EXPERIMENTAL ELICITATION OF AMBIGUITY ATTITUDE USING THE RANDOM INCENTIVE SYSTEM

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Abstract. We demonstrate how the standard usage of the random incentive system in ambiguity experiments is not incentive compatible if the decision maker is ambiguity averse. We propose a slight modification of the procedure in which the randomization takes place before decisions are made and the state is realized and prove that if subjects evaluate the experimental environment in that way (first - risk, second - uncertainty), incentive compatibility may be restored.

1. Introduction

The ambiguity literature has been motivated by thought experiments (Keynes, 1921; Ellsberg, 1961). However, when it comes to experimental implementation, theoretical difficulties arise. This paper concerns the theoretical validity of the most commonly used incentive mechanism - the Random Incentive System (RIS). This mechanism asks a subject to make a series of choices, and then randomly implements only one for payment.

To demonstrate the challenge, consider the basic Ellsberg two-color environment in which a subject is asked to choose between a bet on an uncertain event (e.g. drawing a red ball from an urn with an unknown composition of red and black balls) and a bet on an event with known probability of 0.5 (e.g. drawing a red ball from an urn with 50% red balls and 50% black balls). Facing such a choice, most subjects will choose the risky bet. This is not sufficient as evidence for ambiguity aversion, since we could rationalize the choice by a simple prior over the composition of the unknown urn. For example, the subject might be suspicious that the unknown urn is biased to his disadvantage. Alternatively, he may simply believe that there are more black balls than red balls in the unknown urn.

There are several ways to overcome these difficulties. One is to ask the subject to choose the color to bet on, and only then ask him to choose between the urns (e.g. Halevy, 2007). This may overcome the suspicion concerns, but several drawbacks remain.[1] Indeed, many experimentalists opted for a different solution in which the

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[1]First, a subject preferring the unknown urn may be ambiguity seeking or may have non-symmetric beliefs over the colors. Second, even if the subject prefers a bet on the known urn the experimenter can measure ambiguity attitude only by invoking an identifying assumption that beliefs are symmetric. This is a major concern when symmetry is not a natural assumption (e.g bet on the stock market or on weather).

subject is asked to make two choices: between a bet on red in the urn whose composition is unknown and a bet on red in the urn with 50% red balls, and similarly between a bet on black in the unkwon urn and a bet on black in the urn with 50% black balls. Then, only one of the choices is selected at random to determine the payment, usually using an objective randomization device (e.g. coin toss).

This usage of the Random Incentive System may be problematic for the following reason. Consider an ambiguity averse subject whose underlying preferences are to bet on the two known rather than on the two ambiguous events. However, if he views the two choices as a single decision problem, he may conclude that by choosing bets on the two unknown events he will win with a probability of 0.5, independently of the event that will materialize. Similarly, he would obtain identical probability of winning by choosing the two risky bets. Therefore, he will be indifferent between truthfully reporting his preferences, and reporting the opposite preferences! The underlying reason for this phenomenon is that the random incentive provides the subject a randomization device that allows him to hedge the ambiguity. This point is closely related to Raiffa's (1961) argument against the normative appeal of Ellsbergian behavior, since the decision maker can choose to flip a coin between the two uncertain bets. While the cognitive requirements to implement Raiffa's suggestion may be high, the random incentive system implements this strategy for the subject.

We show in an example that when using RIS where the randomization occurs after the realization of the event, an ambiguity averse subject with maxmin expected utility preferences (Gilboa and Schmeidler, 1989) will exhibit a behavior that can be rationalized by subjective expected utility preferences. This is very different from the existing critiques of the RIS which rely on non-expected utility under risk (Holt, 1986; Karni and Safra, 1987b; Segal, 1988), since our result is obtained even if the subject satisfies expected utility under risk. In that literature, violation of the mixture independence axiom results in non-separability of preferences which invalidates the incentive compatibility of the RIS in general. Following the tradition in models of choice under uncertainty, we will maintain the assumption of expected utility under risk, and concentrate on the additional complications introduced by violations of Savage's (1954) Sure Thing Principle .

We then turn to answer the question whether there exists an implementation of the RIS that will be incentive compatible in eliciting ambiguity preferences. We note that the above result depends on the randomization performed by the RIS occurring *after* the uncertainty is resolved. This induces an Anscombe and Aumann (1963) act which assigns to every state an objective lottery, and where uncertainty can be hedged. If, however, the randomization occurs *before* the resolution of uncertainty and the subject evaluates the experiment in this order, incentive compatibility may be restored. We prove this result assuming expected utility under risk, and show that it extends to some non-expected utility preferences.

Our theoretical results lead to a very practical recommendation for experiments using RIS in ambiguous environments: the experimenter can facilitate this order-perception (risk before uncertainty) by explicitly performing the RIS randomization before choices are made and the uncertainty is resolved. In this case the experimental design is internally consistent with the model under investigation.

2. Description of typical experiments under ambiguity

Let $S = \{s_1, \cdots, s_n\}$ be a finite set *of states of nature. Events* are subsets of S and are typically denoted by E . *Acts* map states into $[0, 1]$ with generic elements denoted by f, g. A *bet on* E is the binary act 1_E0, paying 1 if event E obtains and 0 otherwise. *A constant act*, which assigns the same outcome k to all states of the world, is denoted by its unique outcome.

The first type of experiments we study are those eliciting certainty equivalents. The objective of the experimenter ("she") is to find the sure amount of money that a subject ("he") values as much as an act f. If only one act is studied, the experimenter can use a Becker-DeGroot-Marschak (1964, henceforth, BDM) procedure: she asks the subject to report a number k, then she randomly draws a number $0 \leq \ell \leq 1$, and the subject receives ℓ if $\ell > k$ and f otherwise. Alternatively, she can use a series of choices (e.g. a choice list) and RIS: the subject is asked to choose between a sure amount ℓ and f for many possible realizations of ℓ, and one of the choices is randomly selected for payment.[2] If the subject's preferences satisfy the expected utility assumptions, both procedures (BDM or sequence of choices with RIS) are theoretically equivalent here and would not pose any difficulty if the experimenter considers only one act. Of course, as shown by Holt (1986) and Karni and Safra (1987b), these procedures would be problematic should we not assume expected utility under risk. Here, we focus on a different challenge caused by the fact that the experimenter uses several acts in the experiment, which creates hedging opportunities for the subject. As explained in the introduction, experiments studying ambiguity attitudes have to elicit preferences for bets on more than a single event, and therefore deal with more than a single act. This implies that the experimenter must randomly select which act will be considered for payment (first randomization), and implement a second randomization for this act (the BDM procedure or the randomization over the choices the subject made for this act).

We now turn to formally describe such an incentive scheme: let $\tilde{i}$ and $\tilde{\ell}$ be the random distributions governing the first and the second randomizations, with realizations i and ℓ, respectively.[3]

[2]The highest ℓ for which the act is chosen and the lowest ℓ chosen over the act give an interval for the k elicited in the BDM. See Bardsley et al. (2009) pp.266-267 and 271 for variants of the RIS and the BDM.

[3]In practice, the second randomization is typically based on discrete distributions of ℓ. Implementations of the BDM also mostly use discrete distributions (for instance, ℓ can only take values expressed in cents). For mathematical simplicity, in what follows, we assume a continuous uniform distribution of ℓ.

