Economics 326
Methods of Empirical Research in Economics

Lecture 11: Goodness of fit, estimation of σ^2

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Fitted values

- Consider the multiple regression model with k regressors: $Y_i = \beta_0 + \beta_1 X_{1,i} + \beta_2 X_{2,i} + \ldots + \beta_k X_{k,i} + U_i$.
- Let $\hat{\beta}_0, \hat{\beta}_1, \ldots, \hat{\beta}_k$ be the OLS estimators.

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because when there is an intercept, $\sum_{i=1}^{n} \hat{U}_i = 0$.

Sum-of-Squares

- The total variation of Y in the sample is:

$$SST = \sum_{i=1}^{n} \left(Y_i - \bar{Y} \right)^2 \text{ (Total Sum-of-Squares).}$$

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- If the regression contains an intercept:

$$SST = SSE + SSR.$$

Proof of SST=SSE+SSR

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&= SSE + SSR + 2\sum_{i=1}^{n} \left(\hat{Y}_i - \bar{Y}\right) \hat{U}_i .
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- Next, we will show that $\sum_{i=1}^{n} \left(\hat{Y}_i - \bar{Y}\right) \hat{U}_i = 0$.

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\end{aligned}$$

- The OLS normal equations for a model with an intercept:

$$\sum_{i=1}^{n} \hat{U}_i = \sum_{i=1}^{n} X_{1,i} \hat{U}_i = \ldots = \sum_{i=1}^{n} X_{k,i} \hat{U}_i = 0.$$

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- It follows that $\sum_{i=1}^{n} \left(\hat{Y}_i - \bar{Y}\right) \hat{U}_i = 0$.

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- $0 \leq R^2 \leq 1$.
- R^2 measures the proportion of variation in Y in the sample explained by the X's.

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  and therefore R^2 that corresponds to the regression with one regressor is less or equal than R^2 that corresponds to the regression with two regressors.

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- This can be generalized to the case of k and $k+1$ regressors.

Proof

- Consider

$$\sum_{i=1}^{n}\left(\tilde{U}_i - \hat{U}_i\right)^2 = \sum_{i=1}^{n}\tilde{U}_i^2 + \sum_{i=1}^{n}\hat{U}_i^2 - 2\sum_{i=1}^{n}\tilde{U}_i\hat{U}_i.$$

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- We will show that
$$\sum_{i=1}^{n} \tilde{U}_i \hat{U}_i = \sum_{i=1}^{n} \hat{U}_i^2.$$
- Then,
$$0 \leq \sum_{i=1}^{n} \left(\tilde{U}_i - \hat{U}_i\right)^2 = \sum_{i=1}^{n} \tilde{U}_i^2 - \sum_{i=1}^{n} \hat{U}_i^2,$$

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or

$$\sum_{i=1}^{n} \tilde{U}_i^2 \geq \sum_{i=1}^{n} \hat{U}_i^2.$$

Proof

$$\sum_{i=1}^{n} \tilde{U}_i \hat{U}_i$$

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Proof

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&= \sum_{i=1}^{n} {\color{red} Y_i} \hat{U}_i - \tilde{\beta}_0 \cdot 0 - \tilde{\beta}_1 \cdot 0
\end{aligned}
$$

Proof

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&= \sum_{i=1}^{n} Y_i \hat{U}_i - \tilde{\beta}_0 \cdot 0 - \tilde{\beta}_1 \cdot 0 \\
&= \sum_{i=1}^{n} \left(\hat{\beta}_0 + \hat{\beta}_1 X_{1,i} + \hat{\beta}_2 X_{2,i} + \hat{U}_i \right) \hat{U}_i
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$$\begin{aligned}
\sum_{i=1}^{n} \tilde{U}_i \hat{U}_i &= \sum_{i=1}^{n} \left(Y_i - \tilde{\beta}_0 - \tilde{\beta}_1 X_{1,i} \right) \hat{U}_i \\
&= \sum_{i=1}^{n} Y_i \hat{U}_i - \tilde{\beta}_0 \sum_{i=1}^{n} \hat{U}_i - \tilde{\beta}_1 \sum_{i=1}^{n} X_{1,i} \hat{U}_i \\
&= \sum_{i=1}^{n} Y_i \hat{U}_i - \tilde{\beta}_0 \cdot 0 - \tilde{\beta}_1 \cdot 0 \\
&= \sum_{i=1}^{n} \left(\hat{\beta}_0 + \hat{\beta}_1 X_{1,i} + \hat{\beta}_2 X_{2,i} + \hat{U}_i \right) \hat{U}_i \\
&= \sum_{i=1}^{n} \hat{U}_i \hat{U}_i .
\end{aligned}$$

