Games on Networks

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Section 1

Bramoullé, Kranton, and D'Amours (2014)

Games on networks 1

- Model of Bramoullé, Kranton, and D'Amours (2014)
- N agents, simultaneously choose $x_i \geq 0$
- Payoffs $U_i(x_i, x_{-i}; \delta, G)$
  - Assume U_i depends on x_j iff they are linked
  - $\delta \geq 0$ parameterizes how strongly payoffs depend on one another
- Best reply $x_i = f_i(x_{-i}; \delta, G)$
- Focus on games where $f_i(x_{-i}; \delta, G)$ is linear in x_{-i}

$$f_i(x_{-i}; \delta, G) = \max\{1 - \delta \sum_j g_{ij} x_j, 0\}$$

  - Public goods

$$U_i(x_i, x_{-i}; \delta, G) = b_i(x_i + \delta \sum_j g_{ij} x_j) - \kappa_i x_i$$

Games on networks 2

- Negative externalities (e.g. congestion)

$$U_i(x_i, x_{-i}; \delta, G) = \bar{x}_i x_i - \frac{1}{2}x_i^2 - \delta \sum_j g_{ij} x_i x_j$$

- Cournot competition with linear demand, network of substitutes

$$\Pi_i(x_i, x_{-i}; \delta, G) = x_i \left(a - s \left(x_i + 2\delta \sum_j g_{ij} x_j \right) \right) - d x_i$$

Nash equilibria

- Define G_A = links among active agents with $x_i > 0$
- $G_{N-A,A}$ links connect active to inactive agents
- Actions x is a Nash equilibrium iff
  1. $(I + \delta G_A)x_A = \mathbf{1}$
  2. $\delta G_{N-A,A} x_A \geq \mathbf{1}$
- Main results: equilibria depend on the minimal eigenvalue
  1. If $|\lambda_{\min}(G)| < 1/\delta$, there is a unique Nash equilibrium
  2. An equilibrium is stable iff $|\lambda_{\min}(G_A)| < 1/\delta$
  3. If $|\lambda_{\min}(G)| > 1/\delta$, there may be multiple equilibria and all stable equilibria include inactive agents

Sketch of proof

- Potential games: $\varphi(x_i, x_{-i})$ is a potential for a game with payoffs $v_i(x_i, x_{-i})$ if

$$\varphi(x_i, x_{-i}) - \varphi(x_i', x_{-i}) = v_i(x_i, x_{-i}) - v_i(x_i, x_{-i})$$

- For these games: $\varphi(x; \delta, G) = x^T\mathbf{1} - \frac{1}{2}x^T(I + \delta G)x$
- Key observation: first order conditions for

$$\max_x x^T\mathbf{1} - \frac{1}{2}x^T(I + \delta G)x \text{ s.t. } x \geq 0$$

  are the same as the equilibrium conditions; in fact the set of equilibria are the maxima and any saddle points
- Unique maximum if $I + \delta G$ positive definite
  - Eigenvalues of $I + \delta G = 1 + \delta\lambda(G)$, so unique equilibrium if $1 > -\delta\lambda_{\min}(G)$

Application: R&D free-riding

- Empirical evidence of R&D free-riding
  - Hendricks and Porter (1996): exploratory oil drilling
  - Foster and Rosenzweig (1995): new seeds in agriculture
- Question: how does the network of firms affect free-riding?
  - If $|\lambda_{\min}(G)| < 1/\delta$, unique equilibrium with all firms active
  - Larger $|\lambda_{\min}(G)|$ implies stable equilibrium involves inactive firms
  - Larger $|\lambda_{\min}(G)|$ means more global connections

- Acemoglu et al. (2012) is somewhat similar – how does network structure of sectoral input-output relate to how sectoral shocks translate into aggregate shocks

Section 2

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"Static and dynamic networks in interbank markets"

- Model of interbank lending
- Capture how shock propagation depends on interbank lending network

Static Model

- Leverage constraint:

$$\xi \overbrace{e_i}^{\text{equity}} \geq assets$$

- Interbank loans:

$$\overbrace{q_i}^{\text{loans}} = liabilities_i - \underbrace{X_i}_{\text{cash + loans}}$$

- Balance sheet:

$$\xi e_i \geq X_i + q_i$$

Assets	Liabilities
Cash	Deposits
Loans	Interbank borrowing
Interbank loans = q_i	Equity = e_i

Fig. 1. Balance sheet.

Section 3

König, Liu, and Zenou (2019)

König, Liu, and Zenou (2019)

"R & D Networks: Theory, Empirics and Policy Implications"

- Firms connected in two networks:
  - R & D network
  - Competition network
- Cournot game where firms choose R&D expenditure and quantities
- OECD countries spend more than $50 billion per year on R&D subsidies and tax credits
- Identify key firms, analyze impact of R&D subsidies

Model 1

- Firms $i \in \{1, \ldots, n\}$
- Partitioned product markets $\mathcal{M}_m$, $m = 1, \ldots, M$
- Consumer utility

$$U_m(q) = \alpha_m \sum_{i \in \mathcal{M}_m} q_i - \frac{1}{2} \sum_{i \in \mathcal{M}_m} q_i^2 - \rho \sum_{i \in \mathcal{M}_m} \sum_{j \in \mathcal{M}_m \setminus \{i\}} q_i q_j$$

- Inverse demand

$$p_i = \underbrace{\sum_m \alpha_m 1_{i \in \mathcal{M}_m}}_{\bar{\alpha}_i} - q_i - \rho \sum_{j \in \mathcal{M}_m \setminus \{i\}} q_j$$