Definition 1. Let $\mathfrak{f} = (f_1, ..., f_m)$ be a vector of acts, $\mathfrak{k} = (k_1, \cdots, k_m)$ a vector of reports, $i \in \{1, ..., m\}$ and $\ell \in [0,1]$ are two random numbers following full support distributions $\tilde{i}$ and $\tilde{\ell}$, respectively. Consider a mapping:

$$I(\mathfrak{f}, \mathfrak{k}, i, \ell) = \begin{cases} f_i & \text{if } \ell \leq k_i, \\ \ell & \text{otherwise.} \end{cases}$$

A *Certainty Equivalent Random Incentive Scheme (CERIS)* is defined by $I\left(\mathfrak{f}, \mathfrak{k}, \tilde{i}, \tilde{\ell}\right)$.

A second type of experiments involves probability equivalents. Let $\Delta([0,1])$ be the *set of simple lotteries* over $[0,1]$, and 1_p0 be a *binary lottery*, which pays 1 with probability p and 0 otherwise. If the experimenter elicits the binary lottery 1_p0 that the subject finds as good as an act f, we call p the *probability equivalent* of f. If f is a bet on E, we can think of p as the subjective belief of E. This procedure can be used to detect departures from additivity of belief, and study ambiguity attitudes. Probability equivalents p can be elicited with a BDM procedure (as suggested by Grether, 1981; Holt, 2007; Karni, 2009) or with a series of choices and RIS. As discussed above, difficulties arise when considering experiments eliciting probability equivalents for more than a single act, which necessitates to determine which act will be considered for payment. We now define such an incentive scheme for probability equivalents:

Definition 2. Let $\mathfrak{f} = (f_1, ..., f_m)$ be avector of acts, $\mathfrak{p} = (p_1, \cdots, p_m)$ a vector of reports, $i \in \{1, ..., m\}$ and $q \in [0,1]$ are two random numbers following full support random distributions $\tilde{i}$ and $\tilde{q}$, respectively. Consider a mapping:

$$J(\mathfrak{f}, \mathfrak{p}, i, q) = \begin{cases} f_i & \text{if } q \leq p_i, \\ 1_q0 & \text{otherwise.} \end{cases}$$

A *Probability Equivalent Random Incentive Scheme (PERIS)* is defined by $J\left(\mathfrak{f}, \mathfrak{p}, \tilde{i}, \tilde{\ell}\right)$.

An *incentive scheme* (CERIS or PERIS) designed by the experimenter is characterized by avector of acts and a description of the random processes $\tilde{i}$, and $\tilde{\ell}$ or $\tilde{q}$ underlying i, and ℓ or q. For simplicity, we will assume throughout that these random variables follow uniform distributions (discrete or continuous) over their respective (full) support. With everything clearly defined, we may suppress all arguments and use only I and J to denote the incentive scheme.

We assume that the incentive scheme is fully described to the subject, who is asked to report k (for CERIS) or p (for PERIS). We call the incentive scheme given the subject's report *an experiment* (when all uncertainties have not been resolved). Suppose that the subject's preferences over experiments are represented by the utility function $v(\cdot)$. The subject chooses a report that maximizes his utility of the experiments given by v. The external validity of the incentive scheme is reflected by the fact that the subject employs the same utility function to evaluate acts and experiments.[4] In what follows, v will always be expressed in terms of a utility index u over outcomes (normalized by $u(0) = 0$ and $u(1) = 1$).

For example, suppose that v is subjective expected utility[5] (Savage, 1954). That is, there exists an additive belief μ over S such that the subject evaluates each act by the expected utility of the lottery that μ induces through the act. Imagine PERIS where $f = (1_E 0)$.[6] The subject evaluates the act f by $v(1_E 0) = \mu(E)$, i.e. it is worth to him as much as $1_{\mu(E)}0$ and therefore his probability equivalent should be $p = \mu(E)$. The subject's evaluation of the experiment is more subtle: for each report p he receives the bet $1_E 0$ with probability p, and otherwise a lottery $1_q 0$, where q is in $[p, 1]$. The utility of the experiment is $v(J((1_E 0), (p), (1, \tilde{q}))) = p\mu(E) + \int_p^1 q dq$. Therefore, according to the first order condition, the subject's optimal strategy is to report $p^* = \mu(E)$. That is, under the assumption that the subject's preferences are represented by subjective expected utility, the incentive scheme does not distort his report: he reports his valuation of the act if he optimally responds to the experimental environment. Such an incentive scheme will be referred to as incentive compatible.

We now define the optimal report strategy of a subject in an incentive scheme and then use it to define incentive compatibility for any evaluation function v.

Definition 3. A subject's *optimal report strategy* is a vector of reports $\mathfrak{k}^* = (k_1^*, ..., k_m^*)$ or $\mathfrak{p}^* = (p_1^*, ..., p_m^*)$ that maximizes his evaluation of the experiment induced by his report through the incentive scheme:

$$\mathfrak{k}^* \in argmax_{\mathfrak{k}} v\left(I\left((f_1, \cdots, f_m), (k_1, \cdots, k_m), \left(\tilde{i}, \tilde{\ell}\right)\right)\right)$$

$$\mathfrak{p}^* \in argmax_{\mathfrak{p}} v\left(J\left((f_1, \cdots, f_m), (p_1, \cdots, p_m), \left(\tilde{i}, \tilde{q}\right)\right)\right)$$

We now define incentive compatibility:

Definition 4. CERIS is *incentive compatible* (IC) with respect to v if $v(k_i^*) = v(f_i)$ for all k_i^*. Similarly, PERIS is IC with respect to v if $v\left(1_{p_i^*}0\right) = v(f_i)$ for all p_i^*.

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[4]The domain of v is an extension of the domain used by Seo (2009). A typical element of it has the first two stages of events with given probabilities (risk), followed by one stage of events without given probabilities (ambiguity), and then two stages of risk (Seo had only one stage of risk before and after the ambiguity stage). In the domain of v, acts can be modelled as an element with all risk stages degenerate. In section 3 (4), we will consider only elements whose first (last) two stages are degenerate.

[5]For events with objectively given probabilities, their subjective probabilities are equated to the objectively given ones.

[6]The subject knows E is out of the experimenter's control (e.g. E is related to the weather) and since his evaluation function is subjective expected utility, he is not ambiguity averse. Hence, the explanation that required to use more than a single act does not apply here.

3. Random Incentive after Uncertainty Resolves

We now show that if the randomization used in the incentive scheme occurs after uncertainty is resolved, then CERIS and PERIS may not be incentive compatible. We consider a subject whose preferences are represented by maxmin expected utility (MEU, Gilboa and Schmeidler (1989)), and a simple experiment whose goal is to evaluate the subject's behavior under ambiguity. We prove that the subject's optimal response under CERIS and PERIS is to report as if he had a unique probability measure over the state space, and not a set of priors as the model assumes. As a consequence, the experimenter would underestimate the extent of deviation from ambiguity neutrality in her experiment.

