UBC, ECONOMICS 326 - 003

2011 MIDTERM EXAMINATION

Suggested solution

Question 1 First, since $\bar{Y} = \alpha + \beta\bar{X} + \bar{U}$, we have $\hat{\alpha} = \alpha - (\tilde{\beta} - \beta)\bar{X} + \bar{U}$. Next, $E(\hat{\alpha}|X_1, \ldots, X_n) = \alpha - (E(\tilde{\beta}|X_1, \ldots, X_n) - \beta)\bar{X} + E(\bar{U}|X_1, \ldots, X_n) = \alpha - 0 \cdot \bar{X} + 0$. The equality holds because $\tilde{\beta}$ is conditionally unbiased and therefore $E(\tilde{\beta}|X_1, \ldots, X_n) = \beta$, and because

$$\begin{aligned} E(\bar{U}|X_1, \ldots, X_n) &= E\left(n^{-1} \sum_{i=1}^{n} U_i \middle| X_1, \ldots, X_n\right) \\ &= n^{-1} \sum_{i=1}^{n} E(U_i|X_1, \ldots, X_n) \\ &= 0. \end{aligned}$$

Lastly, by the LIE, $E(\hat{\alpha}) = E(E(\hat{\alpha}|X_1, \ldots, X_n) = E(\alpha) = \alpha$.

Question 2

B. Since the p-value is 0.191 and $n = 23$, the t- statistic value t can be found from the t-table according to $P(t_{21} > t) = 0.191/2 \approx 0.10$, where t_{21} denotes a t-distributed random variable with 21 df's. From the t-table, $t \approx 1.323$.

A. $\text{SE} = \hat{\beta}_1/t \approx 0.091/1.323 \approx 0.069$.

C,D. $CI_{0.95} = \hat{\beta}_1 \pm t_{21,0.975} \times \text{SE} = 0.091 \pm 2.08 \times 0.069 \approx [-0.053, 0.235]$.

Question 3

(a) $\hat{\beta}_1^* = \frac{\sum_i (X_i^* - \bar{X}^*) Y_i^*}{\sum_i (X_i^* - \bar{X}^*)^2} = \frac{\sum_i (c_2 X_i - c_2 \bar{X}) c_1 Y_i}{\sum_i (c_2 X_i - c_2 \bar{X})^2} = \frac{c_1 c_2 \sum_i (X_i - \bar{X}) Y_i}{c_2^2 \sum_i (X - \bar{X})^2} = \frac{c_1}{c_2} \hat{\beta}_1$.

(b) $\hat{\beta}_0^* = \bar{Y}^* - \hat{\beta}_1^* \bar{X}^* = c_1 \bar{Y} - \frac{c_1}{c_2} \hat{\beta}_1 c_2 \bar{X} = c_1 \bar{Y} - c_1 \hat{\beta}_1 \bar{X} = c_1 \hat{\beta}_0$.

(c) First, $\hat{U}_i^* = Y_i^* - \hat{\beta}_0^* - \hat{\beta}_1^* X_i^* = c_1 Y_i - c_1 \hat{\beta}_0 - \frac{c_1}{c_2} \hat{\beta}_1 c_2 X_i = c_1 Y_i - c_1 \hat{\beta}_0 - c_1 \hat{\beta}_1 X_i = c_1 \hat{U}_i$. Next, $s_*^2 = \frac{1}{n-2} \sum_i \left(\hat{U}_i^*\right)^2 = \frac{1}{n-2} \sum_i \left(c_1 \hat{U}_i\right)^2 = c_1^2 s^2$.

(d) For $H_0 : \beta_1^* = 0$, we have

$$\begin{aligned}
T^* &= \hat{\beta}_1^* \Big/ \sqrt{s_*^2 \Big/ \sum_i (X_i^* - \bar{X}^*)^2} \\
&= \frac{c_1}{c_2} \hat{\beta}_1 \Big/ \sqrt{c_1^2 s^2 \Big/ \sum_i (c_2 X_i - c_2 \bar{X})^2} \\
&= \frac{c_1}{c_2} \hat{\beta}_1 \Big/ \sqrt{(c_1/c_2)^2 \, s^2 \Big/ \sum_i (X_i - \bar{X})^2} \\
&= \hat{\beta}_1 \Big/ \sqrt{s^2 \Big/ \sum_i (X_i - \bar{X})^2} \\
&= T.
\end{aligned}$$

Note that T is the test statistic for testing $H_0 : \beta_1 = 0$.

(e) Since $T = T^*$ and df's are the same in both cases, $pval = pval*$. Thus, rescaling the dependent variable and regressor has no effect on testing for significance of the slope parameter.