- R&D collaboration network A

Model 2

- Marginal cost:

$$c_i = \bar{c}_i - e_i - \varphi \sum_{j=1}^{n} a_{ij} e_j$$

- Profits:

$$\pi_i = (\bar{\alpha}_i - \bar{c}_i) q_i - q_i^2 - \rho \sum_{j=1}^{n} b_{ij} q_i q_j + q_i e_i + \varphi q_i \sum_{j=1}^{n} a_{ij} e_j - \frac{1}{2} e_i^2$$

Equilibrium 1

- FOC for e_i: $q_i - e_i = 0$
- Substitute into $\pi(q)$:

$$\pi_i = (\bar{\alpha}_i - \bar{c}_i)q_i - \frac{1}{2}q_i^2 + \sum_{j=1}^{n}(\varphi a_{ij} - \rho b_{ij})q_i q_j$$

- FOC for q:

$$q_i = (\bar{\alpha}_i - \bar{c}_i) + \sum_{j=1}^{n}(\varphi a_{ij} - \rho b_{ij})q_j$$

- Matrix form:

$$(I_n + \rho B - \varphi A)q = (\bar{\alpha} - \bar{c})$$

- Proposition 1: conditions for $(I_n + \rho B - \varphi A)$ to be invertible given special structure of A, B both in general and for special cases

Network & Market effects

- Network effect: through A
  - More connections $\rightarrow$ more e & q
- Market effect: through B
  - More connections (competitors) $\rightarrow$ less e & q

Network & Market effects

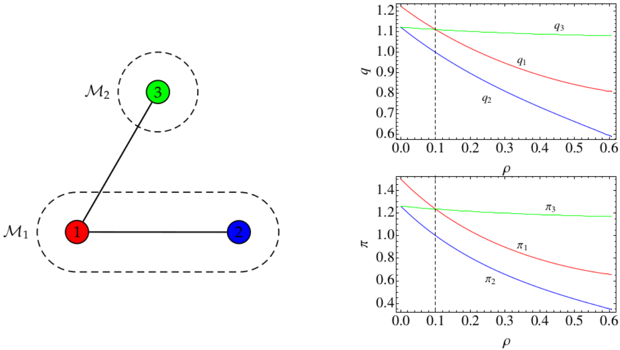


Figure 1: Equilibrium output from Equation (15) and profits for the three firms with varying values of the competition parameter $0 \leq \rho \leq \frac{1}{2}\left(\sqrt{2} - 2\varphi\right)$, $\mu = 1$ and $\varphi = 0.1$. Profits of firms 1 and 3 intersect at $\rho = \varphi$ (indicated with a dashed line).

Welfare

- Welfare = utility + profits

$$W(G) = q^T q + \frac{\rho}{2} q^T B q$$

- Propositions 2-4 : characterize how welfare varies with network, competition effect (ρ) and spillovers (φ)

Key firms

- G^{-i} = network without firm i
- Key firm:
$$i^* = \arg\max_i W(G) - W(G^{-i})$$
- Proposition 5: characterizes i^*
- Key firm need not have highest q_i or highest of any conventional centrality measure (degree, Bonacich, etc)

R & D subsidies

- Subsidize R&D at rate s_i per e_i

$$\pi_i = (\bar{\alpha}_i - \bar{c}_i) q_i - q_i^2 - \rho \sum_{j=1}^n b_{ij} q_i q_j + q_i e_i + \varphi q_i \sum_{j=1}^n a_{ij} e_j - \frac{1}{2} e_i^2 + s_i e_i$$

- Proposition 6: optimal uniform subsidy
- Proposition 7: optimal firm specific subsidies

$$s^* = \arg\max_{s \in \mathbb{R}_+^n} W(G, s)$$

  - Key firm does not necessarily get the largest subsidy

Empirical questions

- Test proposition 1 and disentangle spillover and product rivalry effects of R&D
- Determine key firms
- Estimate optimal subsidies

Data

- MERIT-CATI database on interfirm R&D collaboration
  - 1987-2006, some information going back to 1950
  - 13040 companies
- Matched by firm name with Compustat for balance sheets and income
  - Profit, sales, R&D, employees, capital
  - x_{it} = R&D stock = perpetual inventory of past *R&D* expenditures with 15% depreciation

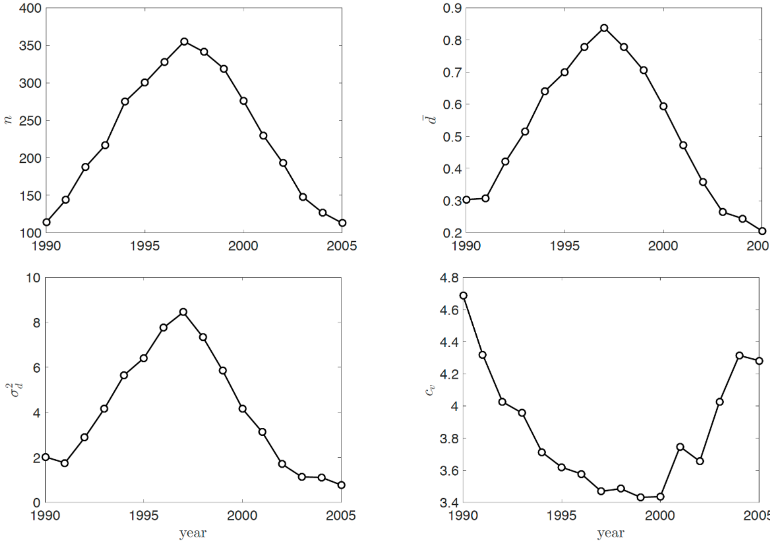


Figure 2: The number of firms, n, participating in an alliance, the average degree, $\bar{d}$, the degree variance, σ_d^2, and the degree coefficient of variation, $c_v = \sigma_d/\bar{d}$.

