

Economics 326
Methods of Empirical Research in Economics

Lecture 9: Hypothesis testing (Part 2)

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The two-sided t -test

- ▶ We are testing $H_0 : \beta_1 = \beta_{1,0}$ against $H_1 : \beta_1 \neq \beta_{1,0}$.
- ▶ When σ^2 is unknown, we replace it with $s^2 = \frac{1}{n-2} \sum_{i=1}^n \hat{U}_i^2$.
- ▶ The t -statistic:

$$T = \frac{\hat{\beta}_1 - \beta_{1,0}}{\sqrt{\widehat{\text{Var}}(\hat{\beta}_1)}} = \frac{\hat{\beta}_1 - \beta_{1,0}}{\sqrt{\frac{s^2}{\sum_{i=1}^n (X_i - \bar{X})^2}}}$$

- ▶ We also replace the standard normal critical values $z_{1-\alpha/2}$ with the t_{n-2} critical values $t_{n-2, 1-\alpha/2}$. However, for large n , $t_{n-2, 1-\alpha/2} \approx z_{1-\alpha/2}$.
- ▶ The two-sided t -test:

Reject H_0 when $|T| > t_{n-2, 1-\alpha/2}$.

The two-sided p -value

- ▶ The decision to accept or reject H_0 depends on the critical value $t_{n-2, 1-\alpha/2}$.
- ▶ If $\alpha_1 > \alpha_2$ then $t_{1-\alpha_1/2} < t_{1-\alpha_2/2}$.
- ▶ Thus, it is easier to reject H_0 with the significance level α_1 since it corresponds to a smaller acceptance region.
- ▶ p -value is the smallest significance level α for which we can reject H_0 .

The two-sided p -value

- ▶ In order to find p -value:

1. Compute T .
2. Find τ such that $|T| = t_{n-2,1-\tau}$.
3. The p -value $= \tau \times 2$.

- ▶ Note that for all $\alpha > p$ -value,

$$|T| = t_{n-2,1-(p\text{-value})/2} > t_{n-2,1-\alpha/2}$$

and we will reject H_0 .

- ▶ For all $\alpha \leq p$ -value,

$$|T| = t_{n-2,1-(p\text{-value})/2} \leq t_{n-2,1-\alpha/2}$$

and we will accept H_0 .

Example of p -value calculation

Suppose a regression with 19 observations produced the following output:

	y	Coef.	Std. Err.	t	P> t	[95%
	x	-.6725304	.5804943	-1.16	0.263	-1.89
	_cons	10.18197	.2509365	40.58	0.000	9.65

- ▶ Here, $\hat{\beta}_1 = -0.6725$, $\beta_{1,0} = 0$, and in the 4th column $t = -0.6725/0.5804 = -1.16$.
- ▶ Thus, $|T| = 1.16$ and $df=17$.
- ▶ From the t -table, the closest critical value is $t_{17,1-0.15} = 1.069$. (The probability that a random variable with t_{17} -distribution lies on the right of 1.16 is ≈ 0.15 .)
- ▶ The p -value is then $\approx 0.15 \times 2 = 0.300$.

- ▶ We can compute critical values and p -values using Stata instead of using the tables.
- ▶ To compute standard normal critical values use:

`display invnormal(τ),`

where τ is a number between 0 and 1.

- ▶ For example: `display invnormal(1-0.05/2)` produces 1.959964.
- ▶ For t critical values use

`display invttail(df , τ),`

where df is the number of degrees of freedom and τ is a number between 0 and 1.

Note that here τ is the right-tail probability!

- ▶ For example, `display invttail(62,0.05/2)` produces 1.9989715.

- ▶ To compute two-sided normal p -values use:

`display 2 * (1-normal (T)) .`

- ▶ For example, `display 2*(1-normal(1.96))` produces 0.04999579.
- ▶ To compute two-sided t -distribution p -values, use

`display 2* (ttail(df, T)) ,`

Note that `ttail` gives the right tail probabilities!

- ▶ For example, `display 2*(ttail(62, 1.96))` produces 0.05449415.

Example

```
. regress rent avginc
```

Source	SS	df	MS			
Model	347069.249	1	347069.249			
Residual	274693.188	62	4430.53529			
Total	621762.438	63	9869.24504			

rent	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]
avginc	.01158	.0013084	8.85	0.000	.0089646 .0141954
_cons	148.7764	32.09787	4.64	0.000	84.6137 212.9392

```
Number of obs = 64  
F( 1, 62) = 78.34  
Prob > F = 0.0000  
R-squared = 0.5582  
Adj R-squared = 0.5511  
Root MSE = 66.562
```

- ▶ Stata report the t -statistics and the p -value for $H_0 : \beta = 0$.
- ▶ To test H_0 whether the coefficient of AvgInc is zero :
 $T = 0.01158/0.0013084 = 8.85$.
- ▶ The p -value is extremely close to zero, (display $2*(\text{ttail}(62, 8.85))$ gives 1.345×10^{-12}), so for all reasonable significance levels α , we reject H_0 that the coefficient of AvgInc is zero.
- ▶ AvgInc is a statistically significant regressor.

Example (continued)

rent	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
avginc	.01158	.0013084	8.85	0.000	.0089646	.0141954
_cons	148.7764	32.09787	4.64	0.000	84.6137	212.9392

- ▶ Consider now testing H_0 that the coefficient of AvgInc is 0.009 against the alternative that it is different from 0.009.
- ▶ $T = (0.01158 - 0.009) / 0.0013084 \approx 1.97$.
- ▶ At 5% significance level, $t_{62,0.975} \approx 1.999 > T$ and we accept H_0 .
- ▶ At 10% significance level, $t_{62,0.95} \approx 1.67 < T$ and we reject H_0 .
- ▶ The two sided p -value is $2*(\text{ttail}(62, 1.97)) \implies \approx 0.053$.
- ▶ For $\alpha \leq 0.053$ we will accept H_0 and for $\alpha > 0.053$ we will reject H_0 .

