A POSITIVE THEORY OF INFLATION AND INFLATION VARIANCE

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Empirically, inflation and the variance of inflation are positively associated. This paper develops a model that provides a potential explanation for this relationship in terms of the incentives facing the policymaker in a “discretionary equilibrium.” The model can also account for an empirical association between inflation and measures of real output instability. There is, however, no direct causal link whatever from the average rate of inflation to either the variance of inflation or that of real output.

I. INTRODUCTION

The debate over the use rules or discretion in monetary policy has been central to macroeconomics for many years (e.g., Simons [1936]; Lucas [1980]; Buitert [1980]). Recently Kydland and Prescott [1977] and Barro and Gordon [1983a,b] have identified “discretion” as the absence of policy commitment in a game between policymakers and the public. They argued that discretionary policymaking will lead governments to create excessive inflation. A policy of low inflation is not consistent with incentives facing governments, and thus will not be believed by the private sector.

Barro and Gordon [1983a] argue that this positive theory of monetary policy can help to explain many features of the trend rate of inflation in modern economies: high and persistent rates of inflation, a positive relationship between inflation and unemployment, and the observed countercyclical behaviour of monetary authorities, among others.

These models are based on the premise that there are significant costs to a high but fully anticipated rate of inflation. Another feature of inflation in modern economies, however, is the well-documented fact that inflation and the variability of inflation are positively associated. This phenomenon has been widely observed over different countries at different times.¹ This has led researchers to question the feasibility of a steady and predictable positive rate of inflation. The finding suggests that high rates of inflation may reduce the ability to forecast future inflation rates. A high average inflation rate may add an unnecessary degree of uncertainty to individual decision making and lead to a misallocation of resources. Friedman [1977] suggests that this may

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cause output instability and possibly raise the average unemployment rate. A related paper by Logue and Sweeney [1981] establishes that there is a positive relationship between inflation and the variability of economic growth for industrial countries. Given this perspective, the welfare costs associated with inflation might be considerably higher than the traditional costs of anticipated inflation. This paper extends the Barro and Gordon [1983a] model of discretionary monetary policy to take account of the relationship between inflation and measures of inflation and output variability. The extension focussed on is to model an endogenous wage-indexing scheme in the labor market. In the discretionary equilibrium, wage setters not only form rational expectations of the future price level, but also choose an optimal degree of wage indexation. This extension to the basic model has the following properties of a discretionary equilibrium.

1. There is a positive association between the mean rate of inflation and the magnitude of real disturbances in the economy, as well as between the mean inflation rate and the degree of output instability in the economy.

2. In an economy where real disturbances are relatively important, there is a positive association between the mean rate of inflation and the variance of inflation.

The explanation behind these results is as follows. The lower is the degree of wage indexation, the greater is the incentive for monetary authorities to cause surprise inflation, and hence the higher is the mean rate of inflation in a discretionary equilibrium. But the degree of wage indexation is negatively related to the variance of real disturbances in an economy which is subject to both real and nominal disturbances. Thus, the mean inflation rate is positively related to the variance of real disturbances. In turn the variance of real disturbances determines the degree of output and inflation variability for the economy.

Thus, a “positive” theory of inflation can potentially account for the stylized facts concerning the relationship between inflation and inflation/output variability. The paper implies that this relationship is the result of the endogenous determination of monetary policy in a discretionary equilibrium as described by Barro and Gordon [1983a].

One of the major implications, however, lies in the causal aspect of this relationship. If high inflation causes high inflation variance at the level of the individual agent, this adds significantly to the welfare costs of inflation. Consequently, the argument for rules rather than discretion in the conduct of monetary policy would have much more force. In the analysis below, there is no causal link whatever from the rate of inflation to inflation variance. The average rate of inflation could be reduced to zero by the imposition of binding rules. However this would not affect either inflation variance or output variance. The benefits of a commitment to rules in this model lie only in its ability to reduce the average rate, and not the variability, of inflation.
II. THE MODEL

The supply relationship used is similar to that of Barro and Gordon [1983a], in that unanticipated movements in prices can affect output and employment. The present model is based on the assumption that nominal wage rates are set by contract and adjusted ex post for unanticipated movements in prices. Employment is then determined by labor demand.\(^2\) The appendix demonstrates that this leads to a supply relationship, where all variables are in logs:

\[
y_t^i = \alpha(P_t - P_t^e) + \delta u_t + \bar{y},
\]