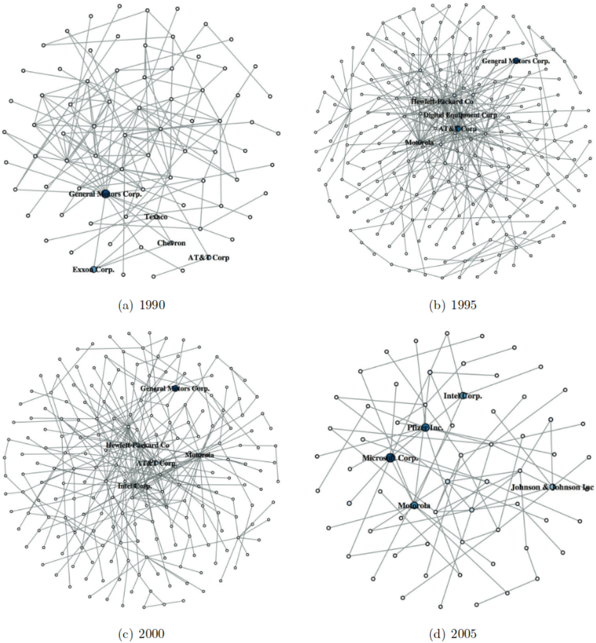


Figure 3: Network snapshots of the largest connected component for the years (a) 1990, (b) 1995, (c) 2000 and (d) 2005. Nodes' sizes and shades indicate their targeted subsidies (see Section 7). The names of the 5 highest subsidized firms are indicated in the network.

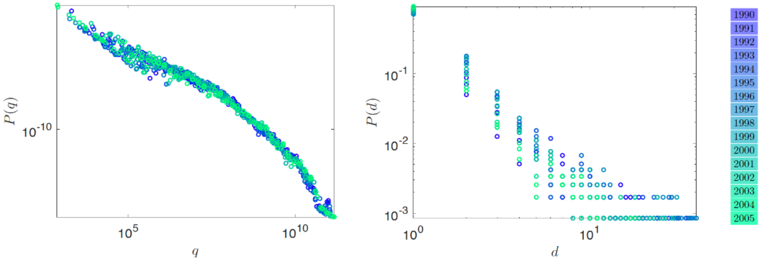


Figure 4: Empirical output distribution $P(q)$ and the distribution of degree $P(d)$ for the years 1990 to 2005. The data for output has been logarithmically binned and non-positive data entries have been discarded. Both distributions are highly skewed.

Econometric Model

- Marginal cost:

$$c_{it} = \eta_i - \epsilon_{it} - x_{it}\beta - e_{it} - \varphi \sum_{j=1}^{n} a_{ij,t} e_{jt}$$

- Inverse demand:

$$p_{it} = \alpha_m + \alpha_t - q_{it} - \rho \sum_{j=1}^{n} b_{ij} q_{jt}$$

- Best response of q

$$q_t = \varphi \sum_{j=1}^{n} a_{ij,t} q_{jt} - \rho \sum_{j=1}^{n} b_{ij} q_{jt} + x_{it}\beta + \eta_i + \kappa_t + \epsilon_{it}$$

Econometric Model

- Instruments:
  - $\sum_{j=1}^{n} a_{ij,t} q_{jt}$, $\sum_{j=1}^{n} b_{ij} q_{jt}$ endogenous
  - Instrument with $\sum_{j=1}^{n} a_{ij,t} x_{it-1}$, $\sum_{j=1}^{n} b_{ij} x_{it-1}$ and/or $\sum_{j=1}^{n} a_{ij,t} taxcredits_{it-1}$, $\sum_{j=1}^{n} b_{ij} taxcredits_{it-1}$
  - Instrument $a_{ij,t}$ with $a_{ij,t-s}$ and/or $\hat{a}_{ij,t-s}$ predicted using time $t-s$ firm characteristics

Table 2: Parameter estimates from a panel regression of Equation (26). Model A includes only time fixed effects, while Model B includes both firm and time fixed effects. The dependent variable is output obtained from deflated sales. Standard errors (in parentheses) are robust to arbitrary heteroskedasticity and allow for first-order serial correlation using the Newey-West procedure. The estimation is based on the observed alliances in the years 1967–2006.

	Model A		Model B	
φ	-0.0118	(0.0075)	0.0106**	(0.0051)
ρ	0.0114***	(0.0015)	0.0189***	(0.0028)
β	0.0053***	(0.0002)	0.0027***	(0.0002)
# firms	1186		1186	
# observations	16924		16924	
Cragg-Donald Wald F stat.	6454.185		7078.856	
firm fixed effects	no		yes	
time fixed effects	yes		yes	

*** Statistically significant at 1% level.
** Statistically significant at 5% level.
* Statistically significant at 10% level.

Table 3: Parameter estimates from a panel regression of Equation (26) with IVs based on time-lagged tax credits. Model C includes only time fixed effects, while Model D includes both firm and time fixed effects. The dependent variable is output obtained from deflated sales. Standard errors (in parentheses) are robust to arbitrary heteroskedasticity and allow for first-order serial correlation using the Newey-West procedure. The estimation is based on the observed alliances in the years 1967–2006.

	Model C		Model D	
φ	-0.0133	(0.0114)	0.0128*	(0.0069)
ρ	0.0182***	(0.0018)	0.0156**	(0.0076)
β	0.0054***	(0.0004)	0.0023***	(0.0006)
# firms	1186		1186	
# observations	16924		16924	
Cragg-Donald Wald F stat.	138.311		78.791	
firm fixed effects	no		yes	
time fixed effects	yes		yes	

*** Statistically significant at 1% level.
** Statistically significant at 5% level.
* Statistically significant at 10% level.