Consider an incentive scheme based on two acts $(1_{E_1}0, 1_{E_2}0)$, such that the events E_1 and E_2 partition the state space. For example, the events may correspond to drawing a red/black ball in the Ellsberg ambiguous urn. Assume that the subject has MEU preferences, and evaluates such acts by the lowest expected utility it might generate over his set of priors C, i.e. $v(1_{E_i}0) = \min_{\mu\in C}[\mu(E_i)u(1) + (1-\mu(E_i))\,u(0)]$. Let $[a,b]$ denote all the values $\mu(E_1)$ can take in C, we obtain $v(1_{E_1}0) = a$ and $v(1_{E_2}0) = 1-b$. The experimenter would like to observe these values, so as to obtain the boundaries of the subject's set of priors. By definition of IC, CERIS is IC only if k_i^* satisfies $v(k_i^*) = v(1_{E_i}0)$ for $i = 1,2$, which implies $k_1^* = u^{-1}(a)$ and $k_2^* = u^{-1}(1-b)$. Similarly, noting that $v(1_{p_i}0) = p_i$,[7] PERIS is IC only if $p_1^* = a$ and $p_2^* = 1-b$.

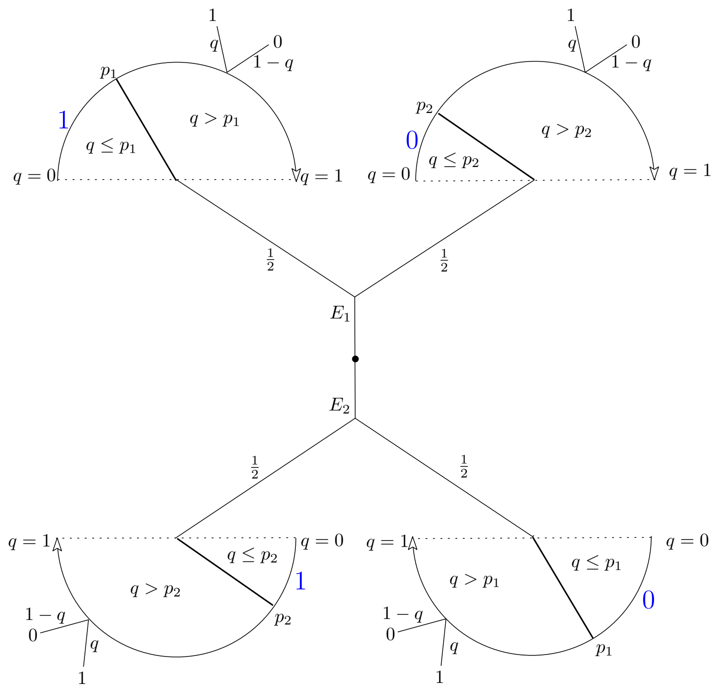
Consider PERIS $J\left((1_{E_1}0, 1_{E_2}0), (p_1,p_2), (\tilde{i},\tilde{q})\right)$. If realization of the events is perceived as occurring before that of the random processes used in the incentive scheme, the subject might perceive the experiment as an *Anscombe-Aumann act*, i.e. a mapping h from S to $\Delta([0,1])$. For such act h, MEU is defined as $v(h) = \min_{\mu\in C}\sum_S[\mu(s)Eu(h(s))]$, where C is the subject's (convex) set of priors and Eu is the expected utility functional.

The subject's perception of the experiment induced by his report through the incentive schemes is depicted in Figure 3.1a.

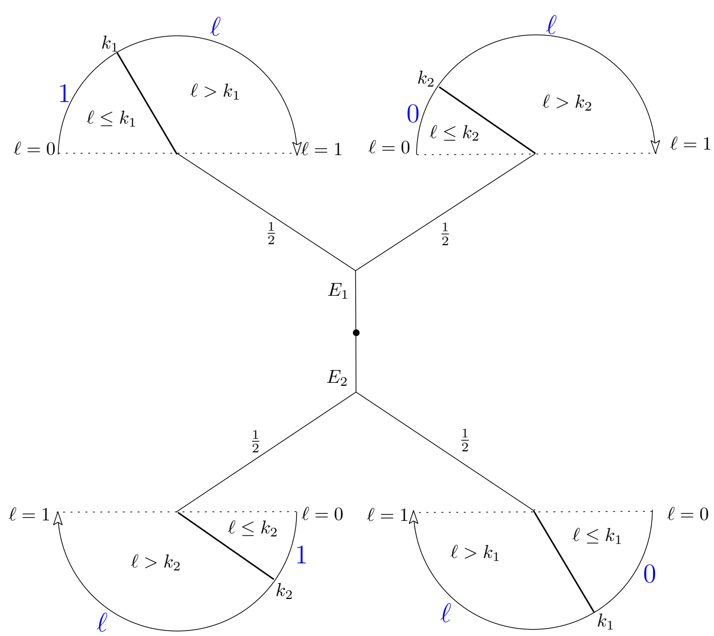
He understands that if E_1 happens, he has a 50% probability to be paid based on the report p_1 for $1_{E_1}0$, where he gets 1 with probability p_1,[8] and otherwise faces a lottery 1_q0 such that $q \in [p_1, 1]$; he also has a 50% probability to be paid based on the report p_2 for $1_{E_2}0$, and would receive 0 with probability p_2 and otherwise a lottery 1_q0 whose winning probability is in $[p_2, 1]$. Following the same reasoning for the case when E_1 does not happen, his utility of the experiment induced by his report (p_1,p_2)through the PERIS is:

[7]Formally, define an event with an objective probability of p_i and require that every prior in C assigns to this event a probability of p_i.

[8]If $q < p_1$ (hence with probability p_1 knowing that $\tilde{q}$ is uniform over $[0,1]$), he gets 1_E0. Since E occurred, he gets 1.



(A) PERIS



(B) CERIS

FIGURE 3.1. The experiments with RIS after resolution of uncertainty

$$v\left(J\left(\left(1_{E_1}0,1_{E_2}0\right),\left(p_1,p_2\right),\left(\tilde{i},\tilde{q}\right)\right)\right)$$
$$= \min_{\mu\in C}\left[\tfrac{1}{2}\left[\left(\mu(E_1)\times p_1+(1-\mu(E_1))\times p_2\right)\right]+\tfrac{1}{2}\left(\int_{p_1}^{1}qdq+\int_{p_2}^{1}qdq\right)\right] \tag{3.1}$$

A similar analysis can be applied to a CERIS experiment, described in Figure 3.1b. The experiment induced by his report of (k_1,k_2) through CERIS is $I\left(\left(1_{E_1}0,1_{E_2}0\right),\left(k_1,k_2\right),\left(\tilde{i},\tilde{\ell}\right)\right)$. Hence, he understands that if E_1 obtains, he has a 50% probability to be paid based on the report k_1 he made for $1_{E_1}0$. If so, he gets 1 with probability k_1,[9] and $\ell\in[k_1,1]$ otherwise; he also has a 50% probability to be paid based on the report k_2 he made for $1_{E_2}0$, and would receive 0 with probability k_2 and otherwise $\ell\in[k_2,1]$. Following the same reasoning for the case when E_1 does not happen, he obtains the evaluation for the experiment :

$$v\left(I\left(\left(1_{E_1}0,1_{E_2}0\right),\left(k_1,k_2\right),\left(\tilde{i},\tilde{\ell}\right)\right)\right)$$
$$= \min_{\mu\in C}\left[\tfrac{1}{2}\left[\left(\mu(E_1)\times k_1+(1-\mu(E_1))\times k_2\right)\right]+\tfrac{1}{2}\left(\int_{k_1}^{1}u(\ell)d\ell+\int_{k_2}^{1}u(\ell)d\ell\right)\right] \tag{3.2}$$

As a maxmin EU decision maker, the subject chooses p_1^* and p_2^* (k_1^* and k_2^*) so as to maximize Eq. 3.1 (Eq. 3.2). The following proposition establishes that PERIS and CERIS are not IC under the assumptions we have made in this section.