(1)

where \(y\) is output, \(P_t\) the price, \(P_t^e\) the expected price, \(u_t\) is an identically and independently distributed (i.i.d.) productivity disturbance with mean zero, and \(\bar{y}\) is a constant. Also \(\alpha = \sigma(1-b)\), where \(\sigma\) is constant and \(b\) is an indexation parameter such that \(0 < b < 1\). For future reference, the output which corresponds to full market clearing with flexible wage rates is written as

\[
\bar{y}_t = \gamma u_t + \bar{y},
\]

(2)

where \(\gamma < \delta\) (as shown in the appendix).

The rest of the model is written as

\[
M_t - P_t = y_t
\]

(3)

\[
M_t = \tilde{M}_t + \omega_t
\]

(4)

Equation (3) is a standard money market equilibrium. For simplicity, an interest rate term is omitted in (3), as the results of the paper are completely unaltered with this factor included.\(^3\) Equation (4) describes the money supply process. \(M_t\) is the total nominal money stock at time \(t\). This is comprised of two elements. The term \(\omega_t\) is an exogenous i.i.d. error in the monetary control process, and is outside the direct control of the monetary authorities. \(\tilde{M}_t\), however, is the money stock currently chosen by the authorities.\(^4\) The difference between \(\tilde{M}_t\) and the final value of the money stock, \(M_t\), is attributable to the \(\omega_t\) disturbance. The value of \(\tilde{M}_t\) is determined below as the outcome of the authorities' optimization problem.

\(^2\) See Gray [1976] and Fischer [1977].

\(^3\) This makes little difference to the results. Cukierman [1980] shows that the effect of wage indexation on the response of the economy to various shocks is not sensitive to the interest rate elasticity of money demand.

\(^4\) The decision process could be reformulated to allow the authorities to choose growth rates of money without changing the results.
In order to proceed, a hypothesis is needed concerning the behaviour of the monetary authorities, as well as the formation of expectations and the determination of indexation by the private sector. Following Barro and Gordon [1983a], it is assumed that the authorities attempt to minimize the social loss function:

\[ LA = E_{t-1}[(y_t - y^*)^2 + \beta(\pi_t)^2], \]  

(5)

where \( \pi_t \) is the current inflation rate, approximated by \( P_t - P_{t-1} \), and \( \beta \) is a constant. At time \( t \), \( P_{t-1} \) is predetermined from the point of view of the monetary authorities. Furthermore, we assume that \( y^*_t = \tilde{y}_t + c\tilde{y} \) where \( c > 0 \). Thus the desired mean output level of the authorities is higher than that of the private sector. This can arise because of taxes and other distortions in the labor market which bias employment downwards, or alternatively, as a result of powerful trade unions forcing up the real wage. The policymaker thus has a natural bias towards inflating the economy.

As discussed in the appendix, wage setters set the contract wage such that the level of employment (and hence output) will be as close as possible to the desired level implicit in equation (2). Thus we approximate the wage setters' loss as

\[ L = E_{t-1}(y_t - \tilde{y}_t)^2. \]  

(6)

Wage setters attempt to minimize this loss by setting nominal wages equal to expected inflation and by indexing wages.\(^6\)

The structure of the game played by monetary authorities and wage setters is as follows. (For a similar setup, see Canzoneri [1985]). At the beginning of time \( t \), the authorities choose \( \tilde{M}_t \) to minimize (5), taking price expectations as given. The expected current price level equals the base set nominal wage and is predetermined from the point of view of the monetary authority. We assume that the authorities must determine \( \tilde{M}_t \) before the values of current disturbances are known. The important element in this decision setup is that expectations are not currently chosen by the authorities, but are determined independently by wage setters.

Base contract wages are set at time \( t-1 \), and are based on the expected value of period \( t \)'s price level. When setting wages, the private sector takes into account not only the distribution of exogenous disturbances, but also the behavior of the monetary authorities. In addition, we assume that the private sector chooses an optimal indexation parameter \( b \), to minimize (6),

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5. Equation (5) indicates that the analysis is restricted to the equilibrium of a one-shot game, as in Barro and Gordon [1983a].

