Economics 326
Methods of Empirical Research in Economics

Lecture 15: Dummy variables

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Interval, Ordinal, and Categorical Variables

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- In some data sets, education is reported as an ordinal variable: only the order between its values matters, but the difference has no meaning. Example: The following two variables are equivalent.

$$Education_i = \begin{cases} 1 & \text{if high-school graduate,} \\ 2 & \text{if college graduate,} \\ 3 & \text{if advanced degree.} \end{cases}$$

$$Education_i = \begin{cases} 1 & \text{if high-school graduate,} \\ 10 & \text{if college graduate,} \\ 234 & \text{if advanced degree.} \end{cases}$$

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$$Gender_i = \begin{cases} 1 & \text{if observation } i \text{ corresponds to a } \textit{woman}, \\ 2 & \text{if observation } i \text{ corresponds to a } \textit{man}. \end{cases}$$

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- Categorical and ordinal variables are also called qualitative.
- Qualitative variables cannot be simply included in regression, because the regression technique assumes that all variables are interval.

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  - $Married_i = \begin{cases} 1 & \text{if married,} \\ 0 & \text{otherwise.} \end{cases}$

Example

TABLE 7.1

A Partial Listing of the Data in WAGE1.RAW

person	*wage*	*educ*	*exper*	*female*	*married*
1	3.10	11	2	1	0
2	3.24	12	22	1	1
3	3.00	11	2	0	0
4	6.00	8	44	0	1
5	5.30	12	7	0	1
⋮	⋮	⋮	⋮	⋮	⋮
525	11.56	16	5	0	1
526	3.50	14	5	1	0

A single dummy independent variable

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  $$E(Wage_i | Female_i = 1, Educ_i, Exper_i, Tenure_i) = \\ = \beta_0 + \delta_0 + \beta_1 Educ_i + \beta_3 Exper_i + \beta_4 Tenure_i.$$

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- Thus,

$$\delta_0 = E(Wage_i | Female_i = 1, Educ_i, Exper_i, Tenure_i) - \\ - E(Wage_i | Female_i = 0, Educ_i, Exper_i, Tenure_i).$$

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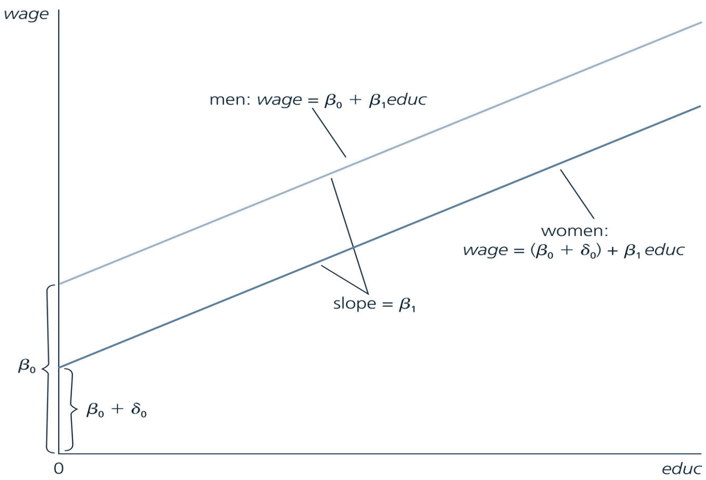
- In this case, men play the role of the base group.
- δ_0 measures the difference relatively to the base group.

An intercept shift

FIGURE 7.1

Graph of $wage = \beta_0 + \delta_0 female + \beta_1 educ$ for $\delta_0 < 0$.



Example

- Estimated equation:

$$\begin{aligned}\widehat{Wage}_i = -\underset{(0.72)}{1.57} - \underset{(0.26)}{1.81}\ Female_i + \underset{(0.049)}{0.572}\ Educ_i + \\ + \underset{(0.012)}{0.025}\ Exper_i + \underset{(0.021)}{0.141}\ Tenure_i.\end{aligned}$$

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- $\hat{\delta}_0 = -1.81$ implies that a women earns \$1.81 less per hour than a man with the same level of education, experience, and tenure. (These are 1976 wages.)
- The difference is also statistically significant.

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- When the dependent variable is in the log form, δ_0 has a percentage interpretation.