Proposition 5. *Consider an incentive scheme with the acts $f=(1_{E_1}0,1_{E_2}0)$ and assume maxmin expected utility with a set of priors such that $\mu(E_1)\in[a,b]$. With CERIS, the subject reports*

$$(k_1^*,k_2^*)=\begin{cases}\left(u^{-1}(b),u^{-1}\left(1-b\right)\right) & a<b<0.5\\ \left(u^{-1}\left(0.5\right),u^{-1}\left(0.5\right)\right) & a\le 0.5\le b\\ \left(u^{-1}\left(a\right),u^{-1}\left(1-a\right)\right) & 0.5<a<b\end{cases}$$

With PERIS, the subject reports

$$(p_1^*,p_2^*)=\begin{cases}(b,1-b) & a<b<0.5\\ (0.5,0.5) & a\le 0.5\le b\\ (a,1-a) & 0.5<a<b\end{cases}$$

Hence, CERIS and PERIS are not IC.

Proof. See Appendix A. □

Proposition 5 shows that neither PERIS nor CERIS is IC when uncertainty is resolved before the implementation of the random incentive. The results for CERIS and PERIS highlight that the experimenter will not be able to observe the set of priors

[9] If $\ell<k_1$ (hence with probability k_1 knowing that $\tilde{\ell}$ is uniform over $[0,1]$), he gets $1_{E_1}0$ and since E_1 occurred, he gets 1.

$[a, b]$. She will conclude that the subject has a degenerate set of priors ($\{b\}$,$\left\{\frac{1}{2}\right\}$, or $\{a\}$), compatible with subjective expected utility or probabilistic sophistication, i.e, ambiguity neutrality. As a consequence, using CERIS or PERIS when the resolution of uncertainty precedes the random incentives leads to underestimating ambiguity aversion if the subject integrates the whole experiment into a single decision problem.

Related observations have recently been made by Bade (2014), Kuzmics (2015) and Oechssler and Roomets (2014). Oechssler and Roomets (2014) showed that if RIS is used, an MEU subject should not display typical ambiguity averse choices in the Ellsberg three-color experiment. Bade (2014) generalized this claim by considering any (possibly uncertain) randomization mechanism. She discussed alternative ways to observe ambiguity preferences but concluded that there is, in most cases, no clear solution. Kuzmics (2015) studies the implication of Wald complete class theorem for ambiguity averse subjects, and argues that an experimenter can never observe the true preferences of ambiguity averse subject who is Waldian and hence satisfies Anscombe and Aumann (1963) axiom of reversal of order (i.e. any subject who is indifferent between a risky lottery over acts or the equivalent act yielding risky lotteries). Azrieli et al. (2015) study a general model of incentives in experiments and show that a condition of *monotonicity*, which is close to Savage's P3, is equivalent to incentive compatibility of a class of experiments that use the RIS. They note that in the domain of uncertainty, reversal of order (which they term *reduction*) together with their condition of monotonicity imply ambiguity neutrality. It follows that if a subject is ambiguity averse and satisfies order-reversal then there exist experiments using the RIS which are not incentive compatible.

Proposition 5 demonstrates that the problem of eliciting ambiguity preferences is pervasive in the elicitation mechanisms used by experimentalists to study ambiguity attitudes. Moreover, we take a descriptive stance in which reversal of order is not guaranteed. There is strong experimental evidence suggesting that ambiguity aversion is related to violation of reduction of compound lotteries (Halevy, 2007; Dean and Orteleva, 2014; Abdellaoui et al., 2015; Chew et al., 2014). As shown by Seo (2009), the relaxation of the reversal of order assumption is required in order to provide an axiomatic foundation to the smooth model that accommodates the violation of reduction of compound lotteries.

In what follows, we relax the reversal of order axiom and show that the elicitation of ambiguity preferences using the RIS is incentive compatible if the latter precedes the ambiguity studied in the experiment. Note that we assume the same level of sophistication as we have assumed so far, i.e. that the subject is able to see the whole experiment as one decision. He will be credibly informed by the experimenter that the RIS is performed prior to the resolution of uncertainty and his perception of the ordering of the various stages changes accordingly.

4. Random Incentive Before Uncertainty Resolves

In this section, we show that if the random incentive system is employed before the resolution of uncertainty, and the subject evaluates the experiment in this order, CERIS and PERIS *are* IC if we assume expected utility under risk. We further study the robustness of this result by relaxing the expected utility assumption. All results below are derived for an arbitrary ambiguity model expressed in u terms with u being the utility index that the subject uses under risk. In other words, v assigns utility $u(k)$ to outcome k.[10]

Consider an experiment on $\mathfrak{f} = (f_1, \cdots, f_m)$ with CERIS and random incentive preceding subjective uncertainty. The experiment can be represented as in Figure 4.1b (for $m = 2$). With probability $\frac{1}{m}$, the subject's payoff depends on his report of k_i (and not on any $k_{j \neq i}$). In this case, he receives f_i (that he values at $v(f_i)$) with probability k_i, and $\ell \in [k_i, 1]$ otherwise. Figure 4.1a illustrates PERIS when the incentive system precedes subjective uncertainty (for $m = 2$). With probability $\frac{1}{m}$, the subject's payoff depends on his report of p_i (and not on any $p_{j \neq i}$). In this case, he receives f_i (that he values at $v(f_i)$) with probability p_i, and the lottery $(1_q 0)$ where $q \in [p_i, 1]$ otherwise.

4.1. Expected utility under risk.

Assuming expected utility, the subject's optimal report of $\mathfrak{k}^*$ in CERIS maximizes

$$\sum_{i=1}^{m} \left(\frac{k_i}{m} v(f_i) + \frac{1}{m} \int_{k_i}^{1} u(\ell) d\ell \right). \tag{4.1}$$

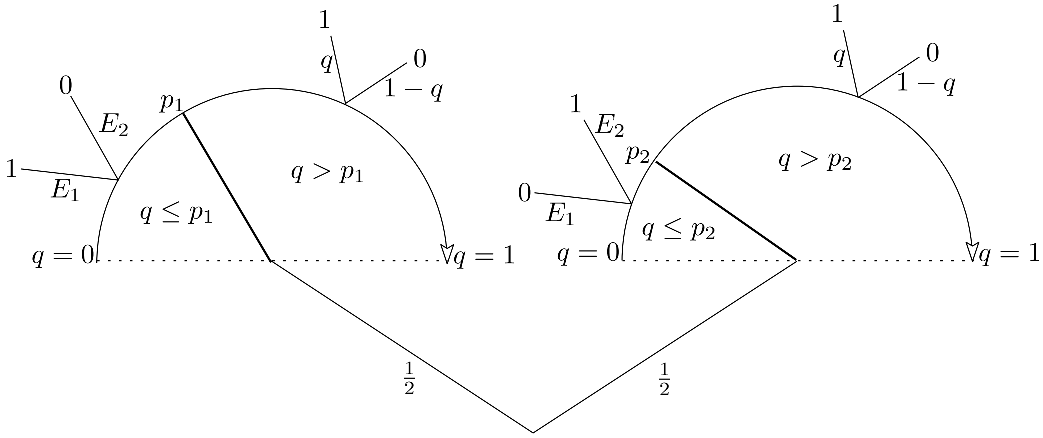
We show that for all i reporting k_i^* such that $u(k_i^*) = v(f_i)$ is the subject's optimal strategy. First, reporting k_i such that $u(k_i) > v(f_i)$ is a dominated strategy since the experiment induced by reporting $k_i > u^{-1}(v(f_i))$ through CERIS can be obtained from that induced by reporting $k_i = u^{-1}(v(f_i))$ by transferring positive probability mass from higher utility values to $v(f_i)$. Conversely, reporting k_i such that $u(k_i) < v(f_i)$ would also be suboptimal as the subject would receive outcomes with utility less than $v(f_i)$ instead of f_i. Therefore CERIS is IC, under the assumption of expected utility under risk.