Table 4: Link formation regression results. Technological similarity, f_{ij}, is measured using either the Jaffe or the Mahalanobis patent similarity measures. The dependent variable $a_{ij,t}$ indicates if an R&D alliance exists between firms i and j at time t. The estimation is based on the observed alliances in the years 1967–2006.

technological similarity	Jaffe	Mahalanobis
Past collaboration	0.5981***	0.5920***
	(0.0150)	(0.0149)
Past common collaborator	0.1162***	0.1164***
	(0.0238)	(0.0236)
$f_{ij,t-s-1}$	13.6977***	6.0864***
	(0.6884)	(0.3323)
$f^2_{ij,t-s-1}$	-20.4083***	-3.9194***
	(1.7408)	(0.4632)
$city_{ij}$	1.1283***	1.1401***
	(0.1017)	(0.1017)
$market_{ij}$	0.8451***	0.8561***
	(0.0424)	(0.0422)
# observations	3,964,120	3,964,120
McFadden's R^2	0.0812	0.0813

*** Statistically significant at 1% level.
** Statistically significant at 5% level.
* Statistically significant at 10% level.

Table 5: Parameter estimates from a panel regression of Equation (26) with endogenous R&D alliance matrix. The IVs are based on the predicted links from the logistic regression reported in Table 4, where technological similarity is measured using either the Jaffe or the Mahalanobis patent similarity measures. The dependent variable is output obtained from deflated sales. Standard errors (in parentheses) are robust to arbitrary heteroskedasticity and allow for first-order serial correlation using the Newey-West procedure. The estimation is based on the observed alliances in the years 1967–2006.

technological similarity	Jaffe		Mahalanobis	
φ	0.0582*	(0.0343)	0.0593*	(0.0341)
ρ	0.0197***	(0.0031)	0.0197***	(0.0031)
β	0.0024***	(0.0002)	0.0024***	(0.0002)
# firms	1186		1186	
# observations	16924		16924	
Cragg-Donald Wald F stat.	48.029		49.960	
firm fixed effects	yes		yes	
time fixed effects	yes		yes	

*** Statistically significant at 1% level.
** Statistically significant at 5% level.
* Statistically significant at 10% level.

Robustness: inter vs intra industry spillovers

Table 6: Parameter estimates from a panel regression of Equation (31) with both firm and time fixed effects. Technological similarity, f_{ij}, is measured using either the Jaffe or the Mahalanobis patent similarity measures. The dependent variable is output obtained from deflated sales. Standard errors (in parentheses) are robust to arbitrary heteroskedasticity and allow for first-order serial correlation using the Newey-West procedure. The estimation is based on the observed alliances in the years 1967–2006.

technological similarity	Jaffe		Mahalanobis	
φ	0.0102**	(0.0049)	0.0102**	(0.0049)
χ	0.0063	(0.0052)	0.0043	(0.0030)
ρ	0.0189***	(0.0028)	0.0192**	(0.0028)
β	0.0027***	(0.0002)	0.0027***	(0.0002)
# firms	1190		1190	
# observations	17105		17105	
Cragg-Donald Wald F stat.	4791.308		4303.563	
firm fixed effects	yes		yes	
time fixed effects	yes		yes	

*** Statistically significant at 1% level.

Optimal firm subsidies

- Targeted subsidies have much larger welfare gain than uniform
- Optimal subsidies cyclical
- Firm subsidy ranking not same as market share or other simple observed firm characteristic

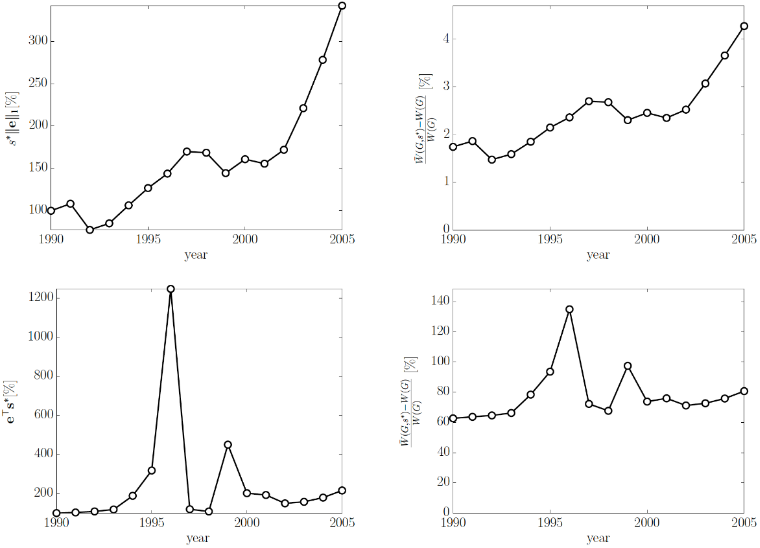


Figure 6: (Top left panel) The total optimal subsidy payments, $s^*\|\mathbf{e}\|_1$, in the homogeneous case over time, using the subsidies in the year 1990 as the base level. (Top right panel) The percentage increase in welfare due to the homogeneous subsidy, s^*, over time. (Bottom left panel) The total subsidy payments, $\mathbf{e}^\top \mathbf{s}^*$, when the subsidies are targeted towards specific firms, using the subsidies in the year 1990 as the base level. (Bottom right panel) The percentage increase in welfare due to the targeted subsidies, $\mathbf{s}^*$, over time.