Adjusted R^2

- Since R^2 cannot decrease when more regressors are added, even if the additional regressors are irrelevant, an alternative measure of goodness-of-fit has been developed.
- Adjusted R^2: the idea is to adjust SSR and SST for degrees of freedom:

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  An estimator for σ^2 is
  $$\begin{aligned} s^2 &= \frac{1}{n-k-1} \sum_{i=1}^{n} \hat{U}_i^2 \\ &= \frac{SSR}{n-k-1}. \end{aligned}$$
- The adjustment $k+1$ is for the number of parameters we have to estimate in order to construct $\hat{U}$'s:
  $$\hat{\beta}_0, \hat{\beta}_1, \ldots, \hat{\beta}_k.$$

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Estimation of σ^2

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- s^2 is an unbiased estimator of σ^2 (i.e. $Es^2 = \sigma^2$) if when the following conditions hold:
  1. $Y_i = \beta_0 + \beta_1 X_{1,i} + \beta_2 X_{2,i} + \ldots + \beta_k X_{k,i} + U_i$.
  2. Conditional on X's, $E\left(U_i\right) = 0$ for all i's.
  3. Conditional on X's, $E\left(U_i^2\right) = \sigma^2$ for all i's (homoskedasticity).
  4. Conditional on X's $E\left(U_i U_j\right) = 0$ for all $i \neq j$.

Stata

```
. regress rent avginc pop enroll

      Source |       SS       df       MS              Number of obs =      64
-------------+------------------------------           F(  3,    60) =   29.05
       Model |  368241.042     3  122747.014           Prob > F      =  0.0000
    Residual |  253521.396    60  4225.35659           R-squared     =  0.5923
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       Total |  621762.438    63  9869.24504           Root MSE      =  65.003

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        rent |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
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      avginc |   .0119416    .001318     9.06   0.000     .0093052     .014578
         pop |  -.0003538   .0001621    -2.18   0.033    -.0006781   -.0000296
      enroll |   .0025595    .001264     2.02   0.047     .0000311    .0050879
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- We have 64 observations ($n = 64$) and 3 regressors ($k = 3$).

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Stata

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- SSR is displayed under Residual SS: 253521.396.
- The Residual df is $n - k - 1$

Stata

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      Source |       SS       df       MS              Number of obs =      64
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```

- SSR is displayed under Residual SS: 253521.396.
- The Residual df is $n - k - 1 = 64 - 3 - 1$

Stata

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      Source |       SS       df       MS              Number of obs =      64
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```

- SSR is displayed under Residual SS: 253521.396.
- The Residual df is $n - k - 1 = 64 - 3 - 1 = 60$.

Stata

```
. regress rent avginc pop enroll

      Source |       SS       df       MS              Number of obs =      64
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       Model |  368241.042     3  122747.014           Prob > F      =  0.0000
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```

- SSR is displayed under Residual SS: 253521.396.
- The Residual df is $n - k - 1 = 64 - 3 - 1 = 60$.
- The Residual MS is $SSR / (n - k - 1) =$

Stata

```
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- SSR is displayed under Residual SS: 253521.396.
- The Residual df is $n - k - 1 = 64 - 3 - 1 = 60$.
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Stata

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      Source |       SS       df       MS              Number of obs =      64
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```

- SSR is displayed under Residual SS: 253521.396.
- The Residual df is $n - k - 1 = 64 - 3 - 1 = 60$.
- The Residual MS is $SSR / (n - k - 1) = s^2$.
- The Residual MS is 253521.396/60=4225.35659.

Stata

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      Source |       SS       df       MS              Number of obs =      64
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```

- SST is displayed under Total SS: 621762.438.

Stata

```
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      Source |       SS       df       MS              Number of obs =      64
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```

- SST is displayed under Total SS: 621762.438.
- The Total df is $n - 1$

Stata

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. regress rent avginc pop enroll

      Source |       SS       df       MS              Number of obs =      64
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```

- SST is displayed under Total SS: 621762.438.
- The Total df is $n - 1 = 64 - 1 = 63$.