Similarly, when determining optimal reports $\mathfrak{p}^*$ for PERIS, assuming expected utility under risk, the subject maximizes

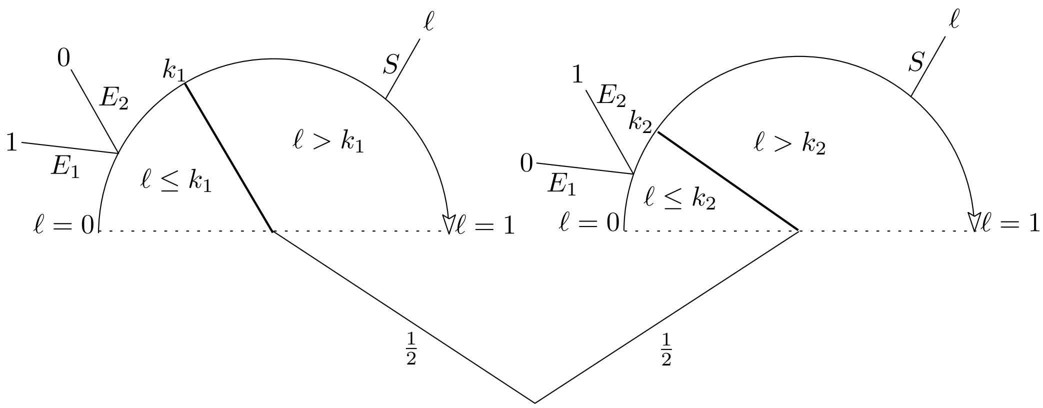
$$\sum_{i=1}^{m} \left(\frac{p_i}{m} v(f_i) + \frac{1}{m} \int_{p_i}^{1} q dq \right). \tag{4.2}$$

by reporting $p_i^* = v(f_i)$. The proof follows identical dominance argument as used above for CERIS.

[10] Virtually all ambiguity models satisfy this property. See for instance the general families of models proposed by Ghirardato and Marinacci (2001), Cerreia-Vioglio et al. (2011), and Grant and Polak (2013).



(A) PERIS



(B) CERIS

FIGURE 4.1. Experiments with RIS before uncertainty ($m = 2$)

Proposition 6. *Assume expected utility under risk with utility index u and an ambiguity model expressed in u terms. If the random incentive system (RIS) precedes the resolution of uncertainty, then CERIS and PERIS are incentive compatible (IC).*

4.2. **Nonexpected utility under risk with compound independence.** So far, we allowed for deviations from (subjective) expected utility for uncertain acts, but not for risky lotteries. We now relax the assumption of expected utility for risk, as non-expected utility models have been shown to pose difficulties with random incentives. We consider all non-expected expected utility models for risk with utility index u that satisfies weak-ordering and stochastic dominance. If an incentive scheme is incentive compatible, the subject should report, under the assumption of weak-ordering and stochastic dominance: $k_i^* = u^{-1}(v(f_i))$ in CERIS, and p_i^* such that $v\left(1_{p_i^*}0\right) = v(f_i)$ in PERIS for all $i \in \{1, \cdots, m\}$.

Consider CERIS on $f = (f_1, \cdots, f_m)$. Figure 4.1b demonstrates that a report of k induces a three-stage experiment through CERIS. The first stage corresponds to resolution of $\tilde{i}$, the random selection of the report k_i on which the subject's payoff depend. The second stage corresponds to the resolution of $\tilde{\ell}$, and the third stage is either the act f_i or a sure amount ℓ. Since expected utility is linear in probabilities, reducing the first two stages into a single stage does not have any impact on the valuation of the experiment. However, when probabilities are weighted, this does not hold anymore. Therefore, the optimal report depends on the subject's evaluation of compound objective lotteries.

In this subsection, we consider a subject who does not reduce multi-stage lotteries but whose behavior satisfies the compound independence axiom (recursivity) instead (Segal, 1990).

Let $(X_1, r_1; ...; X_n, r_n)$ be a (possibly multi-stage) lottery yielding X_i with probability r_i, where X_i may be a (possibly multi-stage) lottery or an act. The experiment I induced by a report of $\mathfrak{k}$ can be written as $I = \left(B_1, \frac{1}{m}; \cdots; B_m, \frac{1}{m}\right)$ where B_i denotes the i^{th} second-stage branch, implied by i being drawn in the first stage and in which the subject's payoff depends on the report k_i. *Compound independence (recursivity)* implies that the evaluation function satisfies:

$$v(I) = v(u^{-1}(v(B_1)), \frac{1}{m}; \cdots; u^{-1}(v(B_m)), \frac{1}{m})$$

That is, the subject substitutes all second-stage branches by their certainty equivalents, and then evaluates the implied one-stage lottery according to his risk preferences.

Since there is no interaction between different branches, maximizing each $v(B_i)$ is sufficient to maximize $v(I)$. With the subject's risk preference satisfying stochastic dominances, following identical reasoning to the one employed for EU, the optimal report to maximize $v(B_i)$ is: $k_i^* = u^{-1}(v(f_i))$. Indeed, the experiment induced by reporting $k_i > u^{-1}(v(f_i))$ (or $k_i < u^{-1}(v(f_i))$) transfers positive probability masses from higher utility values (or $v(f_i)$) to $v(f_i)$ (or lower utility values).

In the case of PERIS, the experiment induced by a report of $\mathfrak{p}$ can be written as $J = \left(B_1', \frac{1}{m}; \cdots; B_m', \frac{1}{m}\right)$ where B_i' denotes the i^{th} second-stage branch, implied by i being drawn in the first stage and in which the subject's payoff depends on the report p_i. Assuming weak ordering, and stochastic dominance, compound independence also implies that maximizing each valuation $v(B_i')$ is sufficient to maximize $v(J)$. It follows that the optimal report is p_i^* such that $v\left(1_{p_i^*}0\right) = v(f_i)$. By stochastic dominance, $v(1_{p_i}0) > v(1_{p_{i*}}0)$ implies $p_i > p_i^*$. Hence, overreporting p_i ($p_i > p_i^*$) transfers positive probability masses from lotteries with higher utility values (e.g. $v(1_{p_i}0)$) to $v(f_i)$. With similar arguments, under-reporting $p_i < p_i^*$ is also suboptimal.

Proposition 7. *Assume weak ordering, stochastic dominance and compound independence under risk with utility index u and an ambiguity model expressed in u terms. If the random incentive system precedes the resolution of uncertainty, then CERIS and PERIS are IC.*

4.3. **Nonexpected utility under risk with reduction of compound lotteries.** In this subsection, we do not assume anymore that the subject's behavior satisfies compound independence. Instead, we analyze the situation in which reduction of compound lotteries (ROCL) applies. Under this assumption, CERIS remains IC. Reducing the first and second stages simply means multiplying each second stage probability (or density) by $\frac{1}{m}$. Over-reporting k_i still implies a transfer of probability mass from outcomes with higher utility values to $v(f_i)$, and under-reporting k_i implies a transfer of probability mass from $v(f_i)$ to outcomes with lower utility values.

