Economics 326
Methods of Empirical Research in Economics

Lecture 9: Hypothesis testing (Part 2)

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The two-sided t-test

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- The two-sided$t$-test:

$$\text{Reject } H_0 \text{ when } |T| > t_{n-2,1-\alpha/2}.$$

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- p-value is the smallest significance level α for which we can reject H_0.

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  and we will accept H_0.

Example of p-value calculation

Suppose a regression with 19 observations produced the following output:

y	Coef.	Std. Err.	t	P>|t|	[95%
x	-.6725304	.5804943	-1.16	0.263	-1.89
_cons	10.18197	.2509365	40.58	0.000	9.65

- Here, $\hat{\beta}_1 = -0.6725, \beta_{1,0} = 0$, and in the 4th column $t = -0.6725/0.5804 = -1.16$.
- Thus, $|T| = 1.16$ and df=17.
- From the t-table, the closest critical value is $t_{17,1-0.15} = 1.069$. (The probability that a random variable with t_{17}-distribution lies on the right of 1.16 is ≈ 0.15.)
- The p-value is then $\approx 0.15 \times 2 = 0.300$.

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  display invttail(df,τ),

  where df is the number of degrees of freedom and τ is a number between 0 and 1.
  Note that here τ is the right-tail probability!
- For example, display invttail(62,0.05/2) produces 1.9989715.

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Example

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. regress rent avginc

      Source |       SS       df       MS              Number of obs =      64
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       Model |  347069.249     1  347069.249           Prob > F      =  0.0000
    Residual |  274693.188    62  4430.53529           R-squared     =  0.5582
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       Total |  621762.438    63  9869.24504           Root MSE      =  66.562

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        rent |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
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- The p-value is extremely close to zero, (display 2*(ttail(62, 8.85)) gives 1.345×10^{-12}), so for all reasonable significance levels α, we reject H_0 that the coefficient of AvgInc is zero.
- AvgInc is a statistically significant regressor.

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- $T =$

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- The two sided p-value is $2*(\text{ttail}(62, 1.97)) \implies \approx 0.053$.
- For $\alpha \leq 0.053$ we will accept H_0 and for $\alpha > 0.053$ we will reject H_0.

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- Thus, for any $\beta_{1,0} \in CI_{1-\alpha}$, we cannot reject $H_0 : \beta_1 = \beta_{1,0}$ against $H_1 : \beta_1 \neq \beta_{1,0}$ at significance level α.

Example

```
. regress rent avginc

      Source |       SS       df       MS              Number of obs =      64
-------------+------------------------------           F(  1,    62) =   78.34
       Model |  347069.249     1  347069.249           Prob > F      =  0.0000
    Residual |  274693.188    62  4430.53529           R-squared     =  0.5582
-------------+------------------------------           Adj R-squared =  0.5511
       Total |  621762.438    63  9869.24504           Root MSE      =  66.562

------------------------------------------------------------------------------
        rent |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
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      avginc |     .01158   .0013084     8.85   0.000     .0089646    .0141954
       _cons |   148.7764   32.09787     4.64   0.000      84.6137    212.9392
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- A significance level 5% test of $H_0 : \beta_1 = \beta_{1,0}$ against $H_1 : \beta_1 \neq \beta_{1,0}$ will not reject H_0 if $\beta_{1,0} \in [0.0089646, 0.0141954]$.

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  for all $\beta_1 \leq \beta_{1,0}$.

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