Economics 326
Methods of Empirical Research in Economics

Lecture 7: Confidence intervals

Vadim Marmer
University of British Columbia

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Point estimation

- Our model:
  1. $Y_i = \beta_0 + \beta_1 X_i + U_i, \qquad i = 1, \ldots, n.$
  2. $E\left(U_i | X_1, \ldots, X_n\right) = 0$ for all i's.
  3. $E\left(U_i^2 | X_1, \ldots, X_n\right) = \sigma^2$ for all i's.
  4. $E\left(U_i U_j | X_1, \ldots, X_n\right) = 0$ for all $i \neq j$.
  5. U's are jointly normally distributed conditional on X's.
- The OLS estimator $\hat{\beta}_1$ is a point estimator of β_1.
- For our model, conditional on X's:

$$\begin{aligned} \hat{\beta}_1 &\sim N\left(\beta_1, Var\left(\hat{\beta}_1\right)\right), \\ Var\left(\hat{\beta}_1\right) &= \frac{\sigma^2}{\sum_{i=1}^n \left(X_i - \bar{X}\right)^2}. \end{aligned}$$

- With probability one, we have that $\hat{\beta}_1 \neq \beta_1$.

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- Sometimes, a CI is defined as $P(\beta_1 \in CI_{1-\alpha}) \geq 1-\alpha$.

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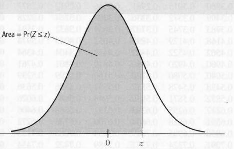
$$\frac{\hat{\beta}_1 - \beta_1}{\sqrt{Var\left(\hat{\beta}_1\right)}} \sim N\left(0, 1\right).$$

Quantiles (percentiles) of the standard normal distribution.

TABLE 1 The Cumulative Standard Normal Distribution Function, $\Phi(z) = \Pr(Z \le z)$



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z	0	1	2	3	4	5	6	7	8	9
–2.9	0.0019	0.0018	0.0018	0.0017	0.0016	0.0016	0.0015	0.0015	0.0014	0.0014
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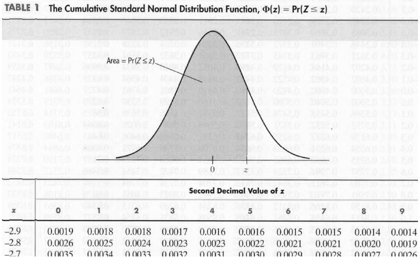

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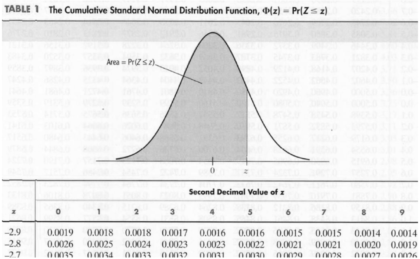

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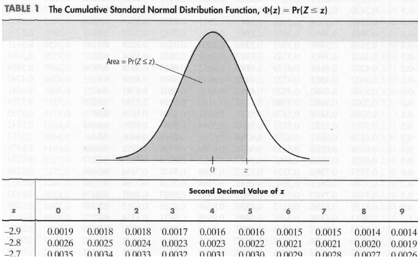

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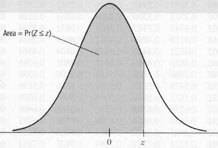

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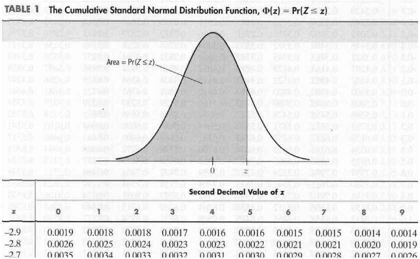

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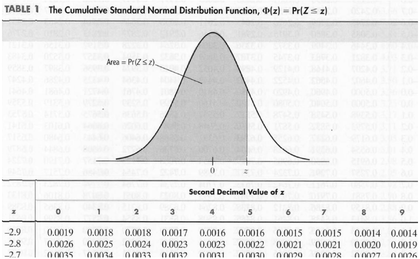

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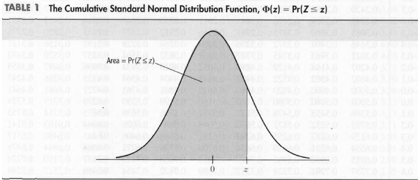