However, in Appendix B we provide a counterexample which demonstrates that PERIS may not be incentive compatible when ROCL is assumed in conjunction with RDU (which satisfies weak-ordering and stochastic dominance). This result originates in the subject possibly being paid a lottery 1_q0. This lottery is compounded with the RIS stages. Therefore, under RDU and ROCL, the RIS stages influence the utility derived from the lottery and affect the subject's optimal report. By contrast, the utility of the outcome ℓ is independent of the RIS stages in CERIS, which retains incentive compatibility.

Proposition 8. *Assume weak ordering, stochastic dominance and with ROCL under risk with utility index u and an ambiguity model expressed in u terms. If the random incentive system precedes the resolution of uncertainty, then CERIS is IC but PERIS may not be.*

For CERIS and PERIS to be incentive compatible, the order of uncertainty resolution and compound independence are the essential conditions to be satisfied. They ensure that subjects treat each choice in isolation and carry out backward induction. For CERIS, ROCL happens to induce the same decision situation as compound independence, whereas for PERIS, ROCL implies violation of compound independence, leading to incentive incompatibility in the same way as highlighted by Karni and Safra (1987a) for experiments under risk.

5. Conclusion

This study demonstrated the theoretical challenges posed by the current usage of a random incentive system in ambiguity experiments. In particular, we showed that when RIS follows the resolution of uncertainty, incentive compatibility may be lost. The reason for this result is that the RIS is incompatible with the structure of the preferences investigated.

We further showed that consistency with the ambiguity models requires that the RIS precedes the resolution of uncertainty. In this case incentive compatibility of CERIS and PERIS is restored if the subject satisfies expected utility under risk.

We extended this result to more general models of choice under risk. We showed that both PERIS and CERIS are incentive compatible if compound independence (recursivity) is satisfied. However, if the subject reduces compound objective lotteries (without compound independence) then only the incentive compatibility of CERIS is guaranteed.

Our theoretical results have practical implications for experimental design. To ensure the internal consistency of the experiment with the decision model under investigation, RIS should precede the resolution of uncertainty. The experimenter can facilitate the subject's perception of this ordering by implementing the RIS before the decision is made and choosing a source of uncertainty whose resolution occurs after the decision. In practice, implementing the RIS before the decision can be implemented easily by letting the subject randomly draw a sealed envelope (containing the choice problem to be implemented) from a pile of envelopes at the beginning of the experiment. At the end of the experiment the envelope is opened and the uncertainty relating to the respective choice problem is resolved to determine the subject's payoff. Such strategy was successfully implemented by Epstein and Halevy (2014), who also included in their protocol choice problems that allowed the experimenter to observe if a subject "reversed the order" and hedged the ambiguity. Johnson et al. (2014) discussed other advantages of similar implementations of this protocol.

Appendix A. Proof of Proposition 5

Proof. **PERIS:** the subject maximizes Eq. 3.1, which can be simplified as:

$$v = \min_{\mu(E_1)\in[a,b]} \frac{1}{2}(p_1\mu(E_1) + p_2(1-\mu(E_1)) + \frac{1-p_1^2}{2} + \frac{1-p_2^2}{2}) \tag{A.1}$$

The DM has three different kinds of strategies: report $p_1 < p_2$, $p_1 > p_2$, or $p_1 = p_2$. Depending on the DM's prior beliefs, he chooses the strategy that maximizes his utility. We first list all available strategies, and then analyze his optimal strategy given different prior beliefs.

Strategy 1: report $p_1 < p_2$. In this case, the valuation is decreasing in $\mu(E_1)$. Therefore a minimum is attained at $\mu(E_1) = b$:

$$v_1 = \frac{1}{2}(bp_1 + (1-b)\,p_2) + \frac{1}{2}\left(\frac{1-p_1^2}{2} + \frac{1-p_2^2}{2}\right)$$

Whenever the constraint $p_1 < p_2$ is satisfied, the optimal strategy, given by the first order condition, is to report $(p_1^*, p_2^*) = (b, 1-b)$ if it satisfies $p_1 < p_2$.

Strategy 2: report $p_1 > p_2$. In this case, $\mu(E_1) = a$ gives the minimal utility:

$$v_2 = \frac{1}{2}(ap_1 + (1-a)p_2) + \frac{1}{2}\left(\frac{1-p_1^2}{2} + \frac{1-p_2^2}{2}\right)$$

and a similar analysis shows that the optimal strategy is to report $(p_1^*, p_2^*) = (a, 1-a)$ if it satisfies $p_1 > p_2$.

Strategy 3: report $p_1 = p_2$. The term $\mu(E_1)$ drops out and the utility becomes:

$$v_3 = \frac{1}{2}(p_1 + 1 - p_1^2)$$

and the optimal strategy is to report $(p_1^*, p_2^*) = \left(\frac{1}{2}, \frac{1}{2}\right)$.

We next show how the DM chooses among the three strategies according to his prior beliefs.

Prior belief 1: $a < b < \frac{1}{2}$

For strategy 1, $p_1 < p_2$ is satisfied since $b < \frac{1}{2}$ implies $b < 1-b$. The attained maximum is: $v_1^* = \frac{1}{2}(\frac{3}{2} + b^2 - b)$, which is minimized at $\frac{1}{2}$. Given that $b < \frac{1}{2}$ then $v_1^* > \frac{5}{8}$,

For strategy 3, the maximum is $v_3^* = \frac{5}{8}$.

For strategy 2, $a < \frac{1}{2}$ implies $a < 1-a$ and therefore $p_1 > p_2$ is not satisfied. We show that when $a < \frac{1}{2}$ then strategy 2 (i.e., $p_1 > p_2$) implies $v_2^* < \frac{5}{8}$:The first inequality is obtained by replacing a by $\frac{1}{2}$ and the second by maximizing the quadratic function in p_1 and p_2. It is maximized at $\left(\frac{1}{2}, \frac{1}{2}\right)$.

Therefore $v_1^* > v_3^* > v_2^*$, implying that when the prior belief is $a < b < \frac{1}{2}$, reporting $(p_1^*, p_2^*) = (b, 1-b)$ is the optimal strategy.

Prior belief 2: $\frac{1}{2} < a < b$In this case, when $b > \frac{1}{2}$ then strategy 1 (i.e., $p_1 < p_2$) implies $v_1^* < \frac{5}{8}$:The first inequality is obtained by replacing b by $\frac{1}{2}$ and the second by maximizing the quadratic function in p_1 and p_2. The maximum of $\frac{5}{8}$ is attained at $\left(\frac{1}{2}, \frac{1}{2}\right)$. As a consequence, $v_1^* < \frac{5}{8}$, $v_2^* = \frac{1}{2}(\frac{3}{2} + a^2 - a)$ (because $a > 1-a$), and $v_3^* = \frac{5}{8}$. Therefore, $v_2^* > v_3^* > v_1^*$, implying that reporting $(p_1^*, p_2^*) = (a, 1-a)$ is the optimal strategy.

