Estimating Production Functions

Introduction

Paul Schrimpf

UBC
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Section 1

Introduction
Why estimate production functions?

- Primitive component of economic model
- Gives estimate of firm productivity — useful for understanding economic growth
  - Stylized facts to inform theory, e.g. Foster, Haltiwanger, and Krizan (2001)
  - Effect of deregulation, e.g. Olley and Pakes (1996)
  - Growth within old firms vs from entry of new firms, e.g. Foster, Haltiwanger, and Krizan (2006)
  - Effect of trade liberalization, e.g. Amiti and Konings (2007)
General references:

• Aguirregabiria (2017) chapter 2
• Ackerberg et al. (2007) section 2
• Van Beveren (2012)
Section 2

Setup
**Setup**

- **Cobb Douglas production**
  \[ Y_{it} = A_{it}K_{it}^{\beta_k}L_{it}^{\beta_l} \]

- In logs,
  \[ y_{it} = \beta_k k_{it} + \beta_l l_{it} + \omega_{it} + \epsilon_{it} \]
  with \( \log A_{it} = \omega_{it} + \epsilon_{it} \), \( \omega_{it} \) known to firm, \( \epsilon_{it} \) not

- **Problems:**
  1. Simultaneity: if firm has information about \( \log A_{it} \) when choosing inputs, then inputs correlated with \( \log A_{it} \), e.g. price \( p \), wage \( w \), perfect information
  \[ L_{it} = \left( \frac{p}{w} \beta_l A_{it} K_{it}^{\beta_k} \right)^{\frac{1}{1-\beta_l}} \]
  2. Selection: firms with low productivity will exit sooner
  3. Others: measurement error, specification
Section 3

Simultaneity
Simultaneity solutions

1. IV
2. Panel data
3. Control functions
Instrumental variables

- Instrument must be
  - Correlated with $k$ and $l$
  - Uncorrelated with $\omega + \epsilon$
- Possible instrument: input prices
  - Correlated with $k$, $l$ through first-order condition
  - Uncorrelated with $\omega$ if input market competitive
- Other possible instruments: output prices (more often endogenous), input supply or output demand shifter (hard to find)
Problems with input prices as IV

- Not available in many data sets
- Average input price of firm could reflect quality as well as price differences
- Need variation across observations
  - If firms use homogeneous inputs, and operate in the same output and input markets, we should not expect to find any significant cross-sectional variation in input prices
  - If firms have different input markets, maybe variation in input prices, but different prices could be due to different average productivity across input markets
  - Variation across time is potentially endogenous because could be driven by time series variation in average productivity
Fixed effects

- Have panel data, so should consider fixed effects
- FE consistent if:
  1. \( \omega_{it} = \eta_i + \delta_t + \omega^*_it \)
  2. \( \omega^*_it \) uncorrelated with \( l_{it} \) and \( k_{it} \), e.g. \( \omega^*_it \) only known to firm after choosing inputs
  3. \( \omega^*_it \) not serially correlated and is strictly exogenous
- Problems:
  - Fixed productivity a strong assumption
  - Estimates often small in practice
  - Worsens measurement error problems

\[
\text{Bias}(\hat{\beta}^{FE}_k) \approx - \frac{\beta_k \text{Var}(\Delta \epsilon)}{\text{Var}(\Delta k) + \text{Var}(\Delta \epsilon)}
\]
Dynamic panel: motivation

• General idea: relax fixed effects assumption, but still exploit panel

• Collinearity problem: Cobb-Douglas production, flexible labor and capital implies log labor and log capital are linear functions of prices and productivity (Bond and Söderbom (2005))

• If observed labor and capital are not collinear then there must be something unobserved that varies across firms (e.g. prices), but that could invalidate monotonicity assumption of control function
Dynamic panel: moment conditions