Stata

```
. regress rent avginc pop enroll

      Source |       SS       df       MS              Number of obs =      64
-------------+------------------------------           F(  3,    60) =   29.05
       Model |  368241.042     3  122747.014           Prob > F      =  0.0000
    Residual |  253521.396    60  4225.35659           R-squared     =  0.5923
-------------+------------------------------           Adj R-squared =  0.5719
       Total |  621762.438    63  9869.24504           Root MSE      =  65.003

------------------------------------------------------------------------------
        rent |      Coef.   Std. Err.      t    P>|t|     [95% Conf. Interval]
-------------+----------------------------------------------------------------
      avginc |   .0119416    .001318     9.06   0.000     .0093052     .014578
         pop |  -.0003538   .0001621    -2.18   0.033    -.0006781   -.0000296
      enroll |   .0025595    .001264     2.02   0.047     .0000311    .0050879
        cons |    120.772   34.53081     3.50   0.001     51.70009    189.8439
```

- SST is displayed under Total SS: 621762.438.
- The Total df is $n - 1 = 64 - 1 = 63$.
- The Total MS is
  $SST / (n - 1)$

Stata

```
. regress rent avginc pop enroll

      Source |       SS       df       MS              Number of obs =      64
-------------+------------------------------           F(  3,    60) =   29.05
       Model |  368241.042     3  122747.014           Prob > F      =  0.0000
    Residual |  253521.396    60  4225.35659           R-squared     =  0.5923
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      avginc |   .0119416    .001318     9.06   0.000     .0093052     .014578
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        cons |    120.772   34.53081     3.50   0.001     51.70009    189.8439
```

- SST is displayed under Total SS: 621762.438.
- The Total df is $n-1=64-1=63$.
- The Total MS is $SST/(n-1)$ =621762.438/63=9869.24504.

Stata

```
. regress rent avginc pop enroll

      Source |       SS       df       MS              Number of obs =      64
-------------+------------------------------           F(  3,    60) =   29.05
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        cons |    120.772   34.53081     3.50   0.001     51.70009    189.8439
```

- $R^2 =$

Stata

```
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      Source |       SS       df       MS              Number of obs =      64
-------------+------------------------------           F(  3,    60) =   29.05
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        cons |    120.772   34.53081     3.50   0.001     51.70009     189.8439
```

- $R^2 = 1 - \frac{SSR}{SST}$

Stata

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```

- $R^2 = 1 - \frac{SSR}{SST} = 1 - \frac{253521.396}{621762.438}$

Stata

```
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      Source |       SS       df       MS              Number of obs =      64
-------------+------------------------------           F(  3,    60) =   29.05
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- $\bar{R}^2 =$

Stata

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```

- $R^2 = 1 - \frac{SSR}{SST} = 1 - \frac{253521.396}{621762.438} = 0.5923.$
- $\bar{R}^2 = 1 - \frac{SSR/(n-k-1)}{SST/(n-1)}$

Stata

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- $\bar{R}^2 = 1 - \frac{SSR/(n-k-1)}{SST/(n-1)} = 1 - \frac{253521.396/60}{621762.438/63}$

Stata

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- $R^2 = 1 - \frac{SSR}{SST} = 1 - \frac{253521.396}{621762.438} =$0.5923.
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- Root MSE (Mean Squared Error) is $s = \sqrt{s^2} =$

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```

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- $\bar{R}^2 = 1 - \frac{SSR/(n-k-1)}{SST/(n-1)} = 1 - \frac{253521.396/60}{621762.438/63}$ =0.5719.
- Root MSE (Mean Squared Error) is $s = \sqrt{s^2} = \sqrt{4225.35659}$

Stata

```
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      Source |       SS       df       MS              Number of obs =      64
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         pop |  -.0003538   .0001621    -2.18   0.033    -.0006781    -.0000296
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- Root MSE (Mean Squared Error) is $s = \sqrt{s^2} = \sqrt{4225.35659} = 65.003$.