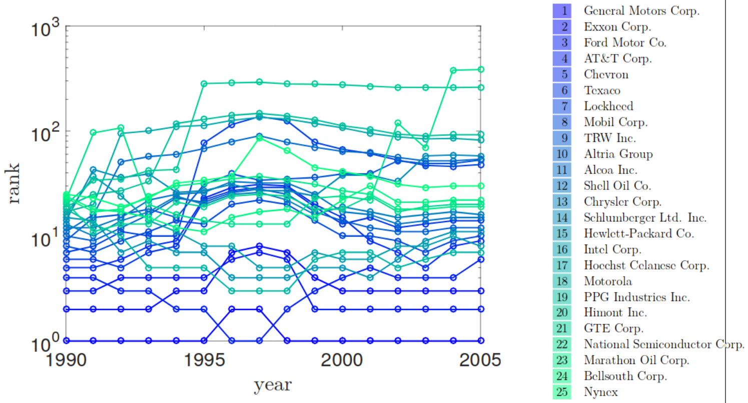


Figure 7: Change in the ranking of the 25 highest subsidized firms (Table 7) from 1990 to 2005.

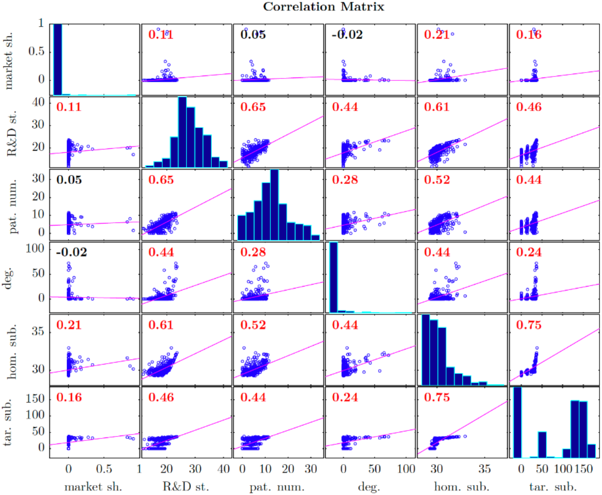


Figure 8: Pair correlation plot of market shares, R&D stocks, the number of patents, the degree, the homogeneous subsidies and the targeted subsidies (cf. Table 8), in the year 2005. The Spearman correlation coefficients are shown for each scatter plot. The data have been log and square root transformed to account for the heterogeneity in across observations.

Table 7: Subsidies ranking for the year 1990 for the first 25 firms.

Firm	Share [%][a]	num pat.	d	$\mathbf{v}_{PF}$	Betweenness[b]	Closeness[c]	$\mathbf{q}$ [%][d]	hom. sub. [%][e]	tar. sub. [%][f]	SIC[g]	Rank
General Motors Corp.	9.2732	76644	88	0.1009	0.0007	0.0493	6.9866	0.0272	0.3027	3711	1
Exxon Corp.	7.7132	21954	22	0.0221	0.0000	0.0365	5.4062	0.0231	0.1731	2911	2
Ford Motor Co.	7.3456	20378	6	0.0003	0.0000	0.0153	3.7301	0.0184	0.0757	3711	3
AT&T Corp.	9.5360	5692	8	0.0024	0.0000	0.0202	3.2272	0.0156	0.0565	4813	4
Chevron	2.8221	12789	23	0.0226	0.0001	0.0369	2.5224	0.0098	0.0418	2911	5
Texaco	2.9896	9134	22	0.0214	0.0000	0.0365	2.4965	0.0095	0.0415	2911	6
Lockheed	42.3696	2	51	0.0891	0.0002	0.0443	1.5639	0.0035	0.0196	3760	7
Mobil Corp.	4.2265	3	0	0.0000	0.0000	0.0000	1.9460	0.0111	0.0191	2911	8
TRW Inc.	5.3686	9438	43	0.0583	0.0002	0.0415	1.4509	0.0027	0.0176	3714	9
Altria Group	43.6382	0	0	0.0000	0.0000	0.0000	1.4665	0.0073	0.0117	2111	10
Alcoa Inc.	11.4121	4546	36	0.0287	0.0002	0.0372	1.2136	0.0032	0.0114	3350	11
Shell Oil Co.	14.6777	9504	0	0.0000	0.0000	0.0000	1.4244	0.0073	0.0109	1311	12
Chrysler Corp.	2.2414	3712	6	0.0017	0.0000	0.0218	1.3935	0.0075	0.0109	3711	13
Schlumberger Ltd. Inc.	25.9218	9	18	0.0437	0.0000	0.0370	1.1208	0.0029	0.0099	1389	14
Hewlett-Packard Co.	7.1106	6606	64	0.1128	0.0002	0.0417	1.1958	0.0047	0.0093	3570	15
Intel Corp.	9.3900	1132	67	0.1260	0.0003	0.0468	1.0152	0.0018	0.0089	3674	16
Hoechst Celanese Corp.	5.6401	516	38	0.0368	0.0002	0.0406	1.0047	0.0021	0.0085	2820	17
Motorola	14.1649	21454	70	0.1186	0.0004	0.0442	1.0274	0.0028	0.0080	3663	18
PPG Industries Inc.	13.3221	24904	20	0.0230	0.0000	0.0366	0.9588	0.0021	0.0077	2851	19
Himont Inc.	0.0000	59	28	0.0173	0.0001	0.0359	0.8827	0.0014	0.0072	2821	20
GTE Corp.	3.1301	4	0	0.0000	0.0000	0.0000	1.1696	0.0067	0.0070	4813	21
National Semiconductor Corp.	4.0752	1642	43	0.0943	0.0001	0.0440	0.8654	0.0012	0.0068	3674	22
Marathon Oil Corp.	7.9828	202	0	0.0000	0.0000	0.0000	1.1306	0.0060	0.0068	1311	23
Bellsouth Corp.	2.4438	3	14	0.0194	0.0000	0.0329	1.0926	0.0060	0.0064	4813	24
Nynex	2.3143	26	24	0.0272	0.0001	0.0340	0.9469	0.0049	0.0052	4813	25

[a] Market share in the primary 4-digit SIC sector in which the firm is operating. In case of missing data the closest year with sales data available has been used.