Example

- Estimated equation:

$$\begin{aligned} \ln\widehat{(Wage_i)} = & \underset{(0.099)}{0.417} - \underset{(0.036)}{0.297} Female_i + \underset{(0.007)}{0.080} Educ_i + \\ & + \underset{(0.005)}{0.029} Exper_i - \underset{(0.00010)}{0.00058} Exper_i^2 + \\ & + \underset{(0.007)}{0.032} Tenure_i - \underset{(0.00023)}{0.00059} Tenure_i^2 . \end{aligned}$$

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& + \underset{(0.007)}{0.032} \; Tenure_i - \underset{(0.00023)}{0.00059} \; Tenure_i^2 .
\end{aligned}$$

- $\hat{\delta}_0 = -0.297$ implies that a woman earns 29.7% less than a man with the same level of education, experience and tenure.

Changing the base group

- Instead of

  $$\ln\left(Wage_i\right) = \beta_0 + \delta_0 Female_i + \beta_1 Educ_i + \beta_3 Exper_i + \beta_4 Tenure_i + U_i$$

  consider:

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- We conclude that $\delta_0 = -\gamma_0$, $\beta_0 = \theta_0 - \delta_0$, $\beta_1 = \theta_1$, and etc.

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- Thus, changing the base group has no effect on the conclusions.

The dummy variable trap

- Consider the equation:

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$$\ln\left(Wage_i\right) = \beta_0 + \delta_0 Female_i + \gamma_0 Male_i + \\ + \beta_1 Educ_i + \beta_3 Exper_i + \beta_4 Tenure_i + U_i.$$

- Recall that the intercept is a regressor that takes the value one for all observations.

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- One cannot include an intercept and dummies for all the groups!

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- Alternatively, one can include both dummies without the intercept: $\ln(Wage_i) = \pi_0 Female_i + \pi_1 Male_i + \beta_1 Educ_i + \beta_3 Exper_i + \beta_4 Tenure_i + U_i$.

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  - In Stata regression with no intercept can be estimated by using the option "no constant":

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  - The coefficients on the dummy variables lose the difference interpretation.

A slope shift and interactions

- We can also allow the returns to education to be different for men and women:

$$\ln\left(Wage_i\right) = \beta_0 + \delta_0 Female_i + \beta_1 Educ_i + \delta_1 \left(Female_i \cdot Educ_i\right) + \\ + \beta_3 Exper_i + \beta_4 Tenure_i + U_i.$$

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- The variable $\left(Female_i \cdot Educ_i\right)$ is called an interaction.
- The equation for men ($Female_i = 0$):

$$\ln\left(Wage_i^M\right) = \beta_0 + \beta_1 Educ_i + \beta_3 Exper_i + \beta_4 Tenure_i + U_i.$$

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$$\ln\left(Wage_i^F\right) = \left(\beta_0 + \delta_0\right) + \left(\beta_1 + \delta_1\right) Educ_i + \\ + \beta_3 Exper_i + \beta_4 Tenure_i + U_i.$$

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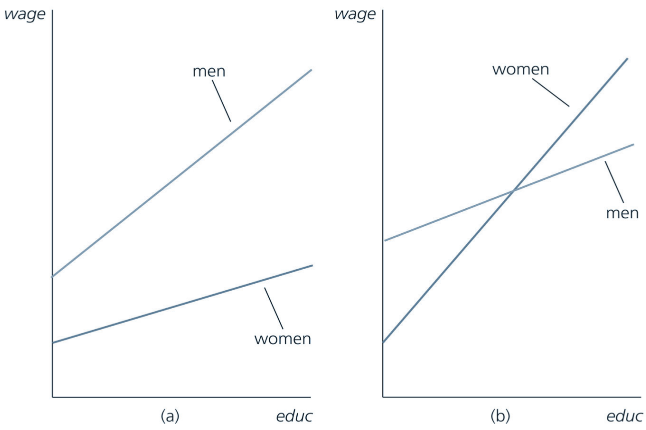
- δ_1 can be interpreted as the difference in return to education between the women and men (the base group) after controlling for experience and tenure.

A slope shift

FIGURE 7.2

Graphs of equation (7.16): (a) $\delta_0 < 0$, $\delta_1 < 0$; (b) $\delta_0 < 0$, $\delta_1 > 0$.



