Econ 326

Assignment 1

The due date for this assignment is Thursday January 19.

1. Prove the following results:

   (a) If c is a constant, then $Cov(X, c) = 0$.

   (b) $Cov(X, X) = Var(X)$.

   (c) $Cov(X, Y) = Cov(Y, X)$.

   (d) $Cov(a_1 + b_1 X, a_2 + b_2 Y) = b_1 b_2 Cov(X, Y)$, where a_1, a_2, b_1, and b_2 are some constants.

   (e) $Var(X + Y) = Var(X) + Var(Y) + 2Cov(X, Y)$.

   (f) $Var(X - Y) = Var(X) + Var(Y) - 2Cov(X, Y)$.

2. Let X and Y be two discrete random variables. The set of possible values for X is $\{x_1, \ldots, x_n\}$, and the set of possible values for Y is $\{y_1, \ldots, y_m\}$. The joint PMF is given by
$$p_{ij}^{X,Y} = P(X = x_i, Y = y_j), \qquad i = 1, \ldots n; j = 1, \ldots, m.$$
Show that if X and Y are independent then $Cov(X, Y) = 0$.

3. Let $\{x_i : i = 1, \ldots, n\}$ and $\{y_i : i = 1, \ldots, n\}$ be two sequences. Define the averages
$$\begin{aligned} \bar{x} &= \frac{1}{n}\sum_{i=1}^{n} x_i, \\ \bar{y} &= \frac{1}{n}\sum_{i=1}^{n} y_i. \end{aligned}$$

   (a) Show that $\sum_{i=1}^{n}(x_i - \bar{x}) = 0$.

   (b) Using the result in part (a), show that
$$\begin{aligned} \sum_{i=1}^{n}(x_i - \bar{x})^2 &= \sum_{i=1}^{n} x_i (x_i - \bar{x}), \text{ and} \\ \sum_{i=1}^{n}(x_i - \bar{x})(y_i - \bar{y}) &= \sum_{i=1}^{n} y_i (x_i - \bar{x}) = \sum_{i=1}^{n} x_i (y_i - \bar{y}). \end{aligned}$$