Chapter 2: The Logic of Atomic Sentences
Arguments & Validity

- The main target of formal logic is to understand the notion of “Logical Consequence” as precisely as possible. (Actually 3 notions: logical truth, logical consequence, logical equivalence)
- In particular, it is to understand the governing rules of valid arguments
- An argument is a piece of reasoning consisting of a series of statements in which one (conclusion) is meant to follow from, or be supported by, the others (premises). Example:
  - All men are mortal (premise)
  - Socrates is a man (premise)
  - Socrates is mortal (conclusion)
- An argument is valid if and only if (IFF) it is impossible for the conclusion to be false while the premises are all true.

Soundness and Fitch format

- An argument is sound IFF
  - it is valid, and
  - all its premises are true.
- So and argument is unsound if either it is invalid, or at least one of its premises is false, or both.
- We will have a canonical way to present arguments by adopting a graphical device that we’ll call “Fitch format” after the logician Frederic Fitch (US, 1908-1987):

- It turns out that this is a valid argument, but arguably it is unsound because the first premise is false.
Validity and Soundness

- Can a valid argument have false premises and a false conclusion?
  - YES
  - (Exp: All cats are dogs; all dogs are fish; ∴ all cats are fish)
- False premises and a true conclusion?
  - YES
  - (Exp: Murat is a postman; all postmen are bitten by a dog; ∴ Murat is bitten by a dog)
- True premises and a false conclusion? NO.
- True premises and a true conclusion? YES.
- Can an invalid argument have true premises and true conclusion?
  - YES
  - Exp 1: Most Jeeps are 4 by 4; Murat owns a Jeep; ∴ Murat’s Jeep is 4 by 4.
  - Exp 2: All dogs are mammals; Fido is a mammal; ∴ Fido is a dog.
  - Exp 3: Venus is a planet; Los Angeles is in CA; ∴ Fido is a dog
- Can there be a valid argument without any premise? YES!
  - Exp: ∴ All bachelors are unmarried.

Logical Consequence, Validity, and PROOF

- How do we show that a statement, C, is a logical consequences of others, P1 ... Pn?
- In other words, how do we show that the following argument is valid?

<table>
<thead>
<tr>
<th>P1</th>
</tr>
</thead>
<tbody>
<tr>
<td>.</td>
</tr>
<tr>
<td>.</td>
</tr>
<tr>
<td>.</td>
</tr>
<tr>
<td>Pn</td>
</tr>
<tr>
<td>C</td>
</tr>
</tbody>
</table>

=> By producing a proof.
**Formal and Informal Proofs**

- A **proof** is a step-by-step demonstration that a conclusion $C$ follows from premises $P_1$ through $P_n$, by establishing a series of intermediate conclusions, each of which is an obvious consequence of the original premises and the intermediate conclusions previously established.

- There is an **informal** way of providing proofs, and a **formal** way. Both, when done right, are equally rigorous. The difference is of **style**. The former is more free-wheeling.
  - **Exp**: Show that the claim that *Socrates sometimes worry about dying* is a logical consequence of the following three claims: *All men are mortal; Socrates is a man; Every person who will eventually die sometimes worry about it.*
  - **Informal Proof**: Since Socrates is a man and all men are mortal, it follows that Socrates is a mortal. But all mortals will eventually die, since that is what it means to be mortal. SO Socrates will eventually die, but we are given that every person who will eventually die sometimes worry about dying. Hence Socrates sometimes worries about dying.
  - **Formal proof**:
    
    1. Cube(c)
    2. c=b
    3. Cube(b) =Elim:1,2

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**Proofs involving the Identity Predicate =**

- **The Indiscernibility of Identicals** (a.k.a., **substitution**, or **Elimination**):
  - If $a$ and $b$ are identical, then they share all their properties.
  