Prior belief 3: $a \leq \frac{1}{2} \leq b$

In this case, $v_1^* < \frac{5}{8}$ and $v_2^* < \frac{5}{8}$. This can be proved as above noticing that the maximum $\frac{5}{8}$ of $\frac{1}{4}\left(2 - p_1^2 - p_2^2 + p_1 + p_2\right)$ is only reached if $p_1 = p_2 = \frac{1}{2}$ and therefore, not if $p_1 < p_2$ or $p_1 > p_2$. Furthermore, we know that $v_3^* = \frac{5}{8}$ and therefore, $v_3^* > v_1^*, v_3^* > v_2^*$. This implies that reporting $(p_1^*, p_2^*) = \left(\frac{1}{2}, \frac{1}{2}\right)$ is the optimal strategy.

CERIS

The subject maximizes his utility of the experiment as represented by

$$v\left(I\left(\left(f_1, f_2\right), \left(k_1, k_2\right), \left(\tilde{i}, \tilde{\ell}\right)\right)\right)$$
$$= \min_{\mu \in C}\left[\tfrac{1}{2}\left[\left(\mu(E_1) \times k_1 + (1-\mu(E_1)) \times k_2\right)\right] + \tfrac{1}{2}\left(\int_{k_1}^1 u(\ell)d\ell + \int_{k_2}^1 u(\ell)d\ell\right)\right]$$

The DM can have three different strategies: reporting $k_1 < k_2$, $k_1 > k_2$ or $k_1 = k_2$. Depending on the prior beliefs of the DM, he chooses the strategy that maximizes his utility. We first give a list of all available strategies, and then analyze his optimal strategy given different prior beliefs.

Strategy 1: report $k_1 < k_2$. In this case, the valuation is decreasing in $\mu(E_1)$ therefore, $\mu(E_1) = b$ gives the minimal valuation:

$$v_1 = \frac{1}{2}\left[\left(bk_1 + (1-b)\,k_2\right)\right] + \frac{1}{2}\left(\int_{k_1}^1 u(\ell)d\ell + \int_{k_2}^1 u(\ell)d\ell\right)$$

Whenever the constraint $k_1 < k_2$ is satisfied, the optimal strategy, given by the first order condition, is to report $(k_1^*, k_2^*) = \left(u^{-1}(b), u^{-1}(1-b)\right)$ if it satisfies $k_1 < k_2$.

Strategy 2: report $k_1 > k_2$. In this case, $\mu(E_1) = a$ gives the minimal valuation:

$$v_2 = \frac{1}{2}\left[\left(ak_1 + (1-a)\,k_2\right)\right] + \frac{1}{2}\left(\int_{k_1}^1 u(\ell)d\ell + \int_{k_2}^1 u(\ell)d\ell\right)$$

and a similar analysis shows that the optimal strategy is to report $(k_1^*, k_2^*) = (u^{-1}(a), u^{-1}(1-a))$ if it satisfies $k_1 > k_2$.

Strategy 3: report $k_1 = k_2$. The $\mu(E_1)$ term drops out and valuation becomes:

$$v_3 = \frac{1}{2}k_1 + \int_{k_1}^{1} u(\ell)d\ell$$

and the optimal strategy is to report $(k_1^*, k_2^*) = \left(u^{-1}\left(\frac{1}{2}\right), u^{-1}\left(\frac{1}{2}\right)\right)$.
Next we analyze the subject's optimal strategy for three different cases of prior beliefs.

Prior belief 1: $a < b < \frac{1}{2}$

Since $b < \frac{1}{2}$, we have $1 - b > b$. Hence, $(k_1^*, k_2^*) = (u^{-1}(b), u^{-1}(1-b))$ satisfies $k_1 < k_2$ (u^{-1} is increasing) and therefore the maximum valuation under strategy 1 is attained at:

$$v_1^* = \frac{1}{2}\left[\left(bu^{-1}(b) + (1-b)\,u^{-1}(1-b)\right)\right] + \frac{1}{2}\left(\int_{u^{-1}(b)}^{1} u(\ell)d\ell + \int_{u^{-1}(1-b)}^{1} u(\ell)d\ell\right)$$

For strategy 3, the maximum is $v_3^* = \int_{u^{-1}(\frac{1}{2})}^{1} u(\ell)d\ell + \frac{1}{2}u^{-1}(\frac{1}{2}))$.

Next we show $v_1^* > v_3^*$. Consider the function:

$$f(x) = \frac{1}{2}\left(\int_{u^{-1}(x)}^{1} u(\ell)d\ell + \int_{u^{-1}(1-x)}^{1} u(\ell)d\ell + xu^{-1}(x) + (1-x)u^{-1}(1-x)\right), \tag{A.2}$$

The first order condition gives:

$$\begin{aligned} -x\left(u^{-1}\right)'(x) - (1-x)\left(u^{-1}\right)'(1-x) + u^{-1}(x) + \\ x\left(u^{-1}\right)'(x) + (1-x)\left(u^{-1}\right)'(1-x) - u^{-1}(1-x) = 0 \end{aligned} \tag{A.3}$$

Simplifying Eq. A.3 gives

$$\frac{1}{2}(u^{-1}(x) - u^{-1}(1-x)) = 0 \tag{A.4}$$

This implies that $u^{-1}(x) = u^{-1}(1-x)$ gives a stationary point. Since u^{-1} increases monotonically, it holds only for $x = 1 - x = \frac{1}{2}$. It is a minimum, since $f'(x) < 0$ for $x < \frac{1}{2}$ and $f'(x) > 0$ for $x > \frac{1}{2}$. Therefore, the minimum is attained at $x = \frac{1}{2}$.

Hence, $v_1^* = f(b) > f(\frac{1}{2}) = v_3^*$ as $b < \frac{1}{2}$.

For strategy 2, $(k_1^*, k_2^*) = (u^{-1}(a), u^{-1}(1-a))$ does not satisfy $k_1 > k_2$ since $a < \frac{1}{2}$ and u^{-1} is increasing. Below we show that v_2 cannot exceed v_3^*:

$$\begin{array}{rcl} v_2 & = & \frac{1}{2}\left[\left(ak_1+\left(1-a\right)k_2\right)\right]+\frac{1}{2}\left(\int_{k_1}^1 u(\ell)d\ell+\int_{k_2}^1 u(\ell)d\ell\right) \\ & < & \frac{1}{4}\left(k_1+k_2\right)+\frac{1}{2}\left(\int_{k_1}^1 u(\ell)d\ell+\int_{k_2}^1 u(\ell)d\ell\right) \\ & \leq & \int_{u^{-1}(\frac{1}{2})}^1 u(\ell)d\ell+\frac{1}{2}u^{-1}(\frac{1}{2})) \end{array}$$

The first inequality is obtained by replacing a by $\frac{1}{2}$ and the second by maximizing the function with respect to k_1 and k_2. The maximum is reached at $\left(u^{-1}\left(\frac{1}{2}\right),u^{-1}\left(\frac{1}{2}\right)\right)$ and it is v_3^*.

Therefore $v_1^* > v_3^* > v_2^*$, implying that reporting $(k_1^*, k_2^*) = (u^{-1}(b), u^{-1}(1-b))$ is the optimal strategy.