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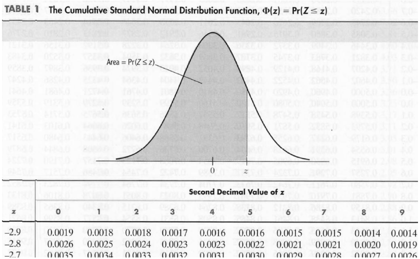

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\end{aligned}
$$

Validity of the infeasible CIs (σ^2 is known)

- We need to show that $P\left(\beta_1 \in \left[\hat{\beta}_1 - z_{1-\alpha/2}\sqrt{Var\left(\hat{\beta}_1\right)}, \hat{\beta}_1 + z_{1-\alpha/2}\sqrt{Var\left(\hat{\beta}_1\right)}\right]\right) = 1 - \alpha$.
- Next,

$$\begin{aligned} & \hat{\beta}_1 - z_{1-\alpha/2}\sqrt{Var\left(\hat{\beta}_1\right)} \leq \beta_1 \leq \hat{\beta}_1 + z_{1-\alpha/2}\sqrt{Var\left(\hat{\beta}_1\right)} \\ \iff & -z_{1-\alpha/2}\sqrt{Var\left(\hat{\beta}_1\right)} \leq \beta_1 - \hat{\beta}_1 \leq z_{1-\alpha/2}\sqrt{Var\left(\hat{\beta}_1\right)} \\ \iff & -z_{1-\alpha/2}\sqrt{Var\left(\hat{\beta}_1\right)} \leq \hat{\beta}_1 - \beta_1 \leq z_{1-\alpha/2}\sqrt{Var\left(\hat{\beta}_1\right)} \\ \iff & -z_{1-\alpha/2} \leq \frac{\hat{\beta}_1 - \beta_1}{\sqrt{Var\left(\hat{\beta}_1\right)}} \leq z_{1-\alpha/2} \end{aligned}$$

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Example: Rent rates and average income

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       Model |  347069.249     1  347069.249           Prob > F      =  0.0000
    Residual |  274693.188    62  4430.53529           R-squared     =  0.5582
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       Total |  621762.438    63  9869.24504           Root MSE      =  66.562

------------------------------------------------------------------------------
        rent |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
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      avginc |     .01158   .0013084     8.85   0.000     .0089646    .0141954
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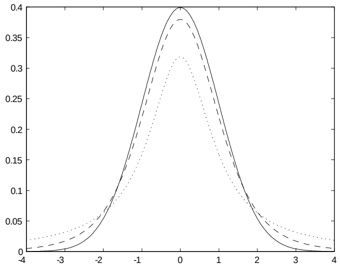
The effect of estimation of σ^2

- The t-distribution has heavier tails than normal.
  The graphs of normal (solid line), t_5 (dashed line), and t_1 (dotted line) PDFs:



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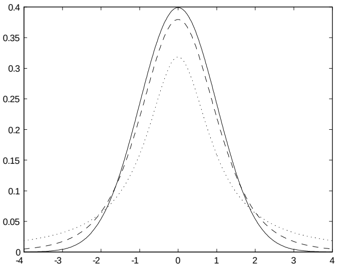

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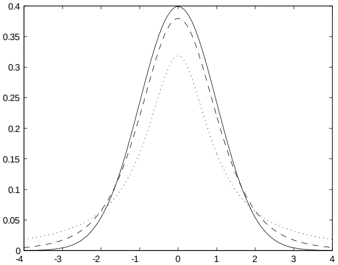

- $t_{df,1-\alpha/2} > z_{1-\alpha/2}$, but as df increases $t_{df,1-\alpha/2} \longrightarrow z_{1-\alpha/2}$.
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Interpretation of confidence intervals

- The confidence interval $CI_{1-\alpha}$ is a function of the sample $\{(Y_i, X_i) : i = 1, \ldots, n\}$, and therefore is random. This allows us to talk about probability of $CI_{1-\alpha}$ containing the true value of β_1.

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- Once the confidence interval is computed given the data, we have its one realization. The realization of $CI_{1-\alpha}$ or (computed confidence interval) is not random, and it does not make sense anymore to talk about the probability that it includes the true β_1.
- Once the confidence interval is computed, it either contains the true value β_1 or it does not.