6. Although governments and wage setters may both agree on the correct social welfare function, because they act as "small agents," wage setters ignore both the effect of their actions on inflation and the distortionary taxes which impinge on labor market equilibrium.
allowing wages to adjust to ex post price movements. This will depend upon the stochastic structure of the model. A Nash equilibrium corresponds to values of $\tilde{M}_t$, $P_t$, $P^e_t$, and $b$ which solve both the problem of the authorities and the private sector, and satisfy market clearing. We might think of wage setters as choosing the indexation rule at more infrequent intervals than expectations are formed. Optimal indexation will depend only on the distribution of stochastic disturbances and the parameters of the economy, and in particular will be independent of the money growth rate chosen by the monetary authorities. This is because the authorities do not use an ex post monetary feedback stabilization rule. There is thus no strategic interaction between the choice of $b$ and the monetary authorities’ decisions.

The rationale for allowing wage setters to index, but ruling out monetary stabilization rules, is similar (in reverse) to the argument of Rogoff [1985]. Once an optimal wage indexation rule is in place, a monetary feedback rule on the price level will have no effect at all on output. Thus the authorities have little incentive to use a feedback rule.\footnote{This discussion presumes that the authorities would choose a monetary feedback rule taking into account the reaction of wage setters, via indexation, to the rule. An alternative scenario might be considered where monetary authorities chose a feedback rule in a “discretionary” manner, i.e., ignoring the response of wage indexation to the chosen rule. The authorities and wage setters would then play a Nash game in feedback rules with the strategies being the monetary rule and wage indexation parameter, respectively. It turns out, however, that in this model the equilibrium monetary feedback rule would be the same in either case. This is because the distortion which introduces a divergence between the objective functions of the authorities and wage setters concerns only the average level of output and not the variance of output. Feedback rules affect only the variance of output; and since both parties agree on the desirable variance, there will be no difference between an equilibrium with commitment and a discretionary equilibrium in the game over feedback rules.}

Following Barro and Gordon [1983a], when the authorities choose $\tilde{M}_t$, taking expectations as given (predetermined), this is denoted as a “discretionary equilibrium.” This equilibrium is determined as follows. From equations (1) to (4), given any $\tilde{M}_t$, $P^e_t$, and $b$, the current price level is written as

$$P_t = (\alpha P^e_t + \tilde{M}_t + \omega_t - \delta u_t)/(1 + \alpha)$$  \hspace{1cm} (7)

Substituting for (7) in (5), the optimal $\tilde{M}_t$ is given by the (first-order condition) equation,

$$(\alpha^2 + \beta)\tilde{M}_t - \alpha(\alpha - \beta)P^e_t - (\alpha^2 + \beta)\bar{y} - c\bar{y}(1 + \alpha) - \beta P_{t-1}(1 + \alpha) = 0.$$  \hspace{1cm} (8)

Price expectations are determined not just by the market-clearing condition, (7), but also by the solution to the authorities’ decision problem, i.e., (8). Wage setters at period $t-1$, understanding the policymakers’ decision, will take this decision into account in the formation of (rational) expectations.
Thus it can be determined that

\[ P_t^e = E_{t-1}P_t = \tilde{M}_t - \bar{y}. \]

Substituting this back into (8),

\[ \tilde{M}_t = \bar{y} + P_{t-1} + c\bar{y}(\alpha/\beta), \]

and thus

\[ \pi_t = c\bar{y}(\alpha/\beta) + [\omega_t-\delta u_t/(1+\alpha)]. \tag{9} \]

The mean inflation rate in the discretionary equilibrium will be positive, as long as the expression \( c\bar{y}(\alpha/\beta) > 0 \). Thus when monetary authorities act with discretion, taking expectations as predetermined, they create an inflationary bias in the economy. Despite this, the level of output and unemployment is completely unaltered by the monetary rule, as it may be established that

\[ P_t - E_{t-1}P_t = (\omega_t-\delta u_t)/(1+\alpha). \tag{10} \]

