

**UBC, ECONOMICS 326 - 003**  
**2010 MIDTERM EXAMINATION**

There are 3 questions (100 points total). You have 80 minutes. Answer all questions.

**Question 1.** Consider a simple linear regression model:

$$\begin{aligned} Y_i &= \beta_0 + \beta_1 X_i + U_i, \quad i = 1, \dots, n; \\ \beta_0 &\neq 0; \\ E(U_i | X_1, \dots, X_n) &= 0. \end{aligned}$$

Define

$$\begin{aligned} \hat{\beta}_1 &= \frac{\sum_{i=1}^n (X_i - \bar{X}) Y_i}{\sum_{i=1}^n (X_i - \bar{X})^2} \text{ and } \hat{\beta}_0 = \bar{Y} - \hat{\beta}_1 \bar{X}, \\ \tilde{\beta}_1 &= \frac{\sum_{i=1}^n X_i Y_i}{\sum_{i=1}^n X_i^2} \text{ and } \tilde{\beta}_0 = 0, \end{aligned}$$

where  $\bar{X} = n^{-1} \sum_{i=1}^n X_i$ . Define also

$$\begin{aligned} \hat{U}_i &= Y_i - \hat{\beta}_0 - \hat{\beta}_1 X_i, \\ \tilde{U}_i &= Y_i - \tilde{\beta}_1 X_i. \end{aligned}$$

For each of the following statements, indicate true or false and explain your answers.

- (a) (6 points)  $\sum_{i=1}^n \hat{U}_i = 0$ .
- (b) (6 points)  $\sum_{i=1}^n \tilde{U}_i = 0$ .
- (c) (6 points)  $\sum_{i=1}^n U_i = 0$ .
- (d) (6 points)  $E(U_i X_i^4) = 0$ .
- (e) (6 points) In this model,  $\hat{\beta}_1$  is the OLS estimator, and therefore the Gauss-Markov Theorem implies that  $Var(\hat{\beta}_1 | X_1, \dots, X_n) \leq Var(\tilde{\beta}_1 | X_1, \dots, X_n)$ . Assume that errors  $U_i$ 's are homoskedastic and there is no serial correlation.

**Question 2. (a) (40 points)** Find A-D in the Stata output below. The number of observations is 29.

**(b) (5 points)** According to the results, does x affect y? Explain.

y	Coef.	Std. Err.	t	P> t	[95% Conf. Interval]	
x	A	.373891	B	C	.2025939	D
_cons	1.261567	.1814382	6.95	0.000	.8892861	1.633847

**Question 3.** Consider the following regression model

$$\begin{aligned}
 Y_i &= \beta X_i + U_i, \quad i = 1, \dots, n; \\
 E(U_i | X_1, \dots, X_n) &= 0; \\
 E(U_i^2 | X_1, \dots, X_n) &= \sigma^2; \\
 E(U_i U_j | X_1, \dots, X_n) &= 0 \text{ for all } i \neq j.
 \end{aligned}$$

Assume that  $\bar{X} \neq 0$  and consider the following estimator of  $\beta$ :

$$\tilde{\beta} = \frac{\bar{Y}}{\bar{X}}, \text{ where } \bar{Y} = \frac{1}{n} \sum_{i=1}^n Y_i \text{ and } \bar{X} = \frac{1}{n} \sum_{i=1}^n X_i.$$

**(a) (10 points)** Show that  $\tilde{\beta}$  is unbiased.

**(b) (10 points)** Show that

$$\text{Var}(\tilde{\beta} | X_1, \dots, X_n) = \frac{\sigma^2}{n(\bar{X})^2}.$$

**(c) (5 points)** Let

$$\hat{\beta} = \frac{\sum_{i=1}^n X_i Y_i}{\sum_{i=1}^n X_i^2},$$

and recall that

$$\text{Var}(\hat{\beta} | X_1, \dots, X_n) = \frac{\sigma^2}{\sum_{i=1}^n X_i^2}.$$

Without using the Gauss-Markov Theorem, show that

$$\text{Var}(\hat{\beta} | X_1, \dots, X_n) \leq \text{Var}(\tilde{\beta} | X_1, \dots, X_n).$$

Hint: What is the relationship between  $\sum_{i=1}^n (X_i - \bar{X})^2$ ,  $\sum_{i=1}^n X_i^2$ , and  $n(\bar{X})^2$ ?