Prior belief 2: $\frac{1}{2} < a < b$

Since $a > \frac{1}{2}$, we have $a > 1-a$. Hence, $(k_1^*, k_2^*) = (u^{-1}(a), u^{-1}(1-a))$ satisfies $k_1 > k_2$ (u^{-1} is increasing) and therefore the maximum valuation under strategy 2 is attained at:

$$v_2^* = \frac{1}{2}\left[\left(au^{-1}(a)+\left(1-a\right)u^{-1}(1-a)\right)\right]+\frac{1}{2}\left(\int_{u^{-1}(a)}^1 u(\ell)d\ell+\int_{u^{-1}(1-a)}^1 u(\ell)d\ell\right)$$

For strategy 3, the maximum is again $v_3^* = \int_{u^{-1}(\frac{1}{2})}^1 u(\ell)d\ell+\frac{1}{2}u^{-1}(\frac{1}{2}))$.

Next we show $v_2^* > v_3^*$. Consider the function f as defined above and its first order condition Eq. A.3. With the same arguments as above, we obtain $v_2^* = f(a) > f(\frac{1}{2}) = v_3^*$ as $a > \frac{1}{2}$.

For strategy 3, $(k_1^*, k_2^*) = (u^{-1}(b), u^{-1}(1-b))$ does not satisfy $k_1 < k_2$ since $b > \frac{1}{2}$ and u^{-1} is increasing. Below we show that v_1 cannot exceed v_3^*:

$$\begin{array}{rcl} v_1 & = & \frac{1}{2}\left[\left(bk_1+\left(1-b\right)k_2\right)\right]+\frac{1}{2}\left(\int_{k_1}^1 u(\ell)d\ell+\int_{k_2}^1 u(\ell)d\ell\right) \\ & < & \frac{1}{4}\left(k_1+k_2\right)+\frac{1}{2}\left(\int_{k_1}^1 u(\ell)d\ell+\int_{k_2}^1 u(\ell)d\ell\right) \\ & \leq & \int_{u^{-1}(\frac{1}{2})}^1 u(\ell)d\ell+\frac{1}{2}u^{-1}(\frac{1}{2})) \end{array}$$

The first inequality is obtained by replacing b by $\frac{1}{2}$ and the second by maximizing the function with respect to k_1 and k_2. The maximum is reached at $\left(u^{-1}\left(\frac{1}{2}\right),u^{-1}\left(\frac{1}{2}\right)\right)$ and it is v_3^*.

Therefore $v_2^* > v_3^* > v_1^*$, implying that reporting $(k_1^*, k_2^*) = (u^{-1}(a), u^{-1}(1-a))$ is the optimal strategy.

Prior belief 3: $a \leq \frac{1}{2} \leq b$

In this case, $v_1^* < v_3^*$ and $v_2^* < v_3^*$. This can be proved as above noticing that the maximum v_3^* of $\frac{1}{4}\left(k_1+k_2\right)+\frac{1}{2}\left(\int_{k_1}^1 u(\ell)d\ell+\int_{k_2}^1 u(\ell)d\ell\right)$ is only reached if $k_1 = k_2 = u^{-1}\left(\frac{1}{2}\right)$ and therefore, not if $k_1 < k_2$ or $k_1 > k_2$. This implies that reporting $(k_1^*, k_2^*) = \left(u^{-1}\left(\frac{1}{2}\right),u^{-1}\left(\frac{1}{2}\right)\right)$ is the optimal strategy. □

Appendix B. PERIS for RDU with ROCL may not be Incentive Compatible : An Example

To give an example where PERIS is not incentive compatible, we consider a subject whose preference for risk is represented by *rank-dependent utility (RDU,* Quiggin, 1982), which also corresponds to *cumulative prospect theory* (Tversky and Kahneman, 1992) restricted to gains.

Let F denote a cumulative distribution function over the outcome set $[0,1]$. The expected utility of F is $\int_0^1 u(x)dF(x)$ which is equivalent to $\int_0^1 (1-F(x))du(x)$. Its RDU value is $\int_0^1 w(1-F(x))du(x)$, where $w(\cdot)$ is a weighting function that is increasing and satisfies $w(0)=0$ and $w(1)=1$. RDU generalizes expected utility by allowing not only outcomes but also probabilities that are subjectively transformed by the subject. The RDU value of a binary lottery 1_p0 is $w(p)$. If an incentive scheme is incentive compatible, the subject should report, under the assumption of RDU: $p_i^* = w^{-1}(v(f_i))$ in PERIS for all $i \in \{1,\cdots,m\}$.

Consider the experiment $J\left((f_1,f_2),(p_1,p_2),(\tilde{i},\tilde{q})\right)$ induced by reporting (p_1,p_2) in PERIS. Assume RDU and ROCL under risk. We show that optimal reports (p_1^*,p_2^*) may deviate from $(w^{-1}(v(f_1)),w^{-1}(v(f_2)))$. The reduction of J can be written as $(f_1,r_1;1,r_2;0,r_3;f_2,r_4)$, where $r_1=\frac{p_1}{2}$, $r_2=\frac{1-p_1^2}{4}+\frac{1-p_2^2}{4}$, $r_3=\frac{(1-p_1)^2}{4}+\frac{(1-p_2)^2}{4}$, and $r_4=\frac{p_2}{2}$.

To determine the RDU value of J, we need to know how the subject ranks the four acts f_1, 1, 0, and f_2. The constant acts 1 and 0 are obviously ranked highest and lowest respectively. The ranks of the other two acts depend on their respective utility. Let's take for instance $v(f_1)=v(f_2)$. The subject's valuation of the experiment is:

$$v(J) = w(r_2) + (w(r_1+r_2+r_4) - w(r_2))\, v(f_1) \tag{B.1}$$

The first order condition for p_1 implies:

$$w'(r_2)\frac{\partial r_2}{\partial p_1} + \left(w'(r_1+r_2+r_4)\frac{\partial(r_1+r_2+r_4)}{\partial p_1} - w'(r_2)\frac{\partial r_2}{\partial p_1}\right) v(f_1) = 0. \tag{B.2}$$

Note that $\frac{\partial r_2}{\partial p_1}=\frac{-p_1}{2}$, $\frac{\partial r_1}{\partial p_1}=\frac{1}{2}$, and $\frac{\partial r_4}{\partial p_1}=0$ by definition. The first order condition then simplified to:

$$\frac{-p_1}{2}w'(r_2) + \left(\frac{1-p_1}{2}w'(r_1+r_2+r_4) - \frac{1-p_1}{2}w'(r_2)\right) v(f_1) = 0. \tag{B.3}$$

Assume $w(r)=r^2$. The first order condition becomes

$$-p_1 r_2 + (1-p_1)(r_1+r_4)\, v(f_1) = 0 \tag{B.4}$$

Suppose PERIS is IC and therefore $(p_i^*)^2 = v(f_i)$ should satisfy the first order condition. Using $(p_i^*)^2 = v(f_i)$ and also $r_1 = \frac{p_1}{2}$, $r_2 = \frac{1-p_1^2}{4} + \frac{1-p_2^2}{4}$, and $r_4 = \frac{p_2}{2}$, the first order condition is now:

$$3p_1^3 - 2p_1^4 - p_1 = 0 \tag{B.5}$$

We show that there exists $v(f_i)$ such that the first order condition does not hold for p_i^* satisfying the IC condition $(p_i^*)^2 = v(f_i)$. Assume for instance $v(f_i) = 0.01$ and therefore p_i^* should be 0.1 for PERIS to be IC. However, Eq. B.5 does not hold in this case. This proves by contradiction that PERIS is not IC.

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