(r_{t+1}^F, r_{t+1}^H) \\ &\quad + \frac{1}{4}\mathbb{C}\mathbb{V}_t(r_{t+1}^N, r_{t+1}^H) + \frac{1}{4}\mathbb{C}\mathbb{V}_t(\hat{r}_{t+1}^N, r_{t+1}^H). \end{aligned} \quad (\text{A5})$$

Second, since both US and ROW households have access to tradable equity issued in both countries, Euler equations in (13) imply that $\mathbb{C}\mathbb{V}_t(w_{t+1}, r_{t+1}^H) = \mathbb{C}\mathbb{V}_t(\hat{w}_{t+1}, r_{t+1}^H)$ and $\mathbb{C}\mathbb{V}_t(w_{t+1}, r_{t+1}^F) = \mathbb{C}\mathbb{V}_t(\hat{w}_{t+1}, r_{t+1}^F)$. Similar steps are applied to derive an expression for $\mathbb{C}\mathbb{V}_t(w_{t+1}, r_{t+1}^F)$. Equipped with these equalities we can substitute the expressions for $\mathbb{C}\mathbb{V}_t(w_{t+1}, r_{t+1}^H)$ and $\mathbb{C}\mathbb{V}_t(w_{t+1}, r_{t+1}^F)$ into equation (19) in the text to get

$$\alpha_t^H = \frac{1}{4} + \frac{1}{\mathbb{V}_t(r_{t+1}^H) \left(1 - \frac{\mathbb{C}\mathbb{V}_t^2(r_{t+1}^H, r_{t+1}^F)}{\mathbb{V}_t(r_{t+1}^H)\mathbb{V}_t(r_{t+1}^F)}\right)} \left[\frac{1}{4} \mathbb{C}\mathbb{V}_t(r_{t+1}^H, \hat{r}_{t+1}^N) - \left(\alpha_t^N - \frac{1}{4}\right) \mathbb{C}\mathbb{V}_t(r_{t+1}^H, r_{t+1}^N) \right. \\ \left. - \frac{\mathbb{C}\mathbb{V}_t(r_{t+1}^H, r_{t+1}^F)}{\mathbb{V}_t(r_{t+1}^F)} \left(\frac{1}{4} \mathbb{C}\mathbb{V}_t(r_{t+1}^F, \hat{r}_{t+1}^N) - \left(\alpha_t^N - \frac{1}{4}\right) \mathbb{C}\mathbb{V}_t(r_{t+1}^F, r_{t+1}^N) \right) \right],$$

or, using the notation in Proposition 1 in the text:

$$\alpha_t^H = \frac{1}{4} + \frac{1}{\sigma_t^2} \left[\frac{1}{4} \mathbb{C}\mathbb{V}_t(r_{t+1}^H, \hat{r}_{t+1}^N) - \left(\alpha_t^N - \frac{1}{4}\right) \mathbb{C}\mathbb{V}_t(r_{t+1}^H, r_{t+1}^N) \right. \\ \left. - \beta_t^{\text{HF}} \left(\frac{1}{4} \mathbb{C}\mathbb{V}_t(r_{t+1}^F, \hat{r}_{t+1}^N) - \left(\alpha_t^N - \frac{1}{4}\right) \mathbb{C}\mathbb{V}_t(r_{t+1}^F, r_{t+1}^N) \right) \right].$$

Under conditions (i)-(iii) in the Proposition, the expression above simplifies to the bias term in the text.