Motivation: Contract design in theory and in practice

- Two paradigms of contract design in economic theory
  - Moral hazard
  - Mechanism design

- Some criticisms of the paradigms
  - Optimal contracts are too complicated.
  - Optimal contracts are not robust.

- Incomplete contracts literature provides an alternative.
Incomplete contracts framework

- Setup of the model
  - Consider a two-period relationship between a buyer and a seller of some good.
  - In the first period, the buyer and the seller make investments $\beta$ and $\sigma$, with costs $h_b(\beta)$ and $h_s(\sigma)$, respectively; both $h_b$ and $h_s$ are increasing and convex.
  - Buyer’s investment $\beta$ stochastically determines the value $v$ for the seller’s good, with distribution functions satisfy $F_b(v|\beta) \leq F_b(v|\tilde{\beta})$ for all $v$ if $\beta > \tilde{\beta}$; seller’s investment $\sigma$ stochastically determines the cost $c$ of producing the good, with distribution functions satisfy $F_s(c|\beta) \geq F_s(c|\tilde{\beta})$ for all $c$ if $\sigma > \tilde{\sigma}$.
  - In the second period, $v$ and $c$ are realized and observed by buyer and seller, and if trade occurs, one unit is produced by seller and transferred to buyer.
  - Contracts are incomplete: $v$ and $c$ are not contractible even though ex post observable, due to unforeseen contingencies, verification or enforcement costs.
  - Buyer and seller are risk-neutral and do not discount.
• Implicit assumptions:
  – Investments $\beta$ and $\sigma$ are specific in the sense that they influence only the value and the cost of the good in the relationship, not the outside options.
  – There is no direct externality in the investments: the value of the good depends only on the buyer’s investment, while the cost depends only on the seller’s investment.

Full information and complete contract benchmarks

• Efficiency (first best) requires both ex ante efficient investments $\sigma^*$ and $\beta^*$, and ex post efficient trading decision of trade with probability 1 if and only if $v \geq c$.
  – Ex post (after investments are made, and $v$ and $c$ are realized) efficiency is achieved, for any given investment levels, when level (probability) of trade $\pi$ equals 1 if and only if $v \geq c$.
  – Ex ante (before the buyer and the seller make investments) efficient investments are $\beta^*$ and $\sigma^*$ that maximize $E_{v,c}[\max\{v - c, 0\}|\beta, \sigma] - h_b(\beta) - h_s(\sigma)$. 

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• The first best can be achieved if the buyer and the seller can write a contract on investment levels and realized value and cost.
  – Participation by both can be ensured through ex ante transfers.

• The first best is still achievable if the value and the cost are contractible.
  – Consider the following contract on realized value \( v \) and cost \( c \): trade is ex post efficient; and for each realization of \((v, c)\)—trade or no trade—the buyer pays seller \( p(v, c) = \int_{\tilde{v} \geq c} \tilde{v} dF_b(\tilde{v}|\beta^*) + \int_{\tilde{c} < v} \tilde{c} dF_s(\tilde{c}|\sigma^*) \), plus some constant.
  – First term in \( p(v, c) \) gives seller all the return to investment \( \sigma \) in reducing \( c \) to below a stochastic value \( v \) distributed according to \( F_b(v|\beta^*) \); second term gives buyer all return to \( \beta \) in increasing \( v \) above \( c \) distributed according to \( F_s(c|\sigma^*) \).
  – Buyer: \( \max_{\beta} \int_c \int_{v > c} v dF_b(v|\beta) dF_s(c|\sigma^*) - \int_v \int_{c < v} c dF_s(c|\sigma^*) dF_b(v|\beta) - h_b(\beta) \), implying optimal \( \beta \) equals \( \beta^* \).
  – Seller: \( \max_{\sigma} - \int_v \int_{c \leq v} c dF_s(c|\sigma) dF_b(v|\beta^*) + \int_c \int_{v > c} v dF_b(v|\beta^*) dF_s(c|\sigma) - h_s(\sigma) \), implying optimal \( \sigma \) equals \( \sigma^* \).
No-contract case: The hold-up problem

- Suppose no contracts are written in the first period.