Confidence intervals and hypothesis testing

- ▶ There is one-to-one correspondence between confidence intervals and hypothesis testing.
- ▶ We cannot reject $H_0 : \beta_1 = \beta_{1,0}$ against a two-sided alternative if $|T| \leq t_{n-2, 1-\alpha/2}$ or if and only if:

$$-t_{n-2, 1-\alpha/2} \leq \frac{\hat{\beta}_1 - \beta_{1,0}}{\sqrt{\widehat{\text{Var}}(\hat{\beta}_1)}} \leq t_{n-2, 1-\alpha/2}$$

$$\iff$$

$$\hat{\beta}_1 - t_{n-2, 1-\alpha/2} \sqrt{\widehat{\text{Var}}(\hat{\beta}_1)} \leq \beta_{1,0} \leq \hat{\beta}_1 + t_{n-2, 1-\alpha/2} \sqrt{\widehat{\text{Var}}(\hat{\beta}_1)}$$

$$\iff$$

$$\beta_{1,0} \in CI_{1-\alpha}.$$

- ▶ Thus, for any $\beta_{1,0} \in CI_{1-\alpha}$, we cannot reject $H_0 : \beta_1 = \beta_{1,0}$ against $H_1 : \beta_1 \neq \beta_{1,0}$ at significance level α .

Example

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Number of obs = 64  
F( 1, 62) = 78.34  
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R-squared = 0.5582  
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Root MSE = 66.562
```

- ▶ The 95% confidence interval for the coefficient of AvgInc is [0.0089646,0.0141954].
- ▶ A significance level 5% test of $H_0 : \beta_1 = \beta_{1,0}$ against $H_1 : \beta_1 \neq \beta_{1,0}$ will not reject H_0 if $\beta_{1,0} \in [0.0089646, 0.0141954]$.

One-sided tests

- ▶ Consider testing $H_0 : \beta_1 \leq \beta_{1,0}$ against $H_1 : \beta_1 > \beta_{1,0}$.
- ▶ It is reasonable to reject H_0 when $\hat{\beta}_1 - \beta_{1,0}$ is large and positive or when

$$T = \frac{\hat{\beta}_1 - \beta_{1,0}}{\sqrt{\widehat{\text{Var}}(\hat{\beta}_1)}} > c_{1-\alpha}$$

where $c_{1-\alpha}$ is a positive constant.

- ▶ The null hypothesis H_0 is composite. The probability of rejection under H_0 depends on β_1 .
- ▶ We pick the critical value $c_{1-\alpha}$ so that

$$P\left(\frac{\hat{\beta}_1 - \beta_{1,0}}{\sqrt{\widehat{\text{Var}}(\hat{\beta}_1)}} > c_{1-\alpha} \mid \beta_1 \leq \beta_{1,0}\right) \leq \alpha$$

for all $\beta_1 \leq \beta_{1,0}$.

One-sided tests

- For all $\beta_1 \leq \beta_{1,0}$,

$$\frac{\beta_1 - \beta_{1,0}}{\sqrt{\widehat{\text{Var}}(\hat{\beta}_1)}} \leq 0,$$

and

$$\begin{aligned} & P\left(\frac{\hat{\beta}_1 - \beta_{1,0}}{\sqrt{\widehat{\text{Var}}(\hat{\beta}_1)}} > c_{1-\alpha} \mid \beta_1 \leq \beta_{1,0}\right) \\ &= P\left(\frac{\hat{\beta}_1 - \beta_1}{\sqrt{\widehat{\text{Var}}(\hat{\beta}_1)}} + \frac{\beta_1 - \beta_{1,0}}{\sqrt{\widehat{\text{Var}}(\hat{\beta}_1)}} > c_{1-\alpha} \mid \beta_1 \leq \beta_{1,0}\right) \\ &\leq P\left(\frac{\hat{\beta}_1 - \beta_1}{\sqrt{\widehat{\text{Var}}(\hat{\beta}_1)}} > c_{1-\alpha} \mid \beta_1 \leq \beta_{1,0}\right) \\ &= \alpha \text{ if } c_{1-\alpha} = t_{n-2, 1-\alpha}. \end{aligned}$$

One-sided tests

- ▶ For size α test, we reject $H_0 : \beta_1 \leq \beta_{1,0}$ against $H_1 : \beta_1 > \beta_{1,0}$ when

$$T = \frac{\hat{\beta}_1 - \beta_{1,0}}{\sqrt{\widehat{\text{Var}}(\hat{\beta}_1)}} > t_{n-2, 1-\alpha}.$$

where $t_{n-2, 1-\alpha}$ is the critical value corresponding to t -distribution with $n - 2$ degrees of freedom.

- ▶ Note that we use $1 - \alpha$ and not $1 - \alpha/2$ for choosing critical values in the case of one-sided testing.
- ▶ For size α test, we reject $H_0 : \beta_1 \geq \beta_{1,0}$ against $H_1 : \beta_1 < \beta_{1,0}$ when

$$T = \frac{\hat{\beta}_1 - \beta_{1,0}}{\sqrt{\widehat{\text{Var}}(\hat{\beta}_1)}} < -t_{n-2, 1-\alpha}.$$

One-sided tests

- ▶ One-sided p -values for $H_0 : \beta_1 \leq \beta_{1,0}$ against $H_1 : \beta_1 > \beta_{1,0}$:
 1. Compute T .
 2. Find τ such that $T = t_{n-2, 1-\tau}$.
 3. The p -value= τ .