The expected rate of inflation has the same properties as in Barro and Gordon [1983a], being higher the greater is the distortion \( c \), and the lower is the authorities' subjective welfare weight on inflation itself. Note that if the authorities could commit themselves in advance to a monetary rule, then by taking expectations into account ex ante, they would choose a zero expected rate of inflation in equilibrium.\(^8\)

Equation (10) shows that the average rate of inflation in the discretionary equilibrium is negatively related to the degree of wage indexation. As wages become more fully indexed, the average rate of inflation is expected to fall. Fischer [1983] presents evidence that more highly indexed countries did not have significantly higher inflationary responses to the oil shocks of the 1970s, and, more important for the present model, that money growth responded to the shocks by less in more highly indexed countries.

However, wage indexation itself is likely to be endogenous. We now assume that the private sector chooses an optimal degree of indexation to minimize (6), given the stochastic structure of the economy. This gives the solution

\[ \sigma(1-b) = \delta(\delta-\gamma)/(V+\delta\gamma) \tag{11} \]

where \( V = (\sigma_{\omega}^2/\sigma_u^2) \). This has the same properties as the well-known indexation model of Gray [1976]. In particular, when there are only nominal disturbances, it is optimal to index the wage rate fully, i.e., set \( b = 1 \).

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\(^8\) As was pointed out in footnote 5, the analysis focuses on the one-shot game outcome. It is well known that alternative equilibria with a lower inflation rate can be supported if the authorities establish a “reputation” for not inflating. Barro and Gordon [1983b] develop such a model and show that the equilibrium inflation rate is between (9) and the full commitment rate. See also Backus and Driffill [1985], and Rogoff [1986].
Substituting (11) into (9) and taking expectations dated \( t-1 \), the complete solution for the mean inflation rate is

\[
E_{t-1}\pi_t = (c\gamma/\beta)e(\delta-\gamma)/(V+\delta\gamma).
\]

(12)

In the next section the properties of equilibrium are examined, in particular the properties of the equilibrium average inflation rate (12).

II. PROPERTIES OF THE EQUILIBRIUM INFLATION RATE

The main results of the paper are contained in equation (12). This indicates that the mean inflation rate in a discretionary equilibrium is positively related to the variance of the real shock, \( \sigma_u^2 \). As \( \sigma_u^2 \) tends to zero, \( E_{t-1}\pi_t \) tends to zero; i.e., the mean inflation rate tends towards the efficient rate. Thus, the inflationary cost of discretion is present only in an economy with random aggregate supply disturbances.

The explanation for this result is straightforward. A rise in \( \sigma_u^2 \) lowers the optimal degree of wage indexation. This flattens out the potential a priori Phillips curve facing the authorities, thereby raising their incentive to cause surprise inflation. In equilibrium, mean inflation will then be higher, despite the fact that the mean inflation rate does not surprise wage setters.

For similar reasons, the mean inflation rate is negatively related to the variance of nominal disturbances in the economy, \( \sigma_o^2 \), in a discretionary equilibrium.

What does the model predict regarding the correlation between mean inflation and the variance of inflation? As discussed above, it is a well-established empirical fact that these two are positively correlated, as documented in Okun [1970], Logue and Willett [1976], Taylor [1981], Fischer [1982], and Logue and Sweeney [1981]. Periods of high inflation rates are associated with a lower degree of predictability of inflation. This holds over a large number of countries.

From equation (9) the conditional variance of inflation may be written as

\[
\text{VAR} \ \pi = E_{t-1}(\pi_t - E_{t-1}\pi_t)^2 = \sigma_u^2(V+\delta\gamma)^2/(V+\delta^2).
\]

(13)

A rise in \( \sigma_u^2 \) will raise the mean rate of inflation. The effects on inflation variance are twofold. First, the direct effect, leading prices to fluctuate more, is positive. But the rise in \( \sigma_u^2 \) also reduces the optimal indexation rate. Following an unanticipated aggregate demand or supply shock, the price will respond by a smaller amount. This tends to \text{reduce} the variance of inflation. The total response is given by

\[
d\text{VAR} \ \pi/d\sigma_u^2 = (\delta(V+\delta\gamma)((V+\delta^2)\gamma - (\delta-\gamma)V)/(V+\delta^2)^2.
\]

(14)

This is positive for \( V+\delta^2 > [(\delta-\gamma)/\gamma]V \). Thus for a low value of \( V \), or a small (large) fraction of nominal (real) disturbances in the economy, a rise
in \( \sigma^2 \) will lead to an increase in both mean inflation and the variance of inflation.\(^9\)

At a minimum, the model seems to be capable of accounting for the observed association between average inflation and its variance.\(^10\) A further question, however, concerns the relationship between the mean rate of inflation and the variability of real output. Friedman [1977] argues that one of the main welfare costs of inflation concerns the detrimental effect of inflation variability on individual decision making under uncertainty. By increasing trading uncertainty, inflation variability may reduce the efficiency of the price system and cause a misallocation of resources. He suggests that this may actually raise the unemployment rate.\(^11\) Logue and Sweeney [1981] discuss reasons why inflation uncertainty may cause instability in production decisions and raise the variability of real output. They provide international evidence of a strong positive association between the mean inflation rate and the variability of real economic growth.

For the present model, the conditional variance of output around its market clearing level is given by

\[
E_{t-1}(y_t - \bar{y})^2 = \sigma_w^2(\delta - \gamma)^2(V/V + \delta^2).
\]

(15)

This is positively related to \( \sigma_w^2 \). Therefore a rise in the variance of the supply shock will simultaneously raise both the mean inflation rate and the variability of output. Furthermore, if (14) is positive this will also be accompanied by a rise in the variance of inflation.

The discretionary equilibrium of the present model therefore predicts a close relationship between the mean rate of inflation, inflation variability, and output variability. One of the most important features of this relationship, however, is the degree of causation involved. If high inflation causes

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9. Both Okun [1971] and Logue and Willett [1976] find a degree of nonlinearity in the relationship. Mean inflation and its variance seem to be strongly correlated for high inflation countries, but relatively weakly correlated for low-inflation countries. It may be established that the present model has a similar prediction. In response to a rise in \( \sigma_w^2 \), inflation variance rises by more (less) than inflation for an initial high (low) rate of inflation.

10. In a recent paper Cukierman and Meltzer [1986] provide an alternative explanation for the correlation between the average inflation rate and its variability based on the incentive structure facing policymakers. In their model the policymakers' preferences shift randomly and are not known with certainty by the private sector. A rise in the variance of the error in money stock control will increase the length of time it takes the private sector to recognize a more expansionary policy. If the policymaker has a positive discount rate, this leads him to raise the current rate of monetary growth. As a result, both average monetary growth and its variability rise simultaneously. Furthermore, countries with more political instability in preferences between inflation and unemployment will desire greater uncertainty in monetary control and thus will exhibit high and variable inflation. If (as in the present model) there is no asymmetric information, however, Cukierman and Meltzer's [1986] model reduces to that of Barro and Gordon [1983a]. Thus the association comes from a quite different channel than the present model. I am grateful to a referee for pointing out this reference.

11. Levi and Makin [1979] test the hypothesis that the inflation variability reduces employment. They find evidence to support this proposition.
high inflation variability and economic instability, then the costs of inflation are greater, and the benefits of reducing inflation greater, than if this is not the case.

Friedman [1977] clearly identifies the increased variability of inflation as one of the costs of running a high rate of inflation. He argues that political constraints lead governments that care about unemployment to reverse bouts of inflation as society becomes more "inflationary minded." Furthermore, inflation has redistributive effects which are socially divisive, and reduce the power of governments to control the instruments of monetary and fiscal policy. This volatility of policy creates a high and variable inflation rate which imposes welfare losses on society. A similar mechanism is suggested by Okun [1970]. Taylor [1981], on the other hand, suggests that for the 1970s the relationship is best explained by a combination of supply shocks and monetary accommodation of these shocks. These explanations (with the possible exception of Taylor) would seem to imply that a low variability of inflation could be achieved by placing direct controls on the monetary authority to enforce a lower average inflation rate.

The implications of the present model are in striking contrast to those of the previous paragraph. In this model there is no direct causal link whatever from inflation to inflation variability (or output instability). Using (1), (9) and (10), the social loss function may be written as

\[ LA = \sigma_n^2(\delta - \gamma)^2(V/V + \delta^2) + \sigma_n^2(V + \delta \gamma)^2/(V + \delta^2) + c^2\bar{\gamma}^2 + (E_{t-1}\pi_t)^2 \]  

(16)

where \( E_{t-1}\pi_t \) is given by (12).

This expression is explained as follows. The first term represents the variance of output around its market-clearing level \( \bar{y}_t \). The second term describes the variance of inflation: \( E_{t-1}(\pi_t - E_{t-1}\pi_t)^2 \). The third term represents the loss arising from the market-clearing level of output being too low relative to the desired level. This is the distortion which leads the monetary authority to overinflate. Finally, the fourth term describes the loss arising from the use of discretion, i.e., the nonzero mean inflation rate.

From (16) it is clear that while both inflation and output variability represent welfare costs, these costs are independent of the costs of discretion. With precommitment, the average inflation rate (the fourth term) would be zero, yet the variance of inflation and output (the first and second terms) would be unchanged. In this particular model, there is no sense at all in which a high average inflation rate causes economic instability. The correlation between inflation and its variance is purely a property of the discretionary equilibrium with supply disturbances and endogenous indexation. Therefore the placing of binding zero money growth rate rules on the monetary authorities would not in any way affect the variance of inflation (or output). Rather it would make the link between inflation and inflation variance disappear.
Indeed, in the present model, causality from inflation to inflation variance cannot be discussed at all because this presupposes that inflation is itself an exogenous variable. In fact, given a "positive" theory of policymaking, inflation is endogenous, determined interdependently with all other variables in the system. In order to establish causality, the fundamental exogenous real and nominal shocks hitting the economy must be separated out. The observed average inflation rate will be a function of these shocks.

IV. CONCLUSIONS

This paper suggests that a "positive" theory of monetary policy may be helpful in explaining some of the observed features of inflation in industrial economies. The observed relationship between inflation and measures of economic stability such as inflation variance and output variance are predicted by the model. However, output and inflation stability are not themselves classifiable as being "costs of inflation." A positive correlation between these and the average inflation rate does not imply a causal link, and suggests that attempts to place binding rules on monetary growth will have no effect on either of these. These results should not, however, be interpreted as suggesting in any way the desirability of activist policy. The "costs of discretion" as outlined by Barro and Gordon [1983a] are still present in the model.

APPENDIX

Following Gray [1976], a simple log-linearized Cobb-Douglas production function, labor demand curve, and labor supply curve are used:

\[ y_t = a l_t + u_t + \log A \]  \hspace{1cm} \text{(A1)}

\[ l^D_t = k + [(p_t-w_t) + u_t]/(1-a) \]  \hspace{1cm} \text{(A2)}

\[ l^i_t = k + c(w_t-p_t) \]  \hspace{1cm} \text{(A3)}

where \( y_t = \log \) of output  \hspace{1cm} \( p_t = \log \) of price level  \\
\( u_t = \text{i.i.d. } (0, \sigma^2_u) \)  \hspace{1cm} \( l_t = \log \) of employment  \\
\( w_t = \log \) of nominal wage  

When the labour market clears \( l^*_t = l^D_t \), and the equilibrium log real wage is \( (w_t-p_t) = u_t/[1+c(1-a)] \). This corresponds to the market-clearing real wage in Gray. Thus the market-clearing output level is given by

\[ y_t = (1+c)/(1+c(1-a)] u_t + \bar{y} \]  \hspace{1cm} \text{(A4)}

where \( \bar{y} = \log A + ak \).
This corresponds to equation (2) of the text. When the nominal wage is set by contract one period ahead, the productivity disturbance \( u_t \) is unknown. Denoting the log of the contract wage by \( l_{-1}w_t \), and allowing for ex post indexation, we have

\[
l_{-1}w_t = \rho_t^{e} + b(p_t - p_t^e).
\]

We may thus substitute directly into (A5) and (A4) to get (A6).

\[
y_t = a/(1-a)(1-b)(p_t - p_t^e) + 1/(1-a)u_t + \bar{y}.
\]

This corresponds to equation (1) of the text. Notice that \( 1/(1-a) > (1+c)/(1+c(1-a)) \) always holds, or, from the text, \( \delta > \gamma \).

REFERENCES