- In the second period, since all relevant information is public, the two sides will always implement the ex post efficient trade decision of \( \pi = 1 \) if and only if \( v \geq c \).
  - Suppose equal sharing of the surplus.
  - Then, no trade if \( v \leq c \), and trade at price \( \frac{1}{2}(v+c) \) if \( v > c \)—so that buyer’s payoff (investment cost is sunk) is \( v - \frac{1}{2}(v+c) \), which is equal to his default payoff of 0 from no trade plus half of the surplus \( v - c \), while seller’s payoff is \( \frac{1}{2}(v+c) - c \), which is equal to his default payoff of 0 from no trade plus half of the surplus \( v - c \).

- Consider a Nash equilibrium \((\beta^{NC}, \sigma^{NC})\) in investments in first period.
  - Given \( \sigma^{NC} \), \( \beta^{NC} \) maximizes \( \frac{1}{2} \mathbb{E}_{\xi,v} \left[ \max \{v - c, 0\} \mid \beta, \sigma^{NC} \right] - h_b(\beta) \).
  - Given \( \beta^{NC} \), \( \sigma^{NC} \) maximizes \( \frac{1}{2} \mathbb{E}_{\xi,v} \left[ \max \{v - c, 0\} \mid \beta^{NC}, \sigma \right] - h_s(\sigma) \).
• Compare above Nash equilibrium conditions with those for first best investments.
  
  – Under standard assumptions on distribution functions and on cost functions, $\beta$ and $\sigma$ are strategic complements (best response functions are upward sloping), implying that $\beta^{NC} < \beta^*$ and $\sigma^{NC} < \sigma^*$ (for example, when the distribution functions $F_b(v|\beta)$ and $F_s(c|\sigma)$ have full support of $[0,1]$ for all $\beta$ and $\sigma$, we have $E_{v,c}[\max\{v-c,0\}|\beta,\sigma] = \int_{0}^{1} (1 - F_b(c|\beta))F_s(c|\sigma)dc$.)
  
  – Holdup problem: an increase in investment $\beta$ or $\sigma$ yields only half of gain in surplus that is generated, compared to all the surplus generated as in efficient investment (or under complete contract).
  
  – Equilibrium investments are inefficiently low with efficient trading ex post.

• Who is holding up whom?
  
  – Consider a special case where only $\beta$ matters: $c(\omega;\sigma) = c(\omega)$, so that $\sigma^* = 0$.
  
  – In any Nash equilibrium with no contract, $\sigma^{NC} = \sigma^* = 0$, and $\beta^{NC} < \beta^*$.
  
  – Seller is holding up buyer.
Ownership and control rights

- Incomplete contracts
  - Investments $\beta$ and $\sigma$ are unobservable (and thus not contractible).
  - State $\omega$, the value $v$ and the cost $c$ are observable in the second period, but not verifiable and hence not contractible in the first period: justified by unforeseen contingencies, or undescrivable characteristics.
  - No contract is an option, but incomplete contract in the form of ownership or control of ex post trading decisions may do better.
  - Why? control rights change the default trading decision and hence the ex post bargaining outcomes for the two parties; as a result, investment incentives of the two parties are affected, and in some situations improved relative to no contract.
  - In practice, contracting on control rights is possible even though contracting on investments or trading decisions is not.
• Suppose that the buyer controls the ex post trading decision.
  
  – Then, default decision is trade, and default payoffs for the buyer and for the seller become $v$ and $-c$ (buyer can impose production on seller).
  
  – Since all information is public in the second period, the two parties will reach the ex post efficient trading decision.
  
  – Assuming equal share of surplus from renegotiating from the default decision of trade.
  
  – If $v \geq c$: default decision is efficient so there is no renegotiation, with payoff is $v$ for the buyer and $-c$ for the seller.
  
  – If $v < c$: default decision is inefficient so it is renegotiated to no trade, with payoff $\frac{1}{2}(v + c)$ for the buyer and $-\frac{1}{2}(v + c)$—total surplus from renegotiating the trading decision is $c - v$, half of which goes to buyer so that his payoff is default payoff of $v$ plus half the surplus, and half of which goes to seller so that his payoff is default payoff of $-c$ plus half of the surplus.
• Consider a Nash equilibrium \((\beta^{BC}, \sigma^{BC})\).