[b] The normalized betweenness centrality is the fraction of all shortest paths in the network that contain a given node, divided by $(n-1)(n-2)$, the maximum number of such paths.

[c] The closeness centrality of node i is computed as $\frac{2}{n-1}\sum_{j=1}^{n} 2^{-\ell_{ij}(G)}$, where $\ell_{ij}(G)$ is the length of the shortest path between i and j in the network G and the factor $\frac{2}{n-1}$ is the maximal centrality attained for the center of a star network.

[d] The relative output of a firm i follows from Proposition 1.

[e] The homogeneous subsidy for each firm i is computed as $e_i^* s^*$, relative to the total homogeneous subsidies $\sum_{j=1}^{n} e_j^* s^*$ (see Proposition 2).

[f] The targeted subsidy for each firm i is computed as $e_i^* s_i^*$, relative to the total targeted subsidies $\sum_{j=1}^{n} e_j^* s_j^*$ (see Proposition 3).

[g] The primary 4-digit SIC code according to Compustat U.S. fundamentals database.

Table 8: Subsidies ranking for the year 2005 for the first 25 firms.

Firm	Share [%][a]	num pat.	d	v_{PF}	Betweenness[b]	Closeness[c]	q [%][d]	hom. sub.[%][e]	tar. sub. [%][f]	SIC[g]	Rank
General Motors Corp.	3.9590	90652	19	0.0067	0.0002	0.0193	4.1128	0.0174	0.2186	3711	1
Ford Motor Co.	3.6818	27452	7	0.0015	0.0000	0.0139	3.4842	0.0153	0.1531	3711	2
Exxon Corp.	4.0259	53215	6	0.0007	0.0001	0.0167	2.9690	0.0132	0.1108	2911	3
Microsoft Corp.	10.9732	10639	62	0.1814	0.0020	0.0386	1.6959	0.0057	0.0421	7372	4
Pfizer Inc.	3.6714	74253	65	0.0298	0.0034	0.0395	1.6796	0.0069	0.0351	2834	5
AT&T Corp.	0.0000	16284	0	0.0000	0.0000	0.0000	1.5740	0.0073	0.0311	4813	6
Motorola	6.6605	70583	66	0.1598	0.0017	0.0356	1.3960	0.0053	0.0282	3663	7
Intel Corp.	5.0169	28513	72	0.2410	0.0011	0.0359	1.3323	0.0050	0.0249	3674	8
Chevron	2.2683	15049	10	0.0017	0.0001	0.0153	1.3295	0.0058	0.0243	2911	9
Hewlett-Packard Co.	14.3777	38597	7	0.0288	0.0000	0.0233	1.1999	0.0055	0.0183	3570	10
Altria Group	20.4890	5	2	0.0000	0.0000	0.0041	1.1753	0.0054	0.0178	2111	11
Johnson & Johnson Inc.	3.6095	31931	40	0.0130	0.0015	0.0346	1.1995	0.0051	0.0173	2834	12
Texaco	0.0000	10729	0	0.0000	0.0000	0.0000	1.0271	0.0055	0.0124	2911	13
Shell Oil Co.	0.0000	12436	0	0.0000	0.0000	0.0000	0.9294	0.0045	0.0108	1311	14
Chrysler Corp.	0.0000	5112	0	0.0000	0.0000	0.0000	0.9352	0.0052	0.0101	3711	15
Bristol-Myers Squibb Co.	1.3746	16	35	0.0052	0.0009	0.0326	0.8022	0.0034	0.0077	2834	16
Merck & Co. Inc.	1.5754	52036	36	0.0023	0.0007	0.0279	0.8252	0.0038	0.0077	2834	17
Marathon Oil Corp.	5.5960	229	0	0.0000	0.0000	0.0000	0.7817	0.0039	0.0076	1311	18
GTE Corp.	0.0000	5	0	0.0000	0.0000	0.0000	0.7751	0.0041	0.0073	4813	19
Pepsico	36.6491	991	0	0.0000	0.0000	0.0000	0.7154	0.0035	0.0066	2080	20
Bellsouth Corp.	0.9081	2129	0	0.0000	0.0000	0.0000	0.7233	0.0039	0.0063	4813	21
Johnson Controls Inc.	22.0636	304	11	0.0027	0.0001	0.0159	0.6084	0.0021	0.0063	2531	22
Dell	18.9098	80	2	0.0190	0.0000	0.0216	0.6586	0.0028	0.0061	3571	23
Eastman Kodak Co	5.5952	109714	17	0.0442	0.0001	0.0262	0.6171	0.0023	0.0060	3861	24
Lockheed	48.9385	9817	44	0.0434	0.0003	0.0223	0.6000	0.0028	0.0049	3760	25

[a] Market share in the primary 4-digit SIC sector in which the firm is operating. In case of missing data the closest year with sales data available has been used.

[b] The normalized betweenness centrality is the fraction of all shortest paths in the network that contain a given node, divided by $(n-1)(n-2)$, the maximum number of such paths.

[c] The closeness centrality of node i is computed as $\frac{2}{n-1}\sum_{j=1}^{n} 2^{-\ell_{ij}(G)}$, where $\ell_{ij}(G)$ is the length of the shortest path between i and j in the network G and the factor $\frac{2}{n-1}$ is the maximal centrality attained for the center of a star network.

[d] The relative output of a firm i follows from Proposition 1.