  *(Historical note: This is sometimes known as Leibniz Law since Leibniz first stated a stronger version, namely: Two things are identical IFF they share all their properties.)*

  **More formally**: For any two objects, $x$ and $y$, and for any property, $F$:
  
  IF $(x=y)$ THEN $(x$ is $F$ IFF $y$ is $F)$

- Other logical relations true of $=$
  - **Reflexivity**: For any object $x$, $x=x$ (a.k.a., $=Intro$)
  - **Symmetry**: For any two objects, $x$ and $y$, $x=y$ IFF $y=x$
  - **Transitivity**: For any three objects, $x$, $y$, and $z$, IF $x=y$ and $y=z$, THEN $x=z$.

- Actually both **Symmetry** and **Transitivity** follow from the first two principles, **Substitution** and **Reflexivity**.
  - **HOW?**
Other Predicates

- Some of these four types of logical relations true of \(=\) are true of many other predicates.
  
  Examples:
  
  Larger than, Less than
  Being the same size as, being in the same row as

- **Inverse relations:**
  
  Example
  
  For any two objects, \(x\) and \(y\),
  
  \(x\) is larger than \(y\) IFF \(y\) is smaller than \(x\);
  
  \(x\) is to the right of \(y\) IFF \(y\) is to the left of \(x\);
  
  \(x\) is in front of \(y\) IFF \(y\) is in the back of \(x\), etc.

- But we’ll keep only the two rules about \(=\), namely:
  
  \(=\)Elim (Indiscernibility of Identicals, Substitution)
  
  \(=\)Intro (Reflexivity)

Deductive System \(\mathcal{F}\), and Formal Proofs

- You get a deductive system when, roughly, you add to a formal language a set of transformation rules used to construct proofs.

- The Fitch-Style deductive system \(\mathcal{F}\), not to be confused with the Fitch format of argument display, not to be confused with the software program called “Fitch”.

- We’ll adopt the Fitch format in \(\mathcal{F}\) (also: Fitch the program uses the Fitch format in constructing and checking proofs in \(\mathcal{F}\)). So the proofs will look like this:

\[
\begin{array}{c}
p \\
q \\
r \\
\vdots \\
S_1 \\
\vdots \\
S_n \\
S \hspace{1cm} \text{Justification n+1} \\
\end{array}
\]
Three formal rules of $\mathcal{F}$

- For any term, $n$ and $m$, and for any sentence $P$:
  
  **Identity Introduction ($\text{Intro}$):**
  
  $\vdash n = n$

  (Read $'P(n)'$ as any sentence in which one or more instances of $'n'$ occur)

  **Identity Elimination ($\text{Elim}$):**
  
  $\vdash P(n)$

  $\vdash n = m$

  $\vdash P(m)$

  **Note what this says:** you may substitute $m$ for $n$, not the other way around!

  Finally we have redundancy rule:

  **Reiteration ($\text{Reit}$):**

  $\vdash P$

  $\vdash P$

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Constructing proofs in $\mathcal{F}$ and in Fitch

- Let’s do the following proof (open up Fitch)

  1. $\text{SameRow}(a, a)$
  2. $b = a$

  $\vdash \text{SameRow}(b, a)$

- **Rules of $\mathcal{F}$ versus Con mechanisms (exp: Ana Con)**
  - Taut Con
  - FO Con
  - Ana Con
Demonstrating nonconsequence

• Proofs of consequence
• Proofs of nonconsequence
• Counterexamples:

To show that a sentence Q is not a consequence of premises P₁,...,Pₙ, we must show that the argument with premises P₁,...,Pₙ and conclusion Q is invalid. This requires us to demonstrate that it is possible for P₁,...,Pₙ to be true while Q is simultaneously false. That is, we must show that there is a possible situation or circumstance in which the premises are all true while the conclusion is false. Such a circumstance is said to be a counterexample to the argument.

• For our Blocks Language, a formal proof of nonconsequence that Q is not a consequence of P₁...Pₙ consists of a sentence file with P₁...Pₙ labeled as premises, Q labeled as conclusion, and a world file that makes each of P₁...Pₙ true and Q false.
• The world depicted in the world file will be called the counterexample to the argument in the sentence file.