  – Anticipating possible renegotiation and given \(\sigma^{BC}\), buyer’s investment \(\beta^{BC}\) maximizes \(E_{v,c} \left[ \frac{1}{2}(v + c) + \frac{1}{2} \max \{v - c, 0\} \mid \beta, \sigma^{BC} \right] - h_b(\beta)\).

  – Anticipating possible renegotiation and given \(\beta^{BC}\), for seller, \(\sigma^{BC}\) maximizes \(E_{v,c} \left[ -\frac{1}{2}(v + c) + \frac{1}{2} \max \{v - c, 0\} \mid \beta^{BC}, \sigma \right] - h_s(\sigma)\).

  – We can make assumptions to ensure \(\beta^{BC} \geq \beta^* \geq \beta^{NC}\) and \(\sigma^{BC} \leq \sigma^{NC} \leq \sigma^*\).

  – Incomplete contract with buyer control is likely to be better than no contract, if seller’s investment is relatively unimportant in terms of reducing the cost compared to buyer’s investment in generating the value.

• Seller control?

  – Default decision is no trade, with default payoff equal to 0 for both buyer and seller.

  – In this environment, incomplete contracts in the form of seller control are same as no contract.
• Application: should client lists be owned by insurance agents or their companies? Answer depends on whether it’s car insurance or life insurance.
  – Control of the ex post trading decision may be interpreted as an ownership arrangement, ownership referring to who should own productive assets.
  – Implication: control rights or ownership enhance the asset owner’s incentives to make investments.

Specific performance contracts

• First best investments can be implemented through appropriate specifications of the default outcome and the bargaining power in the ex post renegotiation, when default trade level and price are contractible.
  – Let \( \hat{\pi} \in (0,1) \) satisfy \( \hat{\pi}(d/d\beta)E_v[v|\beta^*] = (d/d\beta)E_{v,c}[\max\{v-c,0\}|\beta^*,\sigma^*]. \)
  – Let the default outcome in the ex ante contract be trading with probability \( \hat{\pi} \) with a fixed payment of \( \hat{p} \) from the buyer to the seller.
  – Ex ante contract also assigns all bargaining power to seller in any renegotiation.
• Achieving the first best
  - Buyer has no bargaining power: for any realized $v$ and $c$, buyer’s payoff is given by $\hat{\pi}v - \hat{p} - h_b(\beta)$ if the investment is $\beta$.
  - By construction, the buyer makes the first best investment $\beta^*$, if anticipating $\sigma^*$ from seller.
  - Since the seller has all bargaining power in renegotiation, if the seller makes investment $\sigma$, for any realized $v$ and $c$, seller will renegotiate to ex post efficient trading decision of probability of trading equal to 1 if $v \geq c$ and 0 otherwise, and payoff $\max\{v - c, 0\} - (\hat{\pi}v - \hat{p}) - h_s(\sigma)$.
  - Seller becomes the residual claimant of the relationship, and makes the first best investment $\sigma^*$ given that buyer makes first best investment $\beta^*$.

• Such contracts are called “specific performance contracts.”
  - If only $\sigma$ matters, with $v$ constant and $\beta^* = 0$, first best can be achieved if seller makes take-it-or-leave-it offer ex post, with default $\hat{\pi} = 0$. 


Nash bargaining solution

- A bargaining problem between two parties, 1 and 2.
  - $X$ is compact set of agreements, with $d \in X$ the disagreement point.
  - For each $i = 1, 2$, party $i$'s preferences for lotteries over $X$ are represented by expected value of von-Neumann Morgenstern utility function $u_i(x)$.
  - $X$ is non-trivial: $u_i(x) \geq u_i(d)$ for any $x \in X$ and each $i = 1, 2$, with both holding strictly for some $x$.
  - $X$ is non-redundant: if $u_i(\tilde{x}) = u_i(x)$ for each $i = 1, 2$, then $x = \tilde{x}$.
  - $X$ is convex: for each $i = 1, 2$, for any $x, y \in X$ and any $l \in [0, 1]$, there exists $z$ such that $u_i(z) = lu_i(x) + (1 - l)u_i(y)$.