Example

- Estimated equation:

$$\begin{aligned}
\widehat{\ln(Wage_i)} = \underset{(0.119)}{0.389} &- \underset{(0.168)}{0.227} \; Female_i + \\
&+ \underset{(0.008)}{0.082} \; Educ_i - \underset{(0.0131)}{0.0056} \; Female_i \cdot Educ_i \\
&+ \underset{(0.005)}{0.029} \; Exper_i - \underset{(0.00011)}{0.00058} \; Exper_i^2 + \\
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+ \underset{(0.007)}{0.032}\ Tenure_i - \underset{(0.00024)}{0.00059}\ Tenure_i^2.
\end{aligned}
$$

- $\hat{\delta}_1 = -0.0056$ suggesting that the return to education for women is 0.56% less than for men, however it is not statistically significant. Thus, we can conclude that the return to education is the same for men and women.

Multiple categories

- In the previous examples, *Educ* was a quantitative variable: years of education.

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- Suppose now that instead the education variable is ordinal:

$$Education = \begin{cases} 1 & \text{if high-school dropout,} \\ 2 & \text{if high-school graduate,} \\ 3 & \text{if some college,} \\ 4 & \text{if college graduate,} \\ 5 & \text{if advanced degree.} \end{cases}$$

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- Only the order is important, and there is no meaning to the distance between the values.
- Adding such a variable to the regression will give a meaningless result.

Multiple categories

$$Education_i = \begin{cases} 1 & \text{if high-school dropout,} \\ 2 & \text{if high-school graduate,} \\ 3 & \text{if some college,} \\ 4 & \text{if college graduate,} \\ 5 & \text{if advanced degree.} \end{cases}$$

- Define 5 new dummy variables:

$$E_{1,i} = \begin{cases} 1 & \text{if high-school dropout,} \\ 0 & \text{otherwise.} \end{cases} \quad E_{2,i} = \begin{cases} 1 & \text{if high-school graduate,} \\ 0 & \text{otherwise.} \end{cases}$$

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- Group 1 (high-school dropout) becomes the base group.
- δ_2 measures the wage difference between high-school graduates and high-school dropouts.
- δ_3 measures the wage difference between individuals with some college education and high-school dropouts.

Testing for structural breaks or differences in regression functions across groups

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- The model:

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\begin{aligned}
Y_i &= \beta_{1,0} + \beta_{1,1} X_{1,i} + \ldots + \beta_{1,k} X_{k,i} + U_i \text{ if } i \text{ belongs to Group 1} \\
Y_i &= \beta_{2,0} + \beta_{2,1} X_{1,i} + \ldots + \beta_{2,k} X_{k,i} + U_i \text{ if } i \text{ belongs to Group 2}
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- The hypotheses:

$$
\begin{aligned}
H_0 &: \beta_{1,0} = \beta_{2,0}, \beta_{1,1} = \beta_{2,1}, \ldots, \beta_{1,k} = \beta_{2,k}. \\
H_1 &: \beta_{1,j} \neq \beta_{2,j} \text{ at least for one } j \in \{0, 1, \ldots, k\}.
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F^{Chow} = \frac{(SSR_r - SSR_{ur}) \,/\, (k+1)}{SSR_{ur} \,/\, (n - 2\,(k+1))} = \frac{(SSR_r - (SSR_1 + SSR_2)) \,/\, (k+1)}{(SSR_1 + SSR_2) \,/\, (n - 2\,(k+1))},
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where

  - SSR_1 is the SSR obtained by estimating the model using only the observations from Group 1.
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    $$Y_i = \gamma_0 + \gamma_1 X_{1,i} + \ldots + \gamma_k X_{k,i} + U_i \text{ for all } i\text{'s (Groups 1and 2).}$$

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    $Y_i = \gamma_0 + \gamma_1 X_{1,i} + \ldots + \gamma_k X_{k,i} + U_i$ for all i's (Groups 1and 2).

- H_0 of constancy or no structural break is rejected when

  $$F^{Chow} > F_{k+1, n-2(k+1), 1-\alpha}.$$

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- Test:

$$\begin{aligned} H_0 &: \delta_0 = \delta_1 = \ldots = \delta_k = 0. \\ H_1 &: \delta_j \neq 0 \text{ for at least one } j \in \{0, 1, \ldots, k\}. \end{aligned}$$