- See Blundell and Bond (2000)
- Assume $\omega_{it} = \gamma_t + \eta_i + \nu_{it}$ with $\nu_{it} = \rho \nu_{i,t-1} + e_{it}$, so

$$y_{it} = \beta_l l_{it} + \beta_k k_{it} + \gamma_t + \eta_i + \nu_{it} + \epsilon_{it}$$

subtract $\rho y_{i,t-1}$ and rearrange to get

$$y_{it} = \rho y_{i,t-1} + \beta_l (l_{it} - \rho l_{i,t-1}) + \beta_k (k_{it} - \rho k_{i,t-1}) + \gamma_t - \rho \gamma_{t-1} + \eta_i (1 - \rho) + e_{it} + \epsilon_{it} - \rho \epsilon_{i,t-1}$$

= $\eta_i^*$

= $w_{it}$

- Moment conditions:
  - Difference: $E[x_{i,t-s} \Delta w_{it}] = 0$ where $x = (l, k, y)$
  - Level: $E[\Delta x_{i,t-s}(\eta_i^* + w_{it})] = 0$
Dynamic panel: economic model 1

- Adjustment costs

\[ V(K_{t-1}, L_{t-1}) = \max_{I_t, K_t, H_t, L_t} P_t F_t(K_t, L_t) - P^K_t (I_t + G_t(l_t, K_{t-1})) - \]
\[ - W_t (L_t + C_t(H_t, L_{t-1})) + \]
\[ \psi E[V(K_t, L_t)|I_t] \]

s.t. \( K_t = (1 - \delta_k)K_{t-1} + I_t \)
\[ L_t = (1 - \delta_l)L_{t-1} + H_t \]

Implies

\[ P_t \frac{\partial F_t}{\partial L_t} - W_t \frac{\partial C_t}{\partial L_t} = W_t + \lambda_t^L \left( 1 - (1 - \delta_l) \psi E \left[ \frac{\lambda_{t+1}^L}{\lambda_t^L} | I_t \right] \right) \]
\[ P_t \frac{\partial F_t}{\partial K_t} - P^K_t \frac{\partial G_t}{\partial K_t} = \lambda^K_t \left( 1 - (1 - \delta_k) \psi E \left[ \frac{\lambda_{t+1}^K}{\lambda^K_t} | I_t \right] \right) \]
Dynamic panel: economic model 2

- Current productivity shifts \( \frac{\partial F_t}{\partial L_t} \) and (if correlated with future) the shadow value of future labor \( E \left[ \frac{\lambda L}{\lambda_t} \mid I_t \right] \)
- Past labor correlated with current because of adjustment costs
Dynamic panel data: problems

- Problems:
  - Sometimes imprecise (especially if only use difference moment conditions)
  - Differencing worsens measurement error
  - Weak instrument issues if only use difference moment conditions but levels stronger (see Blundell and Bond (2000))
    - Level moments require stronger stationarity assumption
      - $\eta_i$ uncorrelated with $\Delta x_{it}$
Control functions

- From Olley and Pakes (1996) (OP)
- **Control function**: function of data conditional on which endogeneity problem solved
  - E.g. usual 2SLS $y = x\beta + \epsilon, x = z\pi + \nu$, control function is to estimate residual of reduced form, $\hat{\nu}$ and then regress $y$ on $x$ and $\hat{\nu}$. $\hat{\nu}$ is the control function
- Main idea: model choice of inputs to find a control function
OP assumptions

\[ y_{it} = \beta_k k_{it} + \beta_l l_{it} + \omega_{it} + \epsilon_{it} \]

1. \( \omega_{it} \) follows exogenous first order Markov process,
   \[ p(\omega_{it+1}|I_{it}) = p(\omega_{it+1}|\omega_{it}) \]

2. Capital at \( t \) determined by investment at time \( t - 1 \),
   \[ k_t = (1 - \delta)k_{t-1} + i_{it-1} \]

3. Investment is a function of \( \omega \) and other observed variables
   \[ i_{it} = I_t(k_{it}, \omega_{it}) \]

   and is strictly increasing in \( \omega_{it} \)

4. Labor variable and non-dynamic, i.e. chosen each \( t \),
   current choice has no effect on future (can be relaxed)
OP estimation of $\beta_l$

- Invertible investment implies $\omega_{it} = l_t^{-1}(k_{it}, i_{it})$

$$y_{it} = \beta_k k_{it} + \beta_l l_{it} + l_t^{-1}(k_{it}, l_{it}) + \epsilon_{it}$$

$$= \beta_l l_{it} + f_t(k_{it}, i_{it}) + \epsilon_{it}$$

- Partially linear model
  - Estimate by e.g. regress $y_{it}$ on $l_{it}$ and series functions of $t, k_{it}, i_{it}$
  - Gives $\hat{\beta}_l, \hat{f}_t = f_t(k_{it}, i_{it})$
OP estimation of $\beta_k$

- Note: $\hat{f}_t(k_{it}, i_{it}) = \hat{\omega}_{it} + \beta_k k_{it}$
- By assumptions, $\omega_{it} = E[\omega_{it} | \omega_{it-1}] + \xi_{it} = g(\omega_{it-1}) + \xi_{it}$ with $E[\xi_{it} | k_{it}] = 0$
- Use $E[\xi_{it} | k_{it}] = 0$ as moment to estimate $\beta_k$.
  - OP: write production function as
    \[
    y_{it} - \beta_l l_{it} = \beta_k k_{it} + g(\omega_{it-1}) + \xi_{it} + \epsilon_{it} = \beta_k k_{it} + g(f_{it-1} - \beta_k k_{it-1}) + \xi_{it} + \epsilon_{it}
    \]
  - Use $\hat{\beta}_l$ and $\hat{f}_{it}$ in equation above and estimate $\hat{\beta}_k$ by e.g. semi-parametric nonlinear least squares
- Ackerberg, Caves, and Frazer (2015): use
  \[
  E \left[ \hat{\xi}_{it}(\beta_k k_{it}) \right] = 0
  \]
Dynamic panel vs control function

• Both derive moment conditions from assumptions about timing and information set of firm
• Dealing with $\omega$
  • Dynamic panel: AR(1) assumption allows quasi-differencing
  • Control function: makes $\omega$ estimable function of observables
• Dynamic panel allows fixed effects, does not make assumptions about input demand
• Control function allows more flexible process for $\omega_{it}$
Applications

- Olley and Pakes (1996): productivity in telecom after deregulation
- Söderbom, Teal, and Harding (2006): productivity and exit of African manufacturing firms, uses IV
- Levinsohn and Petrin (2003): compare estimation methods using Chilean data
- Javorcik (2004): FDI and productivity, uses OP
- Amiti and Konings (2007): trade liberalization in Indonesia, uses OP
- Aw, Chen, and Roberts (2001): productivity differentials and firm turnover in Taiwan
- Kortum and Lerner (2000): venture capital and innovation
Section 4

Selection
• Let $d_{it} = 1$ if firm in sample.
  • Standard conditions imply $d = 1 \{ \omega \geq \omega^*(k) \}$
• Messes up moment conditions
  • All estimators based on $E[\omega_{it} \text{Something}] = 0$, observed data really use $E[\omega_{it} \text{Something}|d_{it} = 1]$
  • E.g. OLS okay if $E[\omega_{it}|l_{it}, k_{it}] = 0$, but even then,

$$E[\omega_{it}|l_{it}, k_{it}, d_{it} = 1] = E[\omega_{it}|l_{it}, k_{it}, \omega_{it} \geq \omega^*(k_{it})]$$

$$= \lambda(k_{it}) \neq 0$$

• Selection bias negative, larger for capital than labor
Selection in OP model

- Estimate $\beta_l$ as above
- Write
  \[ d_{it} = 1 \{ \tilde{\xi}_{it} \leq \omega^*(k_{it}) - \rho(f_{i,t-1} - \beta_k k_{it-1}) = h(k_{it}, f_{it-1}, k_{it-1}) \} \]
- Propensity score $P_{it} \equiv E[d_{it}|k_{it}, f_{it-1}, k_{it-1}]$
- Similar to before estimate $\beta_k$, from

\[
y_{it} - \beta_l y_{it} = \beta_k k_{it} + \tilde{g}(f_{it-1} - \beta_k k_{it-1}, P_{it}) + \\
+ \xi_{it} + \epsilon_{it}
\]