- Example: buyer with value $v$ and seller with cost $c$ bargain over surplus $v - c > 0$, where $X = \{\pi, p | 0 \leq \pi \leq 1, \pi c \leq p \leq \pi v\}$ with $d = (0, 0)$; and $u_1(\pi, p) = \pi v - p$ and $u_2(\pi, p) = p - \pi c$ for all $(\pi, p) \in X$. 
• Nash bargaining solution is \( x^* = \arg \max_{x \in X} (u_1(x) - u_1(d))(u_2(x) - u_2(d)) \).

  - Nash bargaining solution uniquely satisfies property that every objection by one party to the agreement can be counter-objected by the other party, that is, if \( lu_i(x) + (1 - l)u_i(d) > u_i(x^*) \) for some \( i = 1, 2 \), some \( x \in X \) and some \( l \in [0, 1] \), then \( lu_j(x^*) + (1 - l)u_j(d) \geq u_j(x) \) for \( j \neq i \).

  - Nash solution satisfies the property: \( u_1(x^*) > u_1(d) \) and \( u_2(x^*) > u_2(d) \); if \( lu_i(x) + (1 - l)u_i(d) > u_i(x^*) \), then \( l(u_i(x) - u_i(d)) > u_i(x^*) - u_i(d) \), and so \( l(u_i(x) - u_i(d))(u_j(x^*) - u_j(d)) > (u_i(x) - u_i(d))(u_j(x) - u_j(d)) \), implying \( lu_j(x^*) + (1 - l)u_j(d) > u_j(x) \).

  - Property implies Nash solution: take any \( x \in X \) such that \( u_i(x) > u_i(d) \) for each \( i = 1, 2 \); then, whenever \( lu_i(x) + (1 - l)u_i(d) > u_i(x^*) \) for some \( i = 1, 2 \), or whenever \( l > (u_i(x^*) - u_i(d))/(u_i(x) - u_i(d)) \), by the property we have \( l(u_i(x^*) - u_i(d)) \geq u_i(x) - u_i(d) \), or \( l \geq (u_j(x) - u_j(d))/(u_j(x^*) - u_j(d)) \); thus \( (u_i(x^*) - u_i(d))/(u_i(x) - u_i(d)) \geq (u_j(x) - u_j(d))/(u_j(x^*) - u_j(d)) \), implying \( x^* \) is Nash solution.
• For any $\alpha \in (0, 1)$, define generalized Nash bargaining solution as $x_*$ that solves the problem $\max_{x \in X} (u_1(x) - u_1(d))^\alpha (u_2(x) - u_2(d))^{1-\alpha}$.

  - $\alpha$ models party 1’s relative bargaining power.
  - The generalized solution coincides with the Nash solution when $\alpha = \frac{1}{2}$.

• In the buyer-seller example, the asymmetric Nash bargaining solutions is given by $\pi_* = 1$, and $p_* = (1 - \alpha)v + \alpha c$.

  - First order condition with respect to $p$ implies the objective is increasing in $\pi$ so $\pi_* = 1$, and $u_1(\pi_*, p_*) = \alpha(v - c)$ and $u_2(\pi_*, p_*) = (1 - \alpha)(v - c)$.

  - If $d = (\hat{\pi}, \hat{p})$ for some $\hat{\pi} \in [0, 1)$ and $\hat{p}$, with $u_1(d) = \hat{\pi}v - \hat{p}$ and $u_2(d) = \hat{p} - \hat{\pi}c$, then Nash solution still has $\pi_* = 1$, with $u_1(\pi_*, p_*) = u_1(d) + \alpha(1 - \hat{\pi})(v - c)$ and $u_2(\pi_*, p_*) = u_2(d) + (1 - \alpha)(1 - \hat{\pi})(v - c)$.

• Application: an increase in $\alpha$ reduces the seller’s incentive to make cost-cutting investments, but this is not necessarily true if investments also increase seller’s outside option.