[e] The homogeneous subsidy for each firm i is computed as $e_i^* s^*$, relative to the total homogeneous subsidies $\sum_{j=1}^{n} e_j^* s^*$ (see Proposition 2).

[f] The targeted subsidy for each firm i is computed as $e_i^* s_i^*$, relative to the total targeted subsidies $\sum_{j=1}^{n} e_j^* s_j^*$ (see Proposition 3).

[g] The primary 4-digit SIC code according to Compustat U.S. fundamentals database.

Section 4

Acemoglu, Malekian, and Ozdaglar (2016)

"Network security and contagion" Acemoglu, Malekian, and Ozdaglar (2016)

- Setting:
  - Agents i connected in network
  - Security investment q_i
  - Probability of infection transmitted to i is $1 - q_i$
- Results:
  - Decompose payoff into own effect and externality
  - Characterize relationship between network structure and security investments
  - Compare Nash equilibrium to social optimum

Model

- Agents $V = \{1, \ldots, n\}$
- Undirected network $G = (V, E)$
- Initially, one agent infected with probabilities $\Phi = (\phi_1, \ldots, \phi_n)$
- Before infection i chooses $q_i \in [0, 1]$, q_i is probability of being immune
- Preferences

$$u_i(G, \mathbf{q}, \Phi) = (1 - \mathrm{P}_i(G, \mathbf{q}, \Phi)) - c_i(q_i)$$

  c_i continuously differentiable, increasing, convex, $c(0) = c'(0) = 0$ and $\lim_{q \to 1} c'(q) = \infty$
- Social welfare

$$W(G, \mathbf{q}, \Phi) = \sum_{i \in V} u_i(G, \mathbf{q}, \Phi)$$

Network effect

Proposition

Given network G, security profile $\mathbf{q}$, and attack decision Φ, the infection probability of agent i satisfies

$$\mathrm{P}_i(G, \mathbf{q}, \Phi) = (1 - q_i)\tilde{\mathrm{P}}_i(G, \mathbf{q}_{-i}, \Phi)$$

where $\tilde{\mathrm{P}}_i(G, \mathbf{q}_{-i}, \Phi)$ is the probability of the infection reaching agent i (including the probability of agent i being the seed).

- $\tilde{\mathrm{P}}_i(G, \mathbf{q}_{-i}, \Phi) =$ "network effect on agent i"

Decomposition

Proposition

Given network G, security profile $\mathbf{q}_{-j}$, and attack decision Φ, the probability of the infection reaching agent j, $\tilde{\mathrm{P}}_j(G, \mathbf{q}_{-j}, \Phi)$, satisfies the following: For all $i \in V \setminus \{j\}$,

$$\tilde{\mathrm{P}}_j(G, \mathbf{q}_{-j}, \Phi) = \tilde{\mathrm{P}}_j(G_{-i}, \mathbf{q}_{-\{j,i\}}, \Phi) + (1 - q_i) Q_{ji}(G, q_{-\{j,i\}}, \Phi)$$

where $Q_{ji}(G, q_{-\{j,i\}}, \Phi)$ is the probability of infection reaching agent j only through a path that contains agent i.

- $(1 - q_i) Q_{ji}(G, q_{-\{j,i\}}, \Phi)$ is the externality of i on j

Strategic substitutes

- Propositions 1 & 2 imply

$$\frac{\partial^2 u_i}{\partial q_i \partial q_j} = -Q_{ji}(G, q_{-\{j,i\}}, \Phi) < 0$$

- Agent i invests more if others invest less

Best response 1

- Using propositions 1 & 2:

$$u_i(G, \mathbf{q}, \Phi) = 1 - (1 - q_i)\tilde{\mathrm{P}}_i(G, \mathbf{q}_{-i}, \Phi) - c_i(q_i)$$

and

$$W(G, \mathbf{q}) = 1 - (1 - q_i)\tilde{\mathrm{P}}_i(G, \mathbf{q}_{-i}, \Phi) - c_i(q_i) + \\ + \sum_{j \neq i} 1 - (1 - q_j)\left(\tilde{Pr}_j(G_{-i}, \mathbf{q}_{-\{i,j\}}, \Phi) + (1 - q_i)Q_{ji}(G, \mathbf{q}_{-\{i,j\}}, \Phi)\right) - c_j(q_j)$$

- First order conditions for q_i

$$c_i'(B_i(\mathbf{q}_{-i})) = \tilde{\mathrm{P}}_i(G, \mathbf{q}_{-i}, \Phi)$$

and

$$c_i'(S_i(\mathbf{q}_{-i})) = \tilde{\mathrm{P}}_i(G, \mathbf{q}_{-i}, \Phi) + \sum_{j \neq i} (1 - q_j)Q_{ji}(G, \mathbf{q}_{-\{i,j\}}, \Phi)$$

Best response 2

- So (given $\mathbf{q}_{-i}$)) equilibrium best response is less than social welfare maximizing best response, $B_i(\mathbf{q}_{-i}) \leq S_i(\mathbf{q}_{-i})$
  - If network and costs are symmetric across agents, then $q^e \leq q^s$

Gatekeepers and protection

- Define $a^j_{ik} = \mathbf{1}\{j \text{ included in all paths from } i \text{ to } k\}$ i.e. j is a gatekeeper between i and k
- Protection of j for i is $a^j_i = \frac{1}{n} \sum_k a^j_{ik}$

(a) $a^1_{24} = 1$ (b) $a^1_{24} = 0$

Equilibrium in non-symmetric networks

Proposition

$$\tilde{\mathrm{P}}(G, \mathbf{q}_{-i}) = 1 - \sum_{j \neq i} a_i^j q_j + o(\|\mathbf{q}\|_\infty)$$

Theorem

If $\alpha = c''(0)$ is large, then

$$\begin{aligned} q^e =& (A + \alpha I)^{-1} e + o(1/\alpha^2) \\ =& \frac{1}{\alpha} e - \frac{1}{\alpha^2} A e + o(1/\alpha^2) \end{aligned}$$

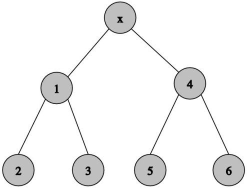
where $A_{ij} = a_i^j$, $A_{ii} = 0$, and $e = (1, 1, \ldots, 1)$

Protection centrality determines equilibrium investment

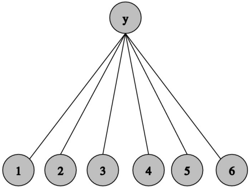
- Theorem implies

$$q_i^e = \frac{1}{\alpha}(1 - \frac{1}{\alpha} a_i) + o(1/\alpha^2)$$

where $a_i = \sum_j a_i^j$ is the *protection centrality* of i



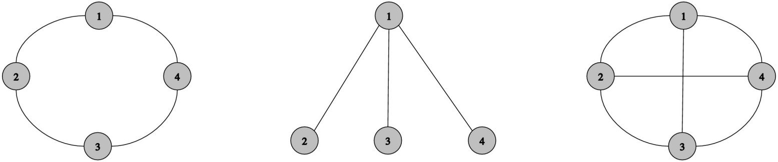
(a) $a_x = \frac{10}{7}$.



(b) $a_y = \frac{6}{7}$.

Gatekeeping centrality and separation

- Gatekeeping centrality, $s_i = \sum_j a_j^i$
- $b_{kt}^{(i,j)} = 1$ if (i,j) is separating pair for k, t, i.e. if neither i or j is a gatekeeper for k, t, but removing both i and j disconnects k and t
- Network separation of i and j, $b_i^j = \sum_{k,t}(a_{kt}^j a_{kt}^i - b_{kt}^{(i,j)})$



(a) $b_{24}^{(1,3)} = 1$ (b) $b_{24}^{(1,3)} = 0$ (c) $b_{24}^{(1,3)} = 0$

Social optimum

Theorem

$$\mathbf{q}^s = (B + \alpha I)^{-1}\mathbf{s} + o(\alpha^{-2})$$
$$= \frac{1}{\alpha}\left(I - \frac{B}{\alpha}\right)\mathbf{s} + o(\alpha^{-2})$$

where $B_{ij} = b_i^j$

Implies

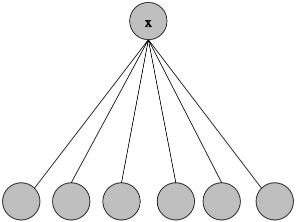
$$q_i^s \approx \frac{1}{\alpha}\left(s_i - \frac{1}{\alpha}\sum_{j \neq i} b_i^j s_j\right)$$

Optimum vs Equilibrium

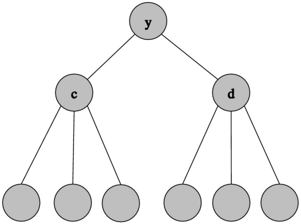
$$q_i^s - q_i^e \approx \frac{1}{\alpha} \sum_{j \neq i} a_j^i - \frac{1}{\alpha^2} \left(\sum_{j \neq i} b_i^j s_j - a_i^j \right)$$

- Equilibrium security investments are smaller than socially optimal security investments
- The node with the largest gatekeeping centrality increases its investment the most in the socially optimal solution compared to the equilibrium
- For all nodes with the same gatekeeping centrality, the gap between socially optimal investment and equilibrium is proportional to $a_i - \sum_j b_i^j s_j$

Optimum vs Equilibrium

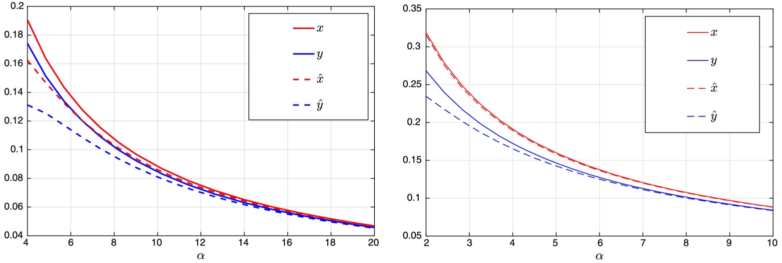


(a) Node x has the highest investment in both equilibrium and social optimum.



(b) Node y has the highest investment in equilibrium. Nodes c and d have highest investments in social optimum.

Approximation accuracy



[0]Equilibrium security investment levels as a function of α for a barbell network of size 20. The solid line plots the exact equilibrium investment. The dashed line in (a) plots the approximation from Eq. (12) and the dashed line in (b) plots the approximation from Theorem 3.

Endogenous Connections

- Allow agents to choose $\mathcal{E}_i \subseteq E$ connections to maintain
- (i, j) maintained iff $(i, j) \in \mathcal{E}_i \cap \mathcal{E}_j$
- Let $\hat{G} =$ network of maintained connections, $C_i(\hat{G}) =$ size of component connected to i

Proposition

Suppose $c(q) = \frac{\alpha}{2}q^2$ and α large. Then agents choose connections to maximize $|\mathcal{E}_i \cap \mathcal{E}_j|(1 - C_i(\hat{G})/n)$

Strategic Attacks

- Instead of Φ fixed, strategic attacker chooses Φ given $\mathbf{q}$
- Relevant for computer network or power transmission network security
- Maybe relevant for social networks and diseases if you are a pessimist
- New externality: investment by one agent shifts attack to others
- Possible to have more investment in equilibrium